Chapter One
Basic Principles :
"Vectors"

* Notation :-

If you have two Vectores $\vec{A} = \hat{i}A_x + \hat{j}A_y + \hat{k}A_z$

$\vec{B} = \hat{i}B_x + \hat{j}B_y + \hat{k}B_z$

then

1- Equality of Vectores

$$\vec{A} = \vec{B}$$

$$A_x = B_x$$

$$A_y = B_y$$

$$A_z = B_z$$

2- Vector Addition

$$\vec{A} + \vec{B} = [A_x + B_x \,,\, A_y + B_y \,,\, A_z + B_z]$$

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3- Multiplication by a scalar

if c is a scalar and A a vector,

$$\therefore c\vec{A} = c[A_x, A_y, A_z] = [cA_x, cA_y, cA_z] = Ac$$

4- Vector subtraction :-

subtraction is defined as follows :-

$$\vec{A} - \vec{B} = [A_x - B_x \,,\, A_y - B_y \,,\, A_z - B_z]$$

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5- The Commutative Law of Addition

$$\vec{A} + \vec{B} = \vec{B} + \vec{A}$$

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6- The Associative Law

$$A + (B + C) = (A + B) + C$$

7 - The Distributive Law

$$c(\vec{A}+\vec{B}) = c\vec{A} + c\vec{B}$$

* Magnitude of a Vector :-

The Magnitude of Vector A, denoted by $|\vec{A}|$

$$|\vec{A}| = (A_x^2 + A_y^2 + A_z^2)^{1/2}$$

* Unit Coordinate Vectors :-

A unit vector is one whose Magnitude is Unity. The three Unit Vectors

$$\hat{i} = [1, 0, 0]$$
$$\hat{j} = [0, 1, 0]$$
$$k = [0, 0, 1]$$

* The Scalar product :-

Given two Vectors $\vec{A}$ and $\vec{B}$, the scalar product or (dot) product, is the scalar defined by the equation :

$$\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}| \cos\theta$$
$$\vec{A} \cdot \vec{B} = AB \cos\theta$$

$$\therefore \quad \cos\theta = \frac{\vec{A} \cdot \vec{B}}{AB}$$

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$$\hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1$$
$$\hat{i} \cdot \hat{j} = \hat{j} \cdot \hat{k}, \ \hat{k} \cdot \hat{i} = \text{zero}$$

(1) $\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$

(2) $\vec{A} \cdot (\vec{B} + \vec{C}) = \vec{A} \cdot \vec{B} + \vec{A} \cdot \vec{C}$

(3) $\vec{A} \cdot \vec{A} = |\vec{A}|^2$

* If the dot product $\vec{A} \cdot \vec{B}$ is equal to zero, then $\vec{A}$ is perpendicular to $\vec{B}$, provided neither $\vec{A}$ nor B is null.

* if θ is the angle between $\vec{A}$ and $\vec{B}$ then the projection of $\vec{B}$ on $\vec{A}$ is just

$$\vec{A} \cdot \vec{B} = AB \cos\theta$$

$$\boxed{B \cos\theta = \frac{\vec{A} \cdot \vec{B}}{A}}$$

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and the projection of $\vec{A}$ on $\vec{B}$

$$\vec{A} \cdot \vec{B} = AB \cos\theta \Rightarrow \boxed{A \cos\theta = \frac{\vec{A} \cdot \vec{B}}{B}}$$

* The Vector product:

Given two Vectors, the Vector product or (Cross product) is given by:

$$\vec{A} \times \vec{B} = AB \sin\theta \, \hat{n}$$
$$\vec{A} \times \vec{B} = |\vec{A} \times \vec{B}| \, \hat{n}$$

where n: is aunit Vector normal to the plane of the two Vectors $\vec{A}$ and $\vec{B}$.

$$\therefore \hat{n} = \frac{\vec{A} \times \vec{B}}{AB \sin\theta}$$

and

$$\vec{A} \times \vec{B} = \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{bmatrix}$$

① $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$

② $\vec{A} \times (\vec{B} + \vec{C}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C}$

③ $m(\vec{A} \times \vec{B}) = (mA) \times B = A \times (mB)$

* $\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = \text{zero}$

$\hat{j} \times \hat{k} = \hat{i}$

$\hat{i} \times \hat{j} = \hat{k}$

$K \times \hat{i} = \hat{j}$

* $\hat{k} \times \hat{j} = -\hat{i}$

$\hat{j} \times \hat{i} = -\hat{k}$

$\hat{i} \times K = -\hat{j}$

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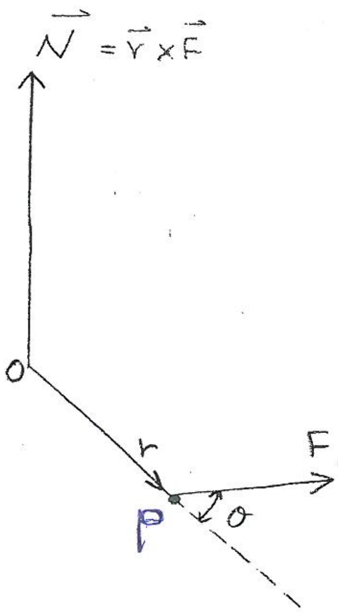
Torque of a Force ($\vec{N}$) :-



Let a Force $\vec{F}$ act at a point $P(x,y,z)$, as shown in Figure and :-

$$\vec{OP} = \vec{r} = \hat{i}x + \hat{j}y + \hat{k}z$$

the torque ($\vec{N}$) is defined as the cross product

$$\vec{N} = \vec{r} \times \vec{F}$$

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ex) Find a unit vector normal to the plane containing the two vectors $\vec{A} = 2\hat{i} + \hat{j} - \hat{k}$ and $\vec{B} = \hat{i} - \hat{j} + 2k$?

$$\vec{A} \times \vec{B} = AB \sin\theta \, \hat{n} = |\vec{A} \times \vec{B}| \, \hat{n}$$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ 1 & -1 & 2 \end{vmatrix} = \hat{i}(2-1) + \hat{j}(-1-4) + \hat{k}(-2-1)$$

$$= 1\hat{i} - 5\hat{j} - 3k$$

$$|\vec{A} \times \vec{B}| = \sqrt{1+25+9} = \sqrt{35}$$

$$\hat{n} = \frac{\vec{A} \times \vec{B}}{|\vec{A} \times \vec{B}|} = \frac{\hat{i} - 3\hat{j} - 3\hat{k}}{\sqrt{35}}$$

ex) Given the two vectors $\vec{A} = 2\hat{i} + \hat{j} - \hat{k}$
$\vec{B} = \hat{i} - \hat{j} + 2\hat{k}$, find

1. $\vec{A} \cdot \vec{B}$, $\vec{A} \times \vec{B}$
2. Find the angle between $\vec{A}$ and B.

1. $\vec{A} \cdot \vec{B} = (2\hat{i} + 1\hat{j} - 1\hat{k}) \cdot (1\hat{i} - 1\hat{j} + 2\hat{k}) = \cancel{2} - 1 - \cancel{2} = -1$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ 1 & -1 & 2 \end{vmatrix} = \hat{i}(2-1) + \hat{j}(-1-4) + \hat{k}(-2-1)$$

$$= \hat{i} - 5\hat{j} - 3\hat{k}$$

2. $$\cos\theta = \frac{\vec{A} \cdot \vec{B}}{AB}$$

$$A = |\vec{A}| = \sqrt{4+1+1} = \sqrt{6}$$
$$B = |\vec{B}| = \sqrt{1+1+4} = \sqrt{6}$$

$$\cos\theta = \frac{-1}{6} \Rightarrow \theta = \cos^{-1}\left(\frac{-1}{6}\right) = 99.6^\circ$$

Problems p.(19)

① Given two Vectors $\vec{A} = \hat{i} + \hat{j} + \hat{k}$

$\vec{B} = \hat{i} + 2\hat{j} - 3\hat{k}$ Find

ⓐ the Value of $|\vec{A} - \vec{B}|$

ⓑ the Value of $\vec{A} \cdot \vec{B}$

ⓒ the angle between $\vec{A}$ and $\vec{B}$

ⓐ $\vec{A} - \vec{B} = (\hat{i} + \hat{j} + \hat{k}) - (\hat{i} + 2\hat{j} - 3\hat{k})$

$= -\hat{j} + 4\hat{k}$

$|\vec{A} - \vec{B}| = \sqrt{(-1)^2 + (4)^2} = \sqrt{17}$

ⓑ $\vec{A} \cdot \vec{B} = (\hat{i} + \hat{j} + \hat{k}) \cdot (\hat{i} + 2\hat{j} - 3\hat{k})$

$= 1 + 2 - 3 = \text{zero}$ ∴ المتجهين متعامدين

ⓒ $\cos\theta = \dfrac{\vec{A} \cdot \vec{B}}{AB}$

$A = |\vec{A}| = \sqrt{1+1+1} = \sqrt{3}$

$B = |\vec{B}| = \sqrt{1+4+9} = \sqrt{15}$

$\because \vec{A} \cdot \vec{B} = \text{zero}$

$\therefore \cos\theta = \text{zero} \rightarrow \therefore$ This Vectors are perpendicular

(2) For the same vectors as in problem (1) express in $\hat{i}\,\hat{j}\,\hat{k}$ Form

(a) $2\vec{A}+3\vec{B} = [2(\hat{i}+\hat{j}+\hat{k}) + 3(\hat{i}+2\hat{j}-3\hat{k})]$

$= [(2\hat{i}+2\hat{j}+2\hat{k}) + (3\hat{i}+6\hat{j}-9\hat{k})]$

$= 5\hat{i}+8\hat{j}-7\hat{k}$

(b) $$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 1 & 2 & -3 \end{vmatrix}$$

$= \hat{i}(-3-2)+\hat{j}(1+3)+\hat{k}(2-1)$

$= -5\hat{i}+4\hat{j}+\hat{k}$

(c) $(\vec{A}+\vec{B})\times(\vec{A}-\vec{B})$

$\vec{A}+\vec{B} = (\hat{i}+\hat{j}+\hat{k})+(\hat{i}+2\hat{j}-3\hat{k})$

$= (2\hat{i}+3\hat{j}-2\hat{k})$

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$\vec{A}-\vec{B} = -\hat{j}+4\hat{k}$

$$(\vec{A}+\vec{B})\times(\vec{A}-\vec{B}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & -2 \\ 0 & -1 & 4 \end{vmatrix}$$

$= \hat{i}(12-2)+\hat{j}(0-8)+\hat{k}(-2-0)$

$= 10\hat{i}-8\hat{j}-2\hat{k}$

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(3) The Vector $\vec{A} = \hat{i} + \hat{j} + 2\hat{k}$ is perpendicular to the Vector $\vec{B} = 2\hat{i} - 3\hat{j} + q\hat{k}$.
what is the value of q.

$\because \vec{A}$ is perpendicular to $\vec{B}$

$\therefore \vec{A} \cdot \vec{B} = 0$

$$\vec{A} \cdot \vec{B} = (\hat{i} + \hat{j} + 2\hat{k}) \cdot (2\hat{i} - 3\hat{j} + q\hat{k})$$

$$2 - 3 + 2q = 0$$

$$-1 + 2q = 0$$

$$2q = 1 \Rightarrow q = \frac{1}{2}$$

(4) Find the length of projection of the Vector $\vec{A} = \hat{i} + \hat{j} + \hat{k}$ on Vector $\vec{B} = 2\hat{i} - 3\hat{j} + 4\hat{k}$

$$\vec{A} \cdot \vec{B} = AB\cos\theta$$

$$A\cos\theta = \frac{\vec{A} \cdot \vec{B}}{B}$$

$$\vec{A} \cdot \vec{B} = (\hat{i} + \hat{j} + \hat{k}) \cdot (2\hat{i} - 3\hat{j} + 4\hat{k})$$

$$= 2 - 3 + 4 = 3$$

$$B = |\vec{B}| = \sqrt{(2)^2 + (-3)^2 + (4)^2} = \sqrt{4 + 9 + 16} = \sqrt{29}$$

$$\therefore A\cos\theta = \frac{3}{\sqrt{29}}$$

5. Find the Value of

$$[(\hat{i}+\hat{j})\times(\hat{i}+\hat{k})]\cdot[(\hat{i}-\hat{j})\times(\hat{j}-\hat{k})]$$

$$(\hat{i}+\hat{j})\times(\hat{i}+\hat{k}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{vmatrix}$$

$$= \hat{i}(1-0)+\hat{j}(0-1)+\hat{k}(0-1)$$

$$= \hat{i}-\hat{j}-\hat{k}$$

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$$(\hat{i}-\hat{j})\times(\hat{j}-\hat{k}) = \begin{vmatrix} \hat{i} & \hat{j} & k \\ 1 & -1 & 0 \\ 0 & 1 & -1 \end{vmatrix}$$

$$= \hat{i}(1-0)+\hat{j}(0+1)+\hat{k}(1-0)$$

$$= \hat{i}+\hat{j}+\hat{k}$$

$$\therefore [(\hat{i}+\hat{j})\times(\hat{i}+\hat{k})]\cdot[(\hat{i}-\hat{j})\times(\hat{j}-\hat{k})] = (\hat{i}-\hat{j}-\hat{k})\cdot(\hat{i}+\hat{j}+\hat{k})$$

$$= 1-1-1$$

$$= -1$$

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(6) Given $\vec{A} = 2\hat{i} - \hat{j}$

$\vec{B} = 2\hat{j} + 3\hat{k}$

$\vec{C} = \hat{i} + \hat{j} + \hat{k}$ Find

(a) $\vec{A} \cdot (\vec{B} \times \vec{C})$ and $(\vec{A} \times \vec{B}) \cdot C$

(b) $\vec{A} \times (\vec{B} \times \vec{C})$ and $(\vec{A} \times \vec{B}) \times \vec{C}$

(a) $$\vec{B} \times \vec{C} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 2 & 3 \\ 1 & 1 & 1 \end{vmatrix} = \hat{i}(2-3) + \hat{j}(3-0) + \hat{k}(0-2) = -\hat{i} + 3\hat{j} - 2\hat{k}$$

$$\therefore \vec{A} \cdot (\vec{B} \times \vec{C}) = (2\hat{i} - \hat{j}) \cdot (-\hat{i} + 3\hat{j} - 2\hat{k})$$
$$= -2 - 3 = -5$$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 0 \\ 0 & 2 & 3 \end{vmatrix} = \hat{i}(-3-0) + \hat{j}(0-6) + \hat{k}(4-0) = -3\hat{i} - 6\hat{j} + 4\hat{k}$$

$$(\vec{A} \times \vec{B}) \cdot \vec{C} = (-3\hat{i} - 6\hat{j} + 4\hat{k}) \cdot (\hat{i} + \hat{j} + \hat{k})$$
$$= -3 - 6 + 4 = -5$$

Not $\Rightarrow \vec{A} \cdot (\vec{B} \times \vec{C}) = (\vec{A} \times \vec{B}) \cdot C$

(b) $\vec{B} \times \vec{C} = -\hat{i} + 3\hat{j} - 2\hat{k}$

$$\bar{A} \times (\vec{B} \times \vec{C}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 0 \\ -1 & 3 & -2 \end{vmatrix}$$

$$= \hat{i}(2-0) + \hat{j}(0+4) + \hat{k}(6-1)$$
$$= 2\hat{i} - \hat{j} + 5\hat{k}$$

(7) A force $\vec{F_1} = \hat{i} + \hat{j}$ is applied to a body at point P_1 where the vector $OP_1 = r_1 = 2\hat{i} + \hat{j}$. A second force $F_2 = \hat{j} - \hat{k}$ is applied at the point $r_2 = \hat{i} + \hat{j} + \hat{k}$. Find the total moment $(\vec{N})$, the magnitude of $\vec{N}$ and the direction cosines of the resultant axis of rotation?

$$* \quad \vec{N_1} = \vec{r_1} \times \vec{F_1} = \begin{vmatrix} \hat{i} & \hat{j} & k \\ 2 & 1 & 0 \\ 1 & 1 & 0 \end{vmatrix} = \hat{i}(0) + \hat{j}(0) + \hat{k}(2-1) = \hat{k}$$

$$* \quad \vec{N_2} = \vec{r_2} \times \vec{F_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 0 & 1 & -1 \end{vmatrix} = \hat{i}(-1-1) + \hat{j}(0+1) + \hat{k}(1-0)$$

$$= -2\hat{i} + \hat{j} + \hat{k}$$

$$* \quad \vec{N}_{total} = \vec{N_1} + \vec{N_2} = -2\hat{i} + \hat{j} + 2\hat{k}$$

* Magnitude of $\vec{N}$

$$N = |\vec{N}| = \sqrt{(-2)^2 + (1)^2 + (2)^2} = \sqrt{9} = 3$$

* the direction cosine

$$n = \frac{\vec{N}}{N}$$

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$$= \frac{-2\hat{i} + \hat{j} + 2\hat{k}}{3} = \frac{-2}{3}\hat{i} + \frac{1}{3}\hat{j} + \frac{2}{3}\hat{k}$$

$$= \hat{i}\cos\alpha + \hat{j}\cos\beta + \hat{k}\cos\gamma$$

$$\cos\alpha = \frac{-2}{3}, \quad \cos\beta = \frac{1}{3}, \quad \cos\gamma = \frac{2}{3}$$

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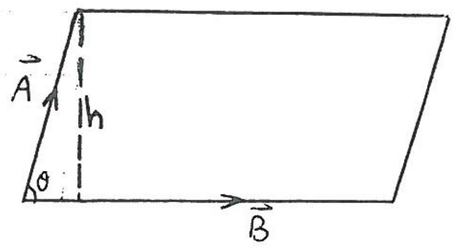
(9) Two vectors $\vec{A}$ and $\vec{B}$ represent concurrent sides of a متوازي الاضلاع parallelogram. Prove that the area of parallelogram is $|\vec{A} \times \vec{B}|$.

$$\sin\theta = \frac{h}{A}$$

$$h = A\sin\theta \quad \text{—} \ (1)$$



$$\vec{A} \times \vec{B} = AB\sin\theta\,\hat{n}$$

$$\vec{A} \times \vec{B} = B(A\sin\theta)\,\hat{n}$$

$$\vec{A} \times \vec{B} = Bh\,\hat{n}$$

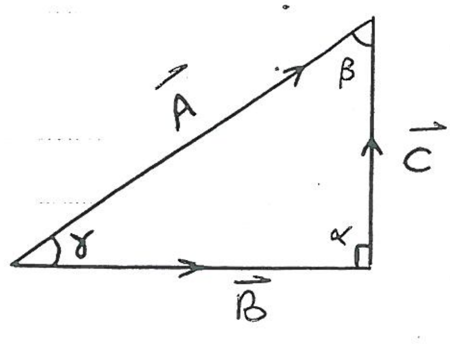
$$|\vec{A} \times \vec{B}| = Bh = \text{the area of parallelogram}$$

مثلث

(11) Prove the trigonometric law of sines using vector methods?

$$\vec{A} = \vec{B} + \vec{C}$$

$$\vec{A} \times \vec{B} = \cancel{\vec{B} \times \vec{B}} + \vec{C} \times \vec{B}$$

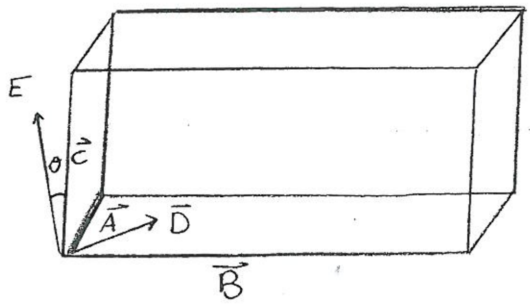


$$A\cancel{B}\sin\gamma\,\hat{n} = C\cancel{B}\sin\alpha\,\hat{n}$$

$$\frac{A}{\sin\alpha} = \frac{C}{\sin\gamma}$$

(12) Three vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ represent concurrent sides of a parallelepiped (متوازي المستطيلات). Show that the volume of the parallelepiped is

$$|\vec{A} \cdot (\vec{B} \times \vec{C})|.$$



$$\vec{A} \times \vec{B} = \vec{D}$$

$$\cos\theta = \frac{E}{C} \Rightarrow E = C\cos\theta$$

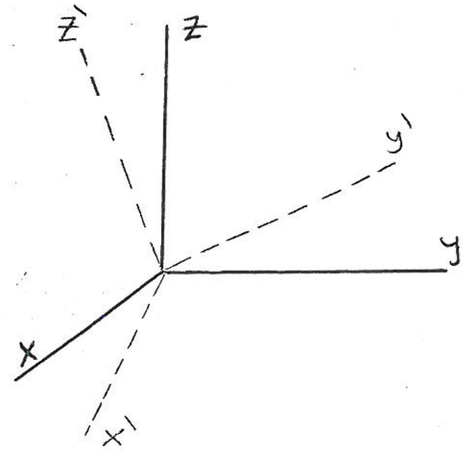
$$\text{Volume} = D\,C\cos\theta$$
$$= \vec{D} \cdot \vec{C}$$

$$= (\vec{A} \times B) \cdot \vec{C}$$

* The Change of Coordinate System

Consider the Vector $\vec{A}$ expressed relative to the $(\hat{i}\,\hat{j}\,\hat{k})$

$$\vec{A} = \hat{i}A_x + \hat{j}A_y + \hat{k}A_z$$



$$\vec{A}\cdot\hat{i}' = A_x' = (\hat{i}\cdot\hat{i}')A_x + (\hat{j}\cdot\hat{i}')A_y + (\hat{k}\cdot\hat{i}')A_z \quad \text{—} (1)$$

$$\vec{A}\cdot\hat{j}' = A_y' = (\hat{i}\cdot\hat{j}')A_x + (\hat{j}\cdot\hat{j}')A_y + (\hat{k}\cdot\hat{j}')A_z \quad \text{—} (2)$$

$$\vec{A}\cdot\hat{k}' = A_z' = (\hat{i}\cdot\hat{k}')A_x + (\hat{j}\cdot\hat{k}')A_y + (\hat{k}\cdot\hat{k}')A_z \quad \text{—} (3)$$

These equations are conveniently expressed in matrix

$$\begin{bmatrix} A_x' \\ A_y' \\ A_z' \end{bmatrix} = \begin{bmatrix} (\hat{i}\cdot\hat{i}') & (\hat{j}\cdot\hat{i}') & (\hat{k}\cdot\hat{i}') \\ (\hat{i}\cdot\hat{j}') & (\hat{j}\cdot\hat{j}') & (\hat{k}\cdot\hat{j}') \\ (\hat{i}\cdot\hat{k}') & (\hat{j}\cdot\hat{k}') & (\hat{k}\cdot\hat{k}') \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

Coefficients of transformation

$$x' = R\ x$$

Relative to a new $(\hat{i}' \; \hat{j}' \; \hat{k}')$ having a different orientation from that of $\hat{i}\,\hat{j}\,\hat{k}$, the same vector $\vec{A}$ is expressed as:.

$$\vec{A} = \hat{i}' A_x' + \hat{j}' A_y' + \hat{k}' A_z'$$

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$$\vec{A} \cdot \hat{i} = A_x$$

$$A_x = \vec{A} \cdot \hat{i} = (\hat{i}' \cdot \hat{i}) A_x' + (\hat{j}' \cdot \hat{i}) A_y' + (\hat{k}' \cdot \hat{i}) A_z' \quad \text{——} (1)$$

$$A_y = \vec{A} \cdot \hat{j} = (\hat{i}' \cdot \hat{j}) A_x' + (\hat{j}' \cdot \hat{j}) A_y' + (\hat{k}' \cdot \hat{j}) A_z' \quad \text{——} (2)$$

$$A_z = \vec{A} \cdot \hat{k} = (\hat{i}' \cdot \hat{k}) A_x' + (\hat{j}' \cdot \hat{k}) A_y' + (\hat{k}' \cdot \hat{k}) A_z' \quad \text{——} (3)$$

These equations are conveniently expressed in Matrix

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} (\hat{i}' \cdot \hat{i}) & (\hat{j}' \cdot \hat{i}) & (\hat{k}' \cdot \hat{i}) \\ (\hat{i}' \cdot \hat{j}) & (\hat{j}' \cdot \hat{j}) & (\hat{k}' \cdot \hat{j}) \\ (\hat{i}' \cdot \hat{k}) & (\hat{j}' \cdot \hat{k}) & (\hat{k}' \cdot \hat{k}) \end{bmatrix} \begin{bmatrix} A_x' \\ A_y' \\ A_z' \end{bmatrix}$$

نجد R^T . يجعل الصف عمود في مصفوفة التحويل R

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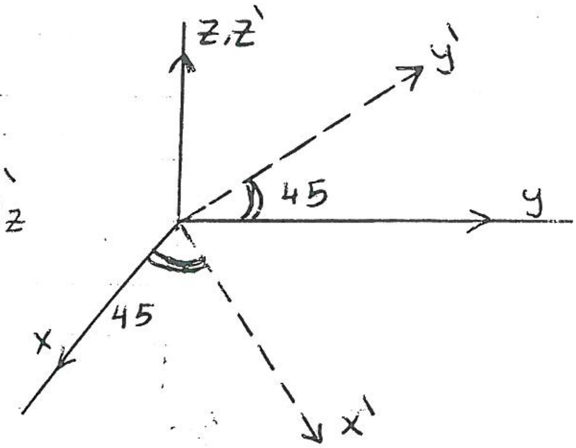
ex)

Express the Vector $\vec{A} = 3\hat{i} + 2\hat{j} + 1\hat{k}$ in terms of the triad $\hat{i}', \hat{j}', \hat{k}'$ where the $x'y'$ axes are rotated (45°) around the (z) axis, the Z and Z' axes Coinciding ?



$$\vec{A} = \hat{i}' A_x' + \hat{j}' A_y' + \hat{k}' A_z'$$

$$\begin{bmatrix} A_x' \\ A_y' \\ A_z' \end{bmatrix} = \begin{bmatrix} (\hat{i}\cdot\hat{i}') & (\hat{j}\cdot\hat{i}') & (\hat{k}\cdot\hat{i}') \\ (\hat{i}\cdot\hat{j}') & (\hat{j}\cdot\hat{j}') & (\hat{k}\cdot\hat{j}') \\ (\hat{i}\cdot\hat{k}') & (\hat{j}\cdot\hat{k}') & (\hat{k}\cdot\hat{k}') \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

$$= \begin{bmatrix} \cos 45 & \cos 45 & \cos 90 \\ \cos(90+45) & \cos 45 & \cos 90 \\ \cos 90 & \cos 90 & \cos 0 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$$

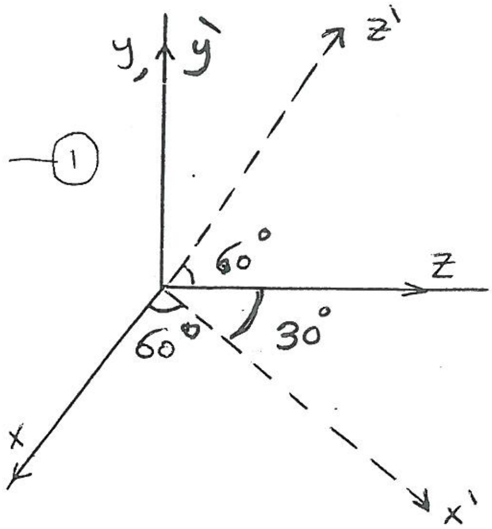
Not $\Rightarrow \cos(90+45) = -\sin 45$

$$= \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{3}{\sqrt{2}} + \frac{2}{\sqrt{2}} + 0 \\ -\frac{3}{\sqrt{2}} + \frac{2}{\sqrt{2}} + 0 \\ 0 + 0 + 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{5}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} \\ 1 \end{bmatrix}, \quad \vec{A} = \frac{5}{\sqrt{2}}\hat{i}' - \frac{1}{\sqrt{2}}\hat{j}' + 1\hat{k}'$$

(13) Express the Vector $\hat{i}+\hat{j}$ interms of the triad $\hat{i}'\hat{j}'\hat{k}'$ where the $x'z'$ axes are rotated about the (y) axis (which coincides with the (y') axis through an angle of (60°)

$$\vec{A} = \hat{i}'A_x' + \hat{j}'A_y' + \hat{k}'A_z' \quad \text{—(1)}$$



$$\begin{bmatrix} A_x' \\ A_y' \\ A_z' \end{bmatrix} = \begin{bmatrix} (\hat{i},\hat{i}') & (\hat{j},\hat{i}') & (\hat{k},\hat{i}') \\ (\hat{i},\hat{j}') & (\hat{j},\hat{j}') & (\hat{k},\hat{j}') \\ (\hat{i},\hat{k}') & (\hat{j},\hat{k}') & (\hat{k},\hat{k}') \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

$$= \begin{bmatrix} \cos 60 & \cos 90 & \cos 30 \\ \cos 90 & \cos 0 & \cos 90 \\ \underset{-\sin 60}{\underbrace{\cos(90+60)}} & \cos 90 & \cos 60 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{2} & 0 & \frac{\sqrt{3}}{2} \\ 0 & 1 & 0 \\ -\frac{\sqrt{3}}{2} & 0 & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{2}+0+0 \\ 0+1+0 \\ \frac{-\sqrt{3}}{2}+0+0 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ 1 \\ \frac{-\sqrt{3}}{2} \end{bmatrix}$$

$$\therefore \vec{A} = \frac{1}{2}\hat{i}' + 1\hat{j}' - \frac{\sqrt{3}}{2}\hat{k}' \Rightarrow |\vec{A}| = \sqrt{\left(\frac{1}{2}\right)^2 + 1 + \left(\frac{-\sqrt{3}}{2}\right)^2}$$

$$= \sqrt{\frac{1}{4} + 1 + \frac{3}{4}} = \sqrt{2}$$

$$(\hat{i},\hat{j},\hat{k}) \Rightarrow |\vec{A}| = \sqrt{(1)^2 + (1)^2} = \sqrt{2}$$

∴ مقدار المتجه لا يتغير سواء في إحداثيات $\hat{i}\,\hat{j}\,\hat{k}$ أو $\hat{i}'\,\hat{j}'\,\hat{k}'$

* Derivative & Integration of a Vectors and Kinematics of a particle

if you have $\vec{A} = \hat{i} A_x + \hat{j} A_y + \hat{k} A_z$

1. $$\frac{d\vec{A}}{dt} = \hat{i}\frac{dA_x}{dt} + \hat{j}\frac{dA_y}{dt} + \hat{k}\frac{dA_z}{dt}$$

2. $$\frac{d}{dt}(\vec{A} + \vec{B}) = \frac{d\vec{A}}{dt} + \frac{d\vec{B}}{dt}$$

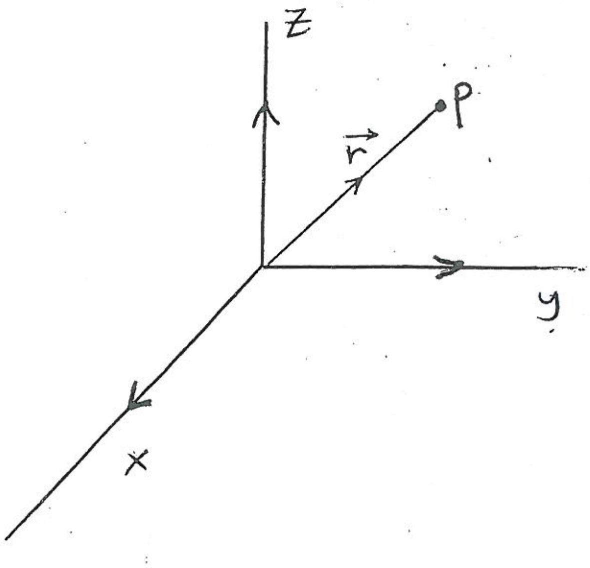
3. $$\frac{d}{dt}(n\vec{A}) = \frac{dn}{dt}\vec{A} + n\frac{d\vec{A}}{dt}$$

4. $$\frac{d}{dt}(\vec{A}\cdot\vec{B}) = \frac{d\vec{A}}{dt}\cdot\vec{B} + \vec{A}\cdot\frac{d\vec{B}}{dt}$$

5. $$\frac{d}{dt}(\vec{A}\times\vec{B}) = \frac{d\vec{A}}{dt}\times\vec{B} + \vec{A}\times\frac{d\vec{B}}{dt}$$

* Position Vector of a particle

$$\vec{r} = \hat{i}x + \hat{j}y + \hat{k}z$$



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* The Velocity Vector :.

$$\vec{V} = \frac{d\vec{r}}{dt} = \frac{d}{dt}(\hat{i}x + \hat{j}y + \hat{k}z)$$

$$\vec{V} = \hat{i}\frac{dx}{dt} + \hat{j}\frac{dy}{dt} + \hat{k}\frac{dz}{dt}$$

$$\vec{V} = \hat{i}\dot{x} + \hat{j}\dot{y} + k\dot{z}$$

$$\dot{x} = \frac{dx}{dt}, \quad \dot{y} = \frac{dy}{dt}, \quad \dot{z} = \frac{dz}{dt}$$

the magnitude of the Velocity is called "Speed"

$$V = |\vec{V}| = \sqrt{\dot{x}^2 + \dot{y}^2 + \dot{z}^2}$$

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* Acceleration Vector :.

The time derivative of the velocity is called the acceleration

$$\vec{a} = \frac{d\vec{V}}{dt} = \frac{d}{dt}(\hat{i}\dot{x} + \hat{j}\dot{y} + \hat{k}\dot{z})$$

$$\vec{a} = \hat{i}\frac{d\dot{x}}{dt} + \hat{j}\frac{d\dot{y}}{dt} + \hat{k}\frac{d\dot{z}}{dt}$$

$$\vec{a} = \hat{i}\ddot{x} + \hat{j}\ddot{y} + k\ddot{z}$$

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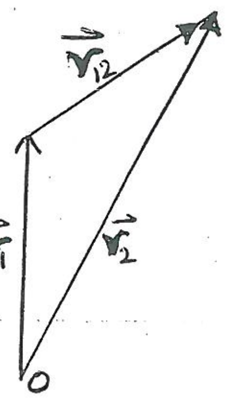
* Vector Integration :

the Integration of $\vec{a}$ is $\Rightarrow$ Velocity $(\vec{V})$

The Integration of $\vec{V}$ is $\Rightarrow$ position Vector $(\vec{r})$

* Relative Velocity :

If you have two particles



r_1 : position Vector for first particle

r_2 : position Vector for second particle

$$\vec{r_1} + \vec{r_{12}} = \vec{r_2}$$

$$\vec{r_{12}} = \vec{r_2} - \vec{r_1}$$

$$\vec{V_{12}} = \frac{d\vec{r_{12}}}{dt} = \frac{d\vec{r_2}}{dt} - \frac{d\vec{r_1}}{dt}$$

$$\boxed{\vec{V_{12}} = \vec{V_2} - \vec{V_1}}$$

$$\boxed{V_2 = V_{12} + V_1}$$ السرعة الحقيقية للجسم الثاني

سرعة الجسم المقاسة بالنسبة لشيء ثابت هي سرعة حقيقية
" " " لشيء متحرك هي سرعة نسبية

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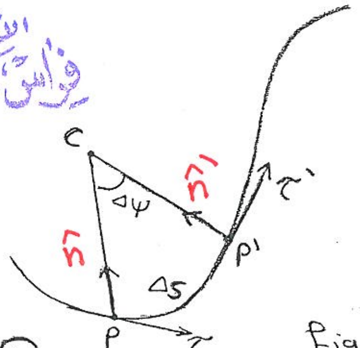
* Tangential and Normal Components of Acceleration

The Velocity Vector of a moving particle $(\vec{V})$ can be written as the product of the particle's Speed (v) and Unit Vector (τ) that gives the direction of the particle's motion. Thus

$$\vec{V} = v\hat{\tau}$$



$$\vec{a} = \frac{d\vec{V}}{dt} = \frac{d}{dt}(v\hat{\tau})$$

$$\vec{a} = \dot{v}\hat{\tau} + v\left(\frac{d\hat{\tau}}{dt}\right) \quad \text{----} (1)$$

Fig (1).

the direction of $\Delta\tau$ become perpendicular to the direction of (τ) in the limit as $\Delta\psi$ and Δs approach zero.

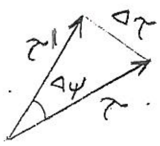


Fig (2)

and $\Delta\tau$ approch from $\Delta\psi$, then

$$\left|\frac{d\tau}{d\psi}\right| = 1 \quad , \quad \frac{d\hat{\tau}}{d\psi} = \hat{n}$$

$\hat{n} \Rightarrow$ unit normal vector

to Find $\left(\frac{d\hat{\tau}}{dt}\right)$

$$\frac{d\hat{\tau}}{dt} = \frac{d\hat{\tau}}{d\psi} \cdot \frac{d\psi}{dt}$$

$$* \frac{d\psi}{d\psi}$$

$$\frac{d\hat{\tau}}{dt} = \hat{n}\frac{d\psi}{dt}$$

$$* \frac{ds}{ds}$$

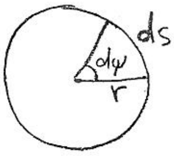


Fig (3)

الزاوية × نق = طول القوس

$$ds = r\,d\psi$$

$$\frac{d\psi}{ds} = \frac{1}{r}$$

$$\frac{d\hat{\tau}}{dt} = \hat{n}\,\frac{d\psi}{ds}\cdot\frac{ds}{dt}$$

$$\frac{d\hat{\tau}}{dt} = \hat{n}\left(\frac{1}{r}\right)v = \frac{v}{r}\,\hat{n}$$

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the above value for $\frac{d\hat{\tau}}{dt}$ is now inserted into eq (1) to yield the final result

$$\vec{a} = \underbrace{\dot{v}\,\tau}_{\text{Tangential Component}} + \underbrace{\left(\frac{v^2}{r}\right)\hat{n}}_{\text{Normal Component}}$$

$$a_n = \frac{v^2}{r}$$

$$a_\tau = \dot{v} = \ddot{r}$$

Normal Component is always directed toward the Center of Curvature on the Concave side of the path of motion.

$$|\vec{a}| = a = \left|\frac{d\vec{v}}{dt}\right| = \left(\dot{v}^2 + \frac{v^4}{r^2}\right)^{1/2}$$

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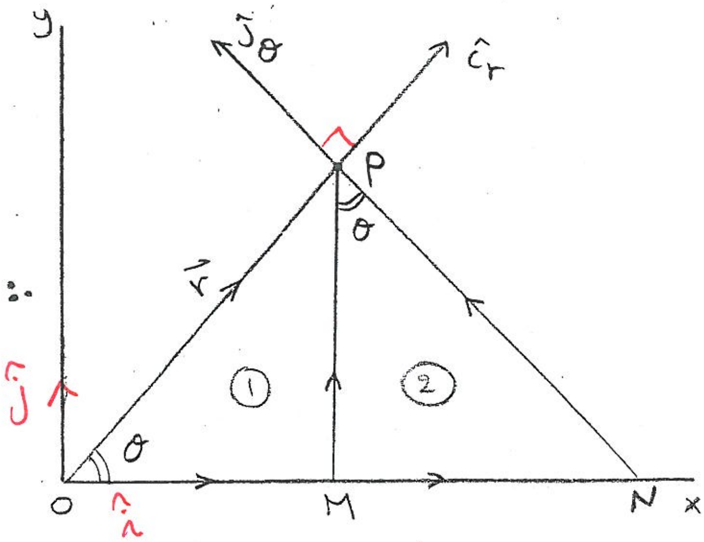
* Velocity and Acceleration in Plane polar Coordinates

Vectorially, the position of the particle can be written as the product of the radial distance (r) by a unit radial vector ($\hat{i}_r$):.



$$\vec{r} = r\,\hat{i}_r$$

$$\vec{V} = \frac{d\vec{r}}{dt} = \frac{d}{dt}(r\,\hat{i}_r)$$

$$\vec{V} = \dot{r}\,\hat{i}_r + r\frac{d\hat{i}_r}{dt} \text{ ------ } (a)$$

$$\vec{OM} = OM\,\hat{i} = (op\cos\theta)\,\hat{i} = r\cos\theta\,\hat{i}$$
$$\vec{Mp} = Mp\,\hat{j} = (op\sin\theta)\,\hat{j} = r\sin\theta\,\hat{j}$$

to Find $\frac{d\hat{i}_r}{dt} \Rightarrow$ in Δ (opM)

$$\vec{op} = \vec{oM} + \vec{Mp}$$
$$\cancel{r}\,\hat{i}_r = \cancel{r}\cos\theta\,\hat{i} + \cancel{r}\sin\theta\,\hat{j}$$
$$\therefore \hat{i}_r = \hat{i}\cos\theta + \hat{j}\sin\theta \text{ ---- } (1)$$

in Δ(MNP)

$$\vec{MP} = \vec{MN} + \vec{NP}$$
$$(Np\cos\theta)\,\hat{j} = (Np\sin\theta)\,\hat{i} + Np\,\hat{j}_\theta$$
$$\hat{j}_\theta = -\hat{i}\sin\theta + \hat{j}\cos\theta \text{ ------ } (2)$$

From eq (1)

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$$\frac{d\hat{i}_r}{dt} = -\hat{i}\sin\theta\, d\theta + \hat{j}\cos\theta\, d\theta$$
$$= \dot{\theta}\,(-\hat{i}\sin\theta + \hat{j}\cos\theta)$$
$$= \dot{\theta}\,\hat{j}_\theta$$

equation (a)

$$\frac{d\hat{i}_r}{dt} = \dot{\theta}\,\hat{j}_\theta$$

$$\frac{d\hat{j}_\theta}{dt} = -\hat{i}_r\dot{\theta}$$

$$\vec{V} = \dot{r}\,\hat{i}_r + r\frac{d\hat{i}_r}{dt}$$

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$$\therefore \vec{V} = \dot{r}\,\hat{i}_r + r\dot{\theta}\,\hat{j}_\theta$$

* Acceleration in plan polar Coordinates

$$\vec{a} = \frac{d\vec{V}}{dt} = \frac{d}{dt}(\dot{r}\,\hat{i}_r + r\dot{\theta}\,\hat{j}_\theta)$$

$$\vec{a} = \ddot{r}\,\hat{i}_r + \dot{r}\frac{d\hat{i}_r}{dt} + \left(\dot{r}\dot{\theta}\,\hat{j}_\theta + \ddot{\theta}\,r\,\hat{j}_\theta + r\dot{\theta}\frac{d\hat{j}_\theta}{dt}\right)$$

eq(2)

$$\hat{j}_\theta = -\hat{i}\sin\theta + \hat{j}\cos\theta$$

$$\frac{d\hat{j}_\theta}{dt} = -\hat{i}\,\dot{\theta}\cos\theta - \hat{j}\,\dot{\theta}\sin\theta = -\dot{\theta}\,\underbrace{(\hat{i}\cos\theta + \hat{j}\sin\theta)}_{=\hat{i}_r}$$

$$\frac{d\hat{j}_\theta}{dt} = -\dot{\theta}\,\hat{i}_r$$

$$\vec{a} = \ddot{r}\,\hat{i}_r + \dot{r}\dot{\theta}\,\hat{j}_\theta + \dot{r}\dot{\theta}\,\hat{j}_\theta + \ddot{\theta}\,r\,\hat{j}_\theta - r\dot{\theta}^2\,\hat{i}_r$$

$$\vec{a} = (\ddot{r} - r\dot{\theta}^2)\,\hat{i}_r + (2\dot{r}\dot{\theta} + \ddot{\theta}r)\,\hat{j}_\theta$$

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Note:-

1. If $r = \text{Constant}$ (circular motion)
   $\theta = \text{Vary}$

then:

$$\vec{V} = r\dot{\theta}\,\hat{j}_\theta$$

$$\vec{a} = -r\dot{\theta}^2\,\hat{i}_r + \ddot{\theta}\,r\,\hat{j}_\theta$$

2. If $r = \text{Vary}$
   $\theta = \text{Constant}$ (general motion) حركه عامه

$$\vec{V} = \dot{r}\,\hat{i}_r$$

$$\vec{a} = \ddot{r}\,\hat{i}_r$$

* Velocity and Acceleration in Cylindrical Coordinates:

- The cylindrical coordinates R, ϕ, Z

- The position vector is written as:

$$\vec{r} = R\hat{i}_R + Z\hat{k}$$

$$\vec{V} = \dot{R}\hat{i}_R + R\underbrace{\left(\frac{d\hat{i}_R}{dt}\right)}_{\dot{\phi} J_\phi} + \dot{Z} K$$

$$\boxed{\vec{V} = \dot{R}\hat{i}_R + R\dot{\phi} J_\phi + \dot{z}\hat{k}}$$

حفظ

$$\vec{a} = \frac{d\vec{V}}{dt} = \frac{d}{dt}\left(\dot{R}\hat{i}_R + R\dot{\phi} J_\phi + \dot{z} k\right)$$

$$\vec{a} = \ddot{R}\hat{i}_R + \dot{R}\underbrace{\left(\frac{d\hat{i}_R}{dt}\right)}_{\dot{\phi} J_\phi} + \dot{R}\dot{\phi} J_\phi + R\ddot{\phi} J_\phi + R\dot{\phi}\underbrace{\left(\frac{dJ_\phi}{dt}\right)}_{-\dot{\phi}\hat{i}_R} + \ddot{z}\hat{k}$$

$$\vec{a} = \ddot{R}\hat{i}_R + \dot{R}\dot{\phi} J_\phi + \dot{R}\dot{\phi} J_\phi + R\ddot{\phi} J_\phi - R\dot{\phi}^2\hat{i}_R + \ddot{z}\hat{k}$$

$$\boxed{\vec{a} = (\ddot{R} - R\dot{\phi}^2)\hat{i}_R + (2\dot{R}\dot{\phi} + R\ddot{\phi}) J_\phi + \ddot{z}\hat{k}}$$

* Spherical Coordinates (r, θ, ϕ)

The position vector is

مهم $\vec{r} = r\,\hat{i}_r$ $\qquad \vec{V} = \dfrac{d\vec{r}}{dt} = \dot{r}\,\hat{i}_r + r\dfrac{d\hat{i}_r}{dt}$

مهم $\vec{V} = \dot{r}\,\hat{i}_r + r\dot{\phi}\sin\theta\,\hat{j}_\phi + r\dot{\theta}\,\hat{k}_\theta$

HoWo

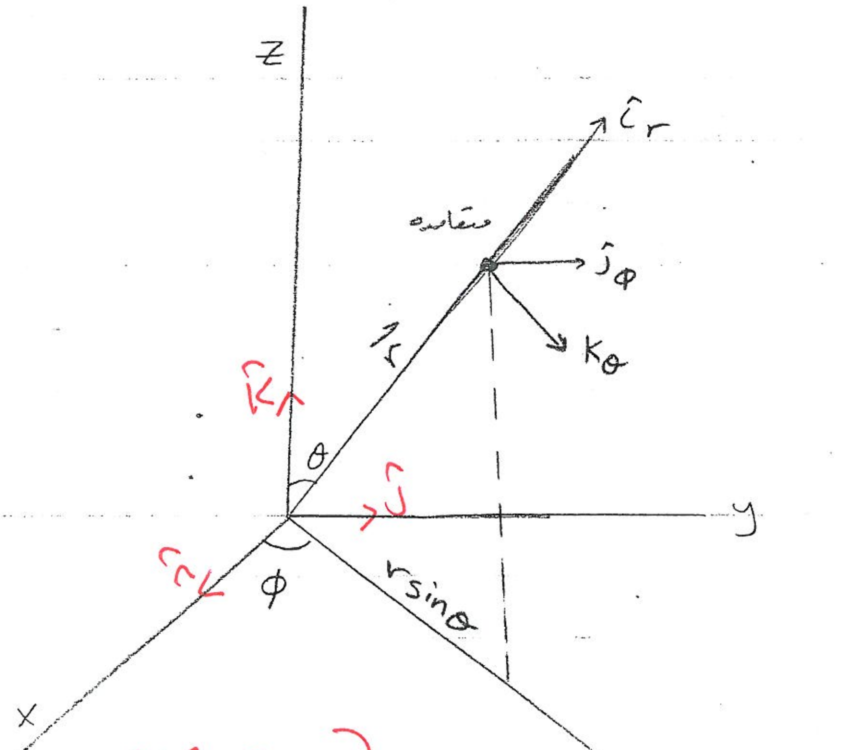
مهم $\vec{a} = (\ddot{r} - r\dot{\phi}^2\sin^2\theta - r\dot{\theta}^2)\,\hat{i}_r + (r\ddot{\phi}\sin\theta + 2\dot{r}\dot{\phi}\sin\theta +$

$2r\dot{\theta}\dot{\phi}\cos\theta)\,\hat{j}_\phi + (r\ddot{\theta} + 2\dot{r}\dot{\theta} - r\dot{\phi}^2\sin\theta\cos\theta)\,k_\theta$

$\hat{i}_r = \hat{i}_R\sin\theta + \hat{k}\cos\theta$

$\hat{j}_\phi = \hat{j}_\phi$

$\hat{k}_\theta = \hat{i}_R\cos\theta - \hat{k}\sin\theta$



$\hat{i}_r = \hat{i}\sin\theta\cos\phi + \hat{j}\sin\theta\sin\phi + k\cos\theta$

$\hat{j}_\phi = -\hat{i}\sin\phi + \hat{j}\cos\phi$

$\hat{k}_\theta = \hat{i}\cos\theta\cos\phi + \hat{j}\cos\theta\sin\phi - \hat{k}\sin\theta$

$\vec{V} = \hat{i}_r\dot{r} + \hat{j}_\phi r\dot{\phi}\sin\theta + \hat{k}_\theta r\dot{\theta}$

$\vec{a} = \hat{i}_r\ddot{r} + \dot{r}\dfrac{d\hat{i}_r}{dt} + \hat{j}_\phi\dfrac{d(r\dot{\phi}\sin\theta)}{dt} + r\dot{\phi}\sin\theta\dfrac{d\hat{j}_\phi}{dt} + \hat{k}_\theta\dfrac{d(r\dot{\theta})}{dt} + r\dot{\theta}\dfrac{d\hat{k}_\theta}{dt}$

eq.1) The position Vector of a moving particle is given by the following expression. Find the velocity, speed, and acceleration as a function of time in each case.

(1) $\vec{r} = \hat{i}\, ct - \hat{j}\, \frac{g}{2} t^2$

(2) $\vec{r} = \hat{i}\, ct + \hat{j}\, b\cos\omega t + \hat{k}\, b\sin\omega t$

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{d}{dt}\left(\hat{i}ct - \hat{j}\,\frac{gt^2}{2}\right)$$

$$= \hat{i}c - \hat{j}\,gt$$

$$v = |\vec{v}| = \sqrt{c^2 + g^2t^2}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = -g\hat{j}$$

(2) $$\vec{v} = \frac{d\vec{r}}{dt} = \frac{d}{dt}\left(\hat{i}ct + \hat{j}\,b\cos\omega t + \hat{k}\,b\sin\omega t\right)$$

$$\vec{v} = c\hat{i} - b\omega\sin\omega t\,\hat{j} + b\omega\cos\omega t\,\hat{k}$$

$$v = |\vec{v}| = \left(c^2 + b^2\omega^2\sin^2\omega t + b^2\omega^2\cos^2\omega t\right)^{1/2}$$
$$= \left(c^2 + b^2\omega^2(\sin^2\omega t + \cos^2\omega t)\right)^{1/2} = \left(c^2 + b^2\omega^2\right)^{1/2}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left(c\hat{i} - b\omega\sin\omega t\,\hat{j} + b\omega\cos\omega t\,\hat{k}\right)$$

$$= -b\omega^2\cos\omega t\,\hat{j} - b\omega^2\sin\omega t\,\hat{k}$$
$$= -b\omega^2\left(\cos\omega t\,\hat{j} + \sin\omega t\,\hat{k}\right)$$

eq-2) The motion of a particle is given by

$$r = \hat{i}\cos wt + 2\hat{j}\sin wt$$

Find the angle between the acceleration vector and the velocity vector at time $\left(t = \frac{\pi}{4w}\right)$

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$$\vec{V}\cdot\vec{a} = va\cos\theta \Rightarrow \cos\theta = \frac{\vec{V}\cdot\vec{a}}{va}$$

$$\vec{V} = \frac{d\vec{r}}{dt} = \frac{d}{dt}(\hat{i}\cos wt + 2\hat{j}\sin wt)$$

$$\vec{V} = (-w\sin wt\,\hat{i} + 2w\cos wt\,\hat{j})$$

$$v = |\vec{V}| = (w^2\sin^2 wt + 4w^2\cos^2 wt)^{1/2}$$

$$\vec{a} = \frac{d\vec{V}}{dt} = \frac{d}{dt}(-w\sin wt\,\hat{i} + 2w\cos wt\,\hat{j})$$

$$= -w^2\cos wt\,\hat{i} - 2w^2\sin wt\,\hat{j}$$

$$a = |\vec{a}| = (w^4\cos^2 wt + 4w^4\sin^2 wt)^{1/2}$$

$$\cos\theta = \frac{\vec{V}\cdot\vec{a}}{va}$$

$$\cos\theta = \frac{(-\hat{i}w\sin wt + 2w\cos wt\,\hat{j})\cdot(-\hat{i}w^2\cos wt - 2w^2\sin wt\,\hat{j})}{(w^2\sin^2 wt + 4w^2\cos^2 wt)^{1/2}(w^4\cos^2 wt + 4w^4\sin^2 wt)^{1/2}}$$

الجواب النهائي $\cos\theta = \frac{-3}{5}$ $\quad\therefore \theta = 127^\circ$

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eq. 3) The position of two particle moving on a common circular path are give by:

$$r_1 = \hat{i}\, b \sin wt + \hat{j}\, b \cos wt$$
$$r_2 = \hat{i}\, b \cos wt - \hat{j}\, b \sin wt$$

Find the relative velocity, the magnitude of the relative velocity, and the time rate of change of the distance between the two particles, all as functions of the time t.

$$\vec{V}_{12} = \vec{V}_2 - \vec{V}_1$$

$$\vec{V}_1 = \frac{dr_1}{dt} = \frac{d}{dt}(\hat{i}\, b \sin wt + \hat{j}\, b \cos wt)$$

$$\vec{V}_1 = \hat{i}\, bw \cos wt - \hat{j}\, bw \sin wt$$

$$\vec{V}_2 = \frac{d\vec{r}_2}{dt} = \frac{d}{dt}(\hat{i}\, b \cos wt - \hat{j}\, b \sin wt)$$

$$\vec{V}_2 = -\hat{i}\, bw \sin wt - \hat{j}\, bw \cos wt$$

$$\vec{V}_{12} = (-\hat{i}\, bw \sin wt - \hat{j}\, bw \cos wt) - (\hat{i}\, bw \cos wt - \hat{j}\, bw \sin wt)$$
$$= \hat{i}(-bw \sin wt - bw \cos wt) + \hat{j}(bw \sin wt - bw \cos wt)$$
$$= \hat{i}(-bw)(\sin wt + \cos wt) + \hat{j}(bw)(\sin wt - \cos wt)$$

الكميات الإتجاهيه لا نستطيع استخراج عامل مشترك منها (إلا للكميات العددية)

$$V_{12} = |\vec{V}_{12}| = \sqrt{b^2 w^2 (\sin wt + \cos wt)^2 + b^2 w^2 (\sin wt - \cos wt)^2}$$

$$= bw \sqrt{\sin^2 wt + \cancel{2 \sin wt \cos wt} + \cos^2 wt + \sin^2 wt \cancel{- 2 \sin wt \cos wt} + \cos^2 wt}$$

$$= bw \sqrt{2(\sin^2 wt + \cos^2 wt} \qquad = bw\sqrt{2}$$

$$\vec{r}_{12} = \vec{r}_2 - \vec{r}_1$$

$$= (\hat{i}\, b\cos wt - \hat{j}\, b\sin wt) - (\hat{i}\, b\sin wt + \hat{j}\, b\cos wt)$$

$$= \hat{i}\, b\cos wt - \hat{j}\, b\sin wt - \hat{i}\, b\sin wt - \hat{j}\, b\cos wt$$

$$= \hat{i}(b\cos wt - b\sin wt) - \hat{j}(b\cos wt + b\sin wt)$$

$$= \hat{i}\, b\,(\cos wt - \sin wt) - \hat{j}\, b\,(\cos wt + \sin wt)$$

$$r_{12} = |\vec{r}_{12}| = \sqrt{b^2(\cos wt - \sin wt)^2 + b^2(\sin wt + \cos wt)^2}$$

$$= b\sqrt{2}$$

$$\frac{dr_{12}}{dt} = 0$$

∴ التغير الزمني للمسافة بين الجسمين = صفر

eq.4) The acceleration of a particle varies with time according to the equation

$$a = \hat{i}At + \hat{j}Bt^2 + \hat{k}Ct^3$$

At time $t=0$, the velocity is (v_0) and the position is (r_0). Find the position vector as a function of t.

$$\vec{a} = \frac{d\vec{v}}{dt} \Rightarrow d\vec{v} = \vec{a}\,dt \Rightarrow \int_{v_0}^{v} dv = \int_0^t (\hat{i}At + \hat{j}Bt^2 + \hat{k}Ct^3)\,dt$$

$$\vec{v} - \vec{v_0} = \hat{i}\frac{At^2}{2}\Big]_0^t + \hat{j}\frac{Bt^3}{3}\Big]_0^t + \hat{k}\frac{Ct^4}{4}\Big]_0^t$$

$$\vec{v} = \hat{i}\frac{At^2}{2} + \hat{j}\frac{Bt^3}{3} + \hat{k}\frac{Ct^4}{4} + v_0$$

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$$\vec{v} = \frac{d\vec{r}}{dt} \Rightarrow \int_{r_0}^{r} dr = \int_0^t \vec{v}\,dt$$

$$\vec{r} - \vec{r_0} = \int_0^t \left(\hat{i}\frac{At^2}{2} + \hat{j}\frac{Bt^3}{3} + \hat{k}\frac{Ct^4}{4} + v_0\right)dt$$

$$\vec{r} - \vec{r_0} = \hat{i}\frac{At^3}{6}\Big]_0^t + \hat{j}\frac{Bt^4}{12}\Big]_0^t + \hat{k}\frac{Ct^5}{20}\Big]_0^t + r_0$$

$$r = \hat{i}\frac{At^3}{6} + \hat{j}\frac{Bt^4}{12} + \hat{k}\frac{Ct^5}{20} + 2r_0$$

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eq.5) A particle moves with constant speed but with continually change direction. show that the acceleration vector is always perpendicular to the velocity?

يجب اثبات أن

$\vec{v} \cdot \vec{a} = 0$

الحل

$$\vec{a} = \left(\dot{v}\hat{\tau} + \frac{v^2}{r}\hat{n} \right)$$

$$\vec{v} \cdot \vec{a} = v\hat{\tau} \cdot \left(\dot{v}\hat{\tau} + \frac{v^2}{r}\hat{n} \right)$$

$$= v\hat{\tau} \cdot \dot{v}\hat{\tau} + \underbrace{v\hat{\tau} \cdot \frac{v^2}{r}\hat{n}}_{\text{zero}}$$

$$= \text{zero}$$

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eq.6) show that the magnitude of tangential Component of the acceleration is given by the expression

$$a_T = \frac{\vec{a}\cdot\vec{V}}{|\vec{V}|}$$, and that of the normal

Component is:- $a_n = (a^2 - a_T^2)^{1/2}$

$$\vec{v} = v\,\hat{\tau}$$

$$\vec{a} = \dot{v}\,\hat{\tau} + \frac{v^2}{r}\,\hat{n}$$

$$\vec{a}\cdot\vec{V} = (\dot{v}\,\hat{\tau} + \frac{v^2}{r}\,\hat{n})\cdot v\,\hat{\tau}$$

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$$= \dot{v}\,\hat{\tau}\cdot v\,\hat{\tau} + \underbrace{\frac{v^2}{r}\,\hat{n}\cdot v\,\hat{\tau}}_{\text{صفر}}$$

$$\vec{a}\cdot\vec{v} = \dot{v}\cdot v = a_T\, v$$ حيث $\dot{v} = a_T$

$$\therefore a_T = \frac{\vec{a}\cdot\vec{v}}{v}$$

2 $\vec{a} = \vec{a_T} + \vec{a_n}$

$$\vec{a} = a_T\,\hat{\tau} + a_n\,\hat{n}$$

بالمقدار $a = (a_T^2 + a_n^2)^{1/2}$

$$a^2 = a_T^2 + a_n^2$$

$$a_n = (a^2 - a_T^2)^{1/2}$$

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eq. 8) A particle moves on a circle of constant radius (b) if the speed of the particle varies with the time (t) according to ($v = At^2$),
For what value, or values, of (t) does the acceleration vector make an angle of (45°) with the velocity vector?

$$\vec{V}\cdot\vec{a} = Va\cos\theta \Rightarrow \cos\theta = \frac{\vec{V}\cdot\vec{a}}{Va} \quad ----(1)$$

$$V = At^2 \Rightarrow \dot{v} = 2At$$

$$\vec{a} = \dot{v}\,\hat{\tau} + \frac{v^2}{b}\hat{n}$$

$$\vec{a} = 2At\,\hat{\tau} + \frac{A^2t^4}{b}\hat{n}$$

$$a = |\vec{a}| = \sqrt{4A^2t^2 + \frac{A^4t^8}{b^2}}$$

$$\vec{V}\cdot\vec{a} = At^2\hat{\tau}\cdot\left(2At\,\hat{\tau} + \frac{A^2t^4}{b}\hat{n}\right)$$

$$\vec{V}\cdot\vec{a} = 2A^2t^3\,\hat{\tau}\cdot\hat{\tau} + \frac{A^3t^6}{b}\underbrace{\hat{\tau}\cdot\hat{n}}_{\text{صفر}}$$

$$\vec{V}\cdot\vec{a} = 2A^2t^3$$

$$\therefore \cos\theta = \frac{2A^2t^3}{(At^2)\sqrt{4A^2t^2 + \frac{A^4t^8}{b^2}}} = \frac{2\cancel{A^2}\cancel{t^3}}{A\cancel{t^2}\cdot\frac{At}{b}(4b^2 + A^2t^4)^{1/2}}$$

$$\cos 45 = \frac{2b}{(4b^2 + A^2t^4)^{1/2}} = \frac{1}{\sqrt{2}}$$

$$4b^2 + A^2t^4 = 8b^2 \Rightarrow 4b^2 = A^2t^4$$

$$t^4 = \frac{4b^2}{A^2} \Rightarrow t^2 = \frac{2b}{A}$$

$$\therefore t = \left(\frac{2b}{A}\right)^{1/2}$$

eq.10) A particle moves on a helical path with cylindrical coordinates varying with time as

لولبي

$$R = A \quad , \phi = Bt^2 \quad , \quad z = Ct^2$$

where A, B, C are Constant. Find the velocity and acceleration vectors as a functions of t.

$R = A$	$\phi = Bt^2$	$z = ct^2$
$\dot{R} = 0$	$\dot{\phi} = 2Bt$	$\dot{z} = 2ct$
$\ddot{R} = 0$	$\ddot{\phi} = 2B$	$\ddot{z} = 2c$

$$\vec{v} = \dot{R}\,\hat{i}_R + R\dot{\phi}\,\hat{j}_\phi + \dot{z}\,\hat{k}$$

$$\vec{v} = 0 + 2ABt\,\hat{j}_\phi + 2ct\,\hat{k}$$

$$\vec{a} = (\ddot{R} - R\dot{\phi}^2)\,\hat{i}_R + (2\dot{R}\dot{\phi} + R\ddot{\phi})\,\hat{j}_\phi + \ddot{z}\,\hat{k}$$

$$= (0 - 4AB^2t^2)\,\hat{i}_R + (0 + 2AB)\,\hat{j}_\phi + 2c\,\hat{k}$$

$$\vec{a}\cdot\vec{v} = av\cos\theta \Rightarrow \cos\theta = \frac{\vec{a}\cdot\vec{v}}{av}$$

نطبقها ثم نستخرج الزاوية

eq. 12) prove that $\vec{A} \cdot \frac{d\vec{A}}{dt} = A \frac{dA}{dt}$

$$\vec{A} \cdot \vec{A} = A^2$$

نأخذ المشتقة للطرفين

$$\frac{d}{dt}(\vec{A} \cdot \vec{A}) = \frac{d}{dt}(A^2)$$

نشتق الطرف الأيسر

$$\frac{d\vec{A}}{dt} \cdot \vec{A} + \vec{A} \cdot \frac{d\vec{A}}{dt} = 2\vec{A} \cdot \frac{d\vec{A}}{dt}$$

نشتق الطرف الأيمن

$$2A \frac{dA}{dt}$$

$$\therefore \cancel{2}\vec{A} \cdot \frac{d\vec{A}}{dt} = \cancel{2}A \frac{dA}{dt} \Longrightarrow \therefore \vec{A} \cdot \frac{d\vec{A}}{dt} = A \frac{dA}{dt}$$

و.هـ.م

eq. 14) prove that $\frac{d}{dt}(\vec{r} \cdot [\vec{v} \times \vec{a}]) = \vec{r} \cdot (\vec{v} \times \dot{\vec{a}})$

$$\frac{d}{dt}(\vec{r} \cdot [\vec{v} \times \vec{a}]) = \vec{r} \cdot \frac{d}{dt}[\vec{v} \times \vec{a}] + [\vec{v} \times \vec{a}] \cdot \dot{r}$$

$$= \vec{r} \cdot [\vec{v} \times \dot{\vec{a}} + \dot{v} \times \vec{a}] + [\vec{v} \times \vec{a}] \cdot \vec{v}$$

$$= \vec{r} \cdot [\vec{v} \times \dot{\vec{a}} + \underbrace{\vec{a} \times \vec{a}}_{\text{صفر}}] + \underbrace{[\vec{v} \times \vec{v}]}_{\text{صفر}} \cdot \vec{a}$$

$$= \vec{r} \cdot [\vec{v} \times \dot{\vec{a}}]$$

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ex) two vectors

$$\vec{A} = 5t^2\,\hat{i} + t\,\hat{j} + t^3\,\hat{k}$$

$$\vec{B} = (\sin t)\,\hat{i} - (\cos t)\,\hat{j}$$

H.W.

Find ① $\frac{d}{dt}(\vec{A}\cdot\vec{B})$, ② $\frac{d}{dt}(\vec{A}\times\vec{B})$

① $\vec{A}\cdot\vec{B} = 5t^2\sin t - t\cos t + 0$

$$\frac{d}{dt}(\vec{A}\cdot\vec{B}) = 5t^2\cos t + 10t\sin t - [-t\sin t + \cos t]$$

$$= 5t^2\cos t + 10t\sin t + t\sin t - \cos t$$

$$= 5t^2\cos t + 11t\sin t - \cos t$$

$$= \cos t\,(5t^2 - 1) + 11t\sin t$$

② $\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & k \\ 5t^2 & t & t^3 \\ \sin t & -\cos t & 0 \end{vmatrix} =$

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$$\hat{i}\,(0 + t^3\cos t) - \hat{j}\,(t^3\sin t) + \hat{k}\,(-5t^2\cos t - t\sin t)$$

تم الاشتقاق

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ex) Suppose the position vector of a particle is given by

$$\vec{r} = \hat{i}\, b \sin \omega t + \hat{j}\, b \cos \omega t + \hat{k}\, c \quad , \text{ Find}$$

(1) The distance from this position to origin

(2) Velocity vector and the speed

(3) acceleration Vector (4) The figure of path

---

(1) $$r = |\vec{r}| = \sqrt{b^2 \sin^2 \omega t + b^2 \cos^2 \omega t + c^2}$$

$$= \sqrt{b^2(\sin^2 \omega t + \cos^2 \omega t) + c^2} = \sqrt{b^2 + c^2}$$

(2) $$\vec{V} = \frac{d\vec{r}}{dt} = \frac{d}{dt}(\hat{i}\, b \sin \omega t + \hat{j}\, b \cos \omega t + \hat{k}\, c)$$

$$= \hat{i}\, b\omega \cos \omega t - \hat{j}\, b\omega \sin \omega t + k(0)$$

$$V = |\vec{V}| = \sqrt{\omega^2 b^2 \cos^2 \omega t + b^2 \omega^2 \sin^2 \omega t} = \sqrt{b^2\omega^2(\sin^2 \omega t + \cos^2 \omega t)}$$

$$= b\omega$$

نستنتج ان متجه السرعة موازي لـ (xy) لان مركبة K تساوي صفر والانطلاق ثابت مقداره (bω)

(3) $$\vec{a} = \frac{d\vec{V}}{dt} = \frac{d}{dt}(\hat{i}\, b\omega \cos \omega t - \hat{j}\, b\omega \sin \omega t)$$

$$\vec{a} = -\hat{i}\, b\omega^2 \sin \omega t - \hat{j}\, b\omega^2 \cos \omega t)$$

$$a = |\vec{a}| = \sqrt{b^2\omega^4 \sin^2 \omega t + b^2 \omega^4 \cos^2 \omega t}$$

$$= \sqrt{b^2 \omega^4 (\sin^2 \omega t + \cos^2 \omega t)} = b\omega^2$$

(4) $\Rightarrow$

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Ex) A particle moves on spiral path such that the position in polar Coordinats is given by::

حلزوني

$r = bt^2$, $\theta = ct$, c, b Constant

Find the velocity and acceleration as a function of (t)?

$\theta = ct$ $\qquad r = bt^2$

$\dot{\theta} = c$ $\qquad \dot{r} = 2bt$

$\ddot{\theta} = \text{zero}$ $\qquad \ddot{r} = 2b$

$\vec{V} = \dot{r}\,\hat{i}_r + r\dot{\theta}\,\hat{j}_\theta$

$\vec{V} = 2bt\,\hat{i}_r + cbt^2\,\hat{j}_\theta$

$\vec{a} = (\ddot{r} - r\dot{\theta}^2)\,\hat{i}_r + (2\dot{r}\dot{\theta} + r\ddot{\theta})\,\hat{j}_\theta$

$= (2b - bt^2c^2)\,\hat{i}_r + 4bct\,\hat{j}_\theta$

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4

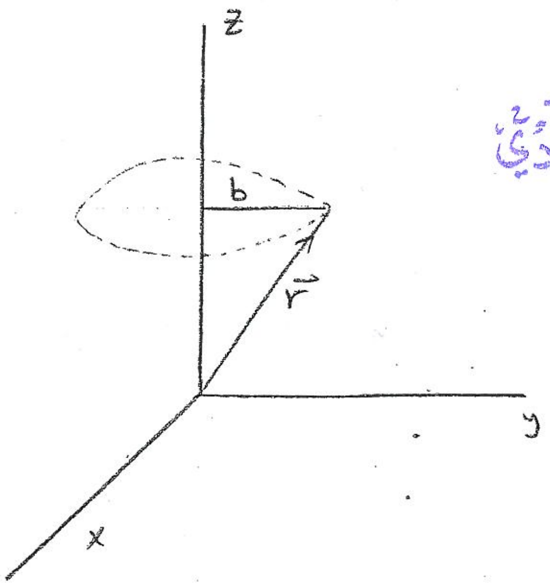
لايجاد شكل المسار نجد $\vec{a}\cdot\vec{v}$

$$\vec{a}\cdot\vec{v} = (-\hat{i}\, b w^2 \sin wt - \hat{j}\, b w^2 \cos wt)\cdot(\hat{i}\, b w \cos wt - \hat{j}\, b w \sin wt + k(0))$$

$$-b w^2 \sin wt \cdot b w \cos wt + b w^2 \cos wt \cdot b w \sin wt$$

$$= \text{zero}$$

بما ان $\vec{a}\cdot\vec{v} = 0$ نستنتج ان التعجيل عمودي على السرعة



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