Chapter one:the nature of light

Fundamentals of optics

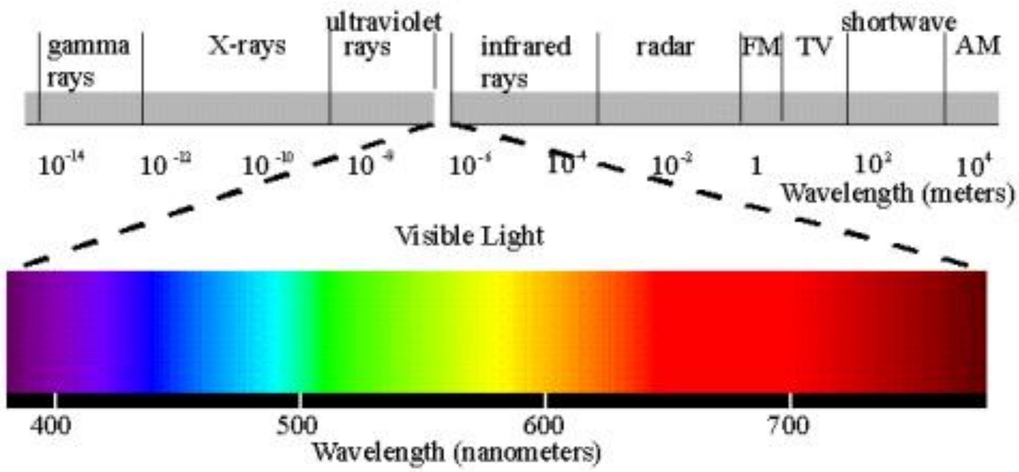
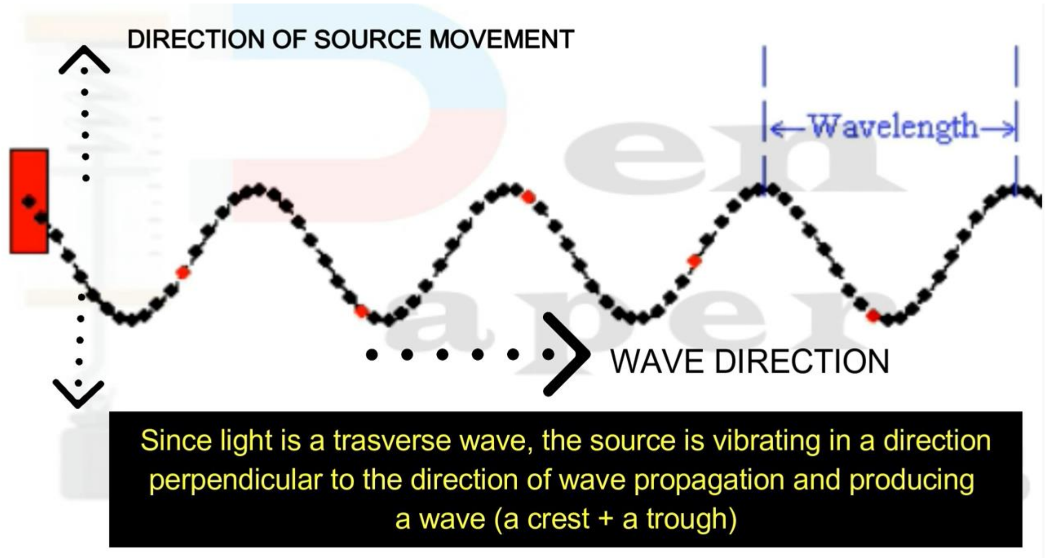
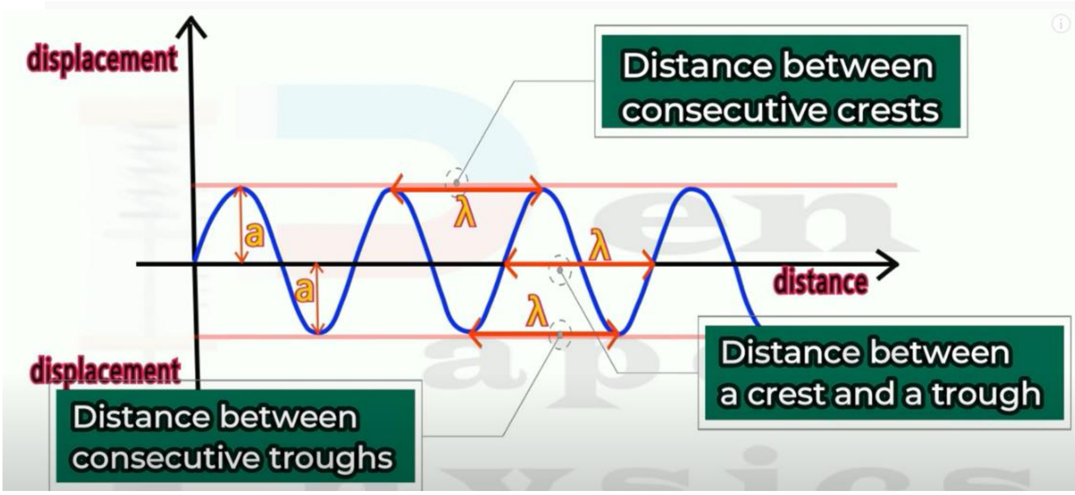
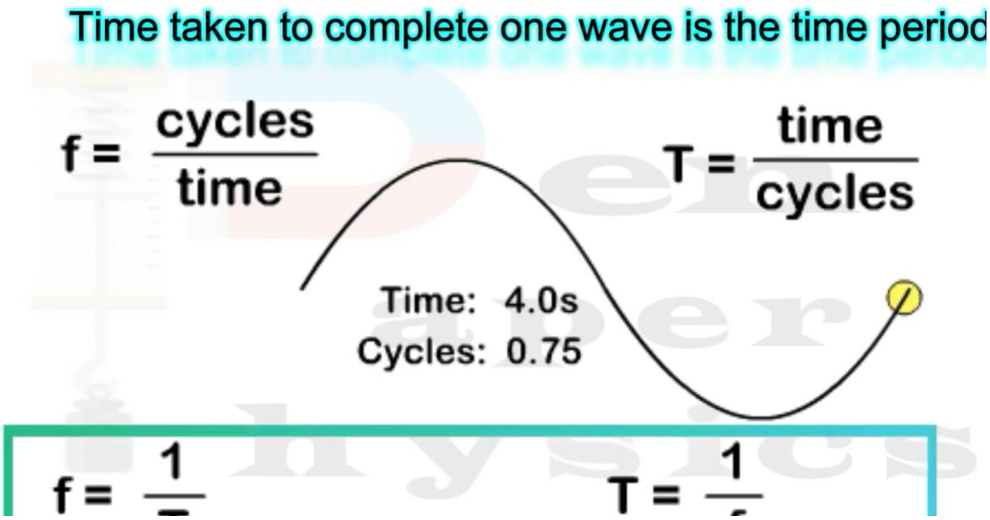


Table (1) : represented the wavelength and frequency ranges of the divisions of the electromagnetic spectrum.

Category	Range of Wavelengths (nm)	Range of Frequencies (Hz)
gamma rays	<1	$>3\times10^{17}$ $>3\times10^{17}$
X-rays	1–10	3×10^{16}–3×10^{17}
ultraviolet light	10–400	$7{,}5\times10^{14}$–3×10
visible light	400–700	$4{,}3\times10^{14}$–$7{,}5\times10^{14}$
Infrared	700–10^{5}	3×10^{12}–$4{,}3\times10^{14}$
microwave	10^{5}–10^{8}	3×10^{9}–3×10^{12}
radio waves	$>10^{8}$	$<3\times10^{9}$


DIRECTION OF SOURCE MOVEMENT
Wavelength
WAVE DIRECTION
Since light is a trasverse wave, the source is vibrating in a direction perpendicular to the direction of wave propagation and producing a wave (a crest + a trough)


displacement
Distance between consecutive crests
a
a
λ
λ
λ
distance
displacement
Distance between a crest and a trough
Distance between consecutive troughs



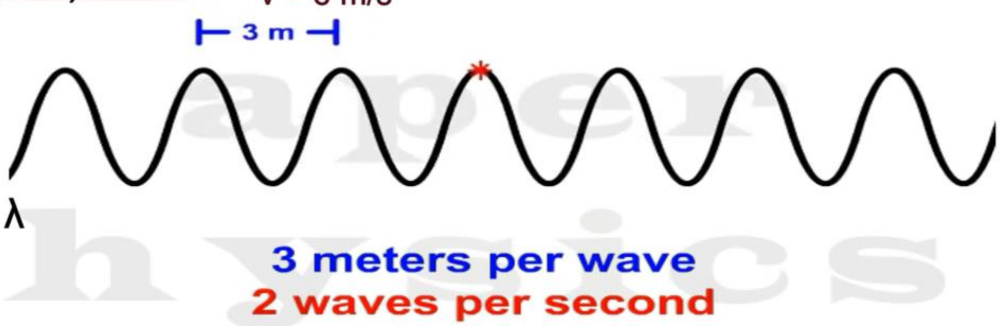
distance travelled by one wave = λ

Time to complete one wave = T

Wave velocity (v) = λ / T

f = (1 / T) or n = (1/ T)

n (no. of waves per second) = 2
wavelength (distance travelled by one wave) = 3 m
v = 6 m/s



v = f . λ or v = n . λ

Wavelength is defined as the distance between two most near points in phase with each other. Hence, two adjacent peaks or troughs on a wave are separated by a distance of a single complete wavelength. Mostly, we use the letter lambda **(λ)** to describe the wavelength of a wave.

$$c = \lambda f$$

speed of light c: $2.998 \times 10^{8}\ ms^{-1}$
wavelength λ: (in m)
frequency f: (in s^{-1} or Hz)

What is the frequency of violet light with wavelength 400 nm?

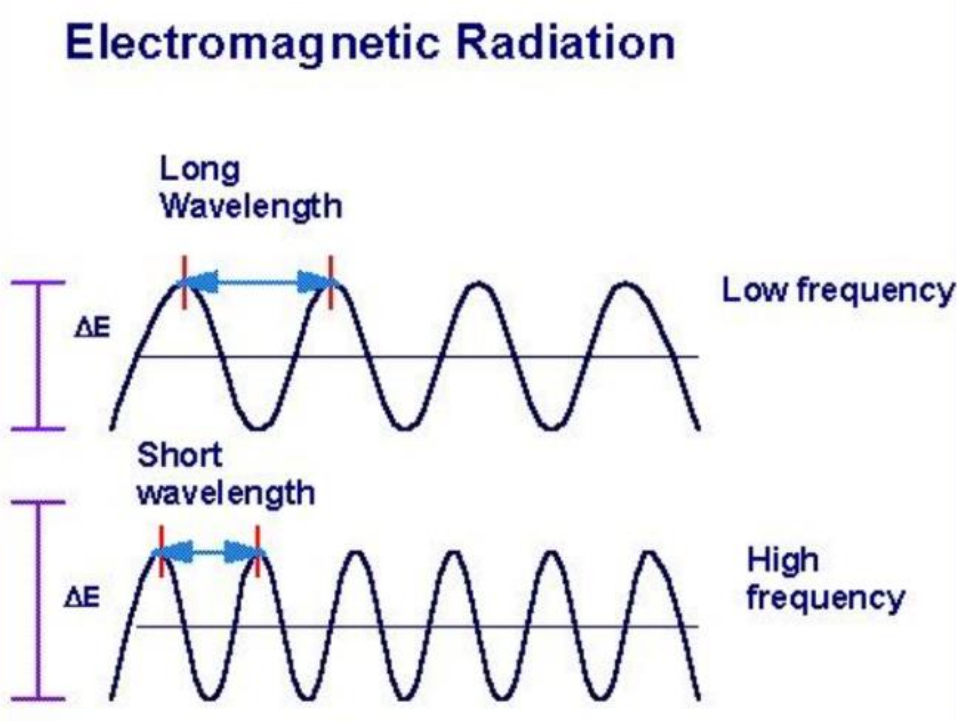
$$\lambda f = c$$

$$f = \frac{c}{\lambda}$$

$c = 2.998 \times 10^{8}\ ms^{-1}$
$\lambda = 400\ nm = 400 \times 10^{-9}\ m$

$$= \frac{2.998 \times 10^{8}\ \cancel{m}s^{-1}}{400 \times 10^{-9}\ \cancel{m}}$$

$$= 7.50 \times 10^{14}\ s^{-1}$$



Please see the link below to get whole idea about the frequency. wavelength and velocity of the waves: https://www.youtube.com/watch?v=9UPnWfBYf28

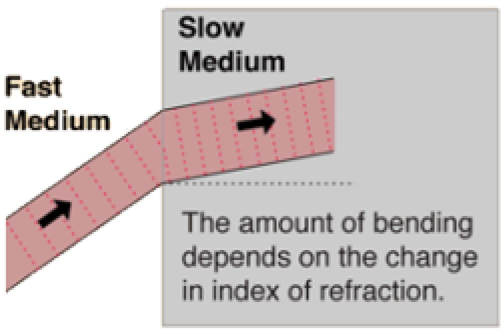
Refractive index.

Refractive index is measure of how light propagates through a material or measure of the bending of a ray of light when passing from one medium into another. The higher the refractive index the slower the light travels, which causes a correspondingly increased change in the direction of the light within the material.

The refractive index can be seen as the factor by which the speed and the wavelength of the radiation are reduced with respect to their vacuum values: The speed of light in a medium is $v = c/n$, and similarly the wavelength in that medium is $\lambda = \lambda_0/n$, where λ_0 is the wavelength of that light in vacuum.

THE REFRACTIVE INDEX

The index of refraction, or refractive index, of any optical medium is defined as the ratio between the speed of light in a vacuum and the speed of light in the medium:



index of refraction

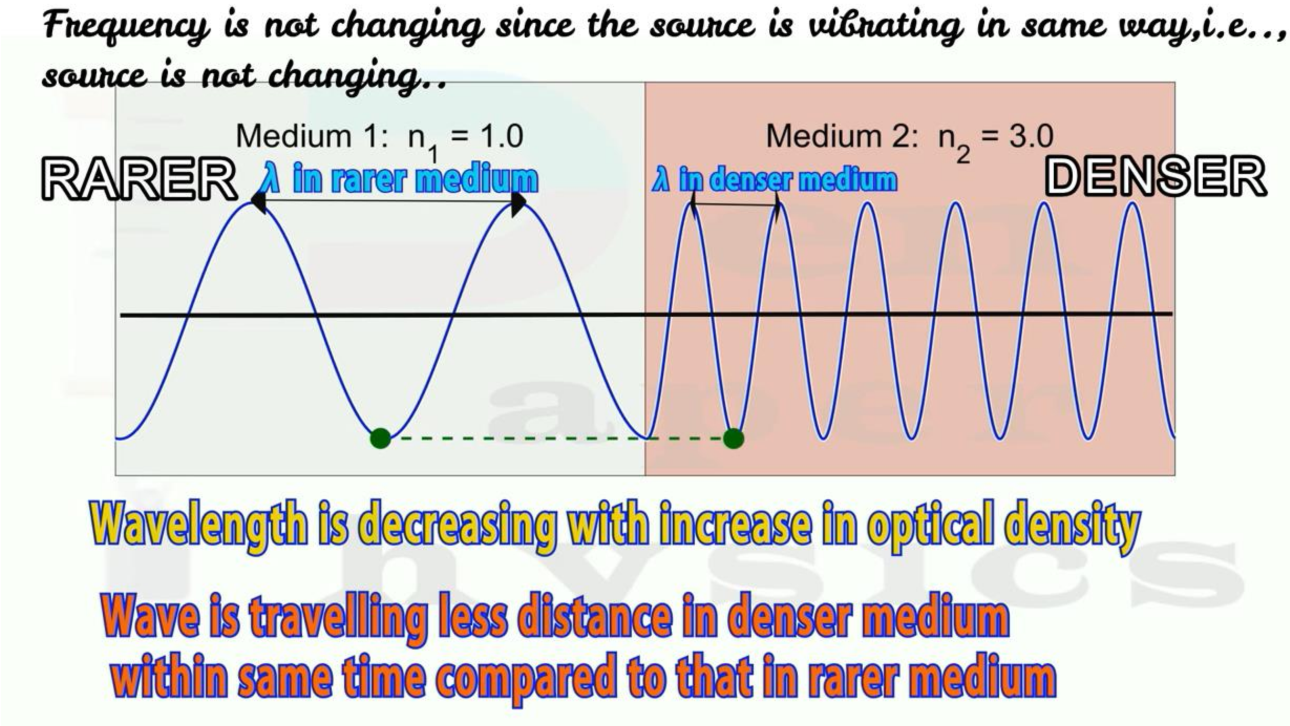
$$n = \frac{c}{v}$$

velocity of light in vacuum

velocity of light in the medium

Table (1-1) :shows different refractive index with various materials.

Material	n	Material	n
Vacuum	1.000	Ethyl alcohol	1.362
Air	1.000277	Glycerine	1.473
Water	4/3	Ice	1.31
Carbon disulfide	1.63	Polystyrene	1.59
Methylene iodide	1.74	Crown glass	1.50-1.62
Diamond	2.417	Flint glass	1.57-1.75


Frequency is not changing since the source is vibrating in same way,i.e..,
source is not changing..
Medium 1: n_1 = 1.0
Medium 2: n_2 = 3.0
RARER
λ in rarer medium
λ in denser medium
DENSER
Wavelength is decreasing with increase in optical density
Wave is travelling less distance in denser medium
within same time compared to that in rarer medium

- Chapter one:the nature of light

Fundamentals of optics

Example (1): The speed of light in an unknown medium is measured to be 2.76 x 10^8 m/s. What is the index of refraction of the medium?

Solution:

The index is found to be

$n = c/v$ = (3.00 x 108 m/s)/(2.76 x 108 m/s) = 1.09.

(2) Optical path

In a medium of constant refractive index, *n*, the optical path length OPL for a path of physical length *d* ,to define a quantity called the *optical path*. The path *d* of a ray of light in any medium is given by the product *velocity* times *time:*

$$d = vt$$

Since by definition $n = c/v$, which gives $v = c/n$, we can write the product *nd* is called the *optical path* Δ:

$$nd = ct$$

$$\Delta = n\, d$$

The optical path represents the distance light travels in a vacuum in the same time it travels a distance *d* in the medium. If a light ray travels through a series of optical media of thickness *d, d', d"* ... and refractive indices *n, n', n"*, ... , the total optical path is just the sum of the separate values:

.If the refractive index varies along the path, the OPL is given by

$$Opt = n_1 d_1 + n_2 d_2$$

$$\text{Now, speed} = \frac{\text{distance}}{\text{time}}$$

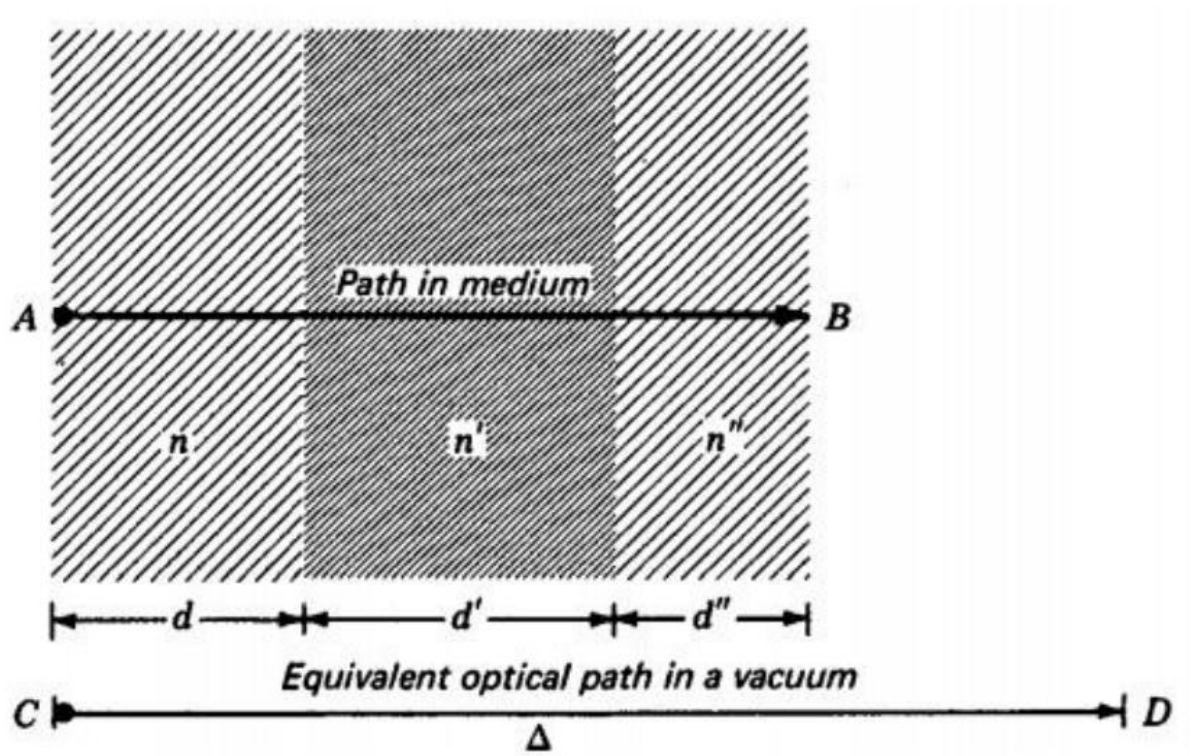
$$\therefore \quad \text{time} = \frac{\text{distance}}{\text{speed}}$$

$$\therefore \quad t = \frac{d_{\text{medium}}}{v_{\text{medium}}} = \frac{d_{\text{vacuum}}}{v_{\text{vacuum}}}$$

$$\text{Hence, the optical path} = d_{\text{vacuum}}$$

$$= \frac{v_{\text{vacuum}}}{v_{\text{medium}}} \times d_{\text{medium}}$$

$$= n \times d_{\text{medium}}$$


Path in medium
A
B
n
n'
n''
d
d'
d''
Equivalent optical path in a vacuum
C
D
Δ

Problems chapter one

From (FUNDAMENTALS OF OPTICS, **Francis A. Jenkins)**

1.4 If the refractive index for a piece of optical glass is 1.5250, calculate the speed of light in the glass. *Ans.* 1.9659 x 108 *m/s*

1.5 Calculate the difference between the speed of light in kilometers per second in a vacuum and the speed of light in air if the refractive index of air is 1.0002340. Use velocity values to seven significant figures.
1.6 If the moon's distance from the earth is 3.840 x 105 km, how long will it take microwaves to travel from the earth to the moon and back again?

1.7 How long does it take light from the sun to reach the earth? Assume the earth's distance from the sun to be 1.50×10^8 km. *Ans.* 500 s, or 8 min 20 s

1.8 A beam of light passes through a block of glass 10.0 cm thick, then through water for a distance of 30.5 cm, and finally through another block of glass 5.0 cm thick. If the refractive index of both pieces of glass is 1.5250 and of water is 1.3330, find the total optical path.

1.9 A water tank is 62.0 cm long inside and has glass ends which are each 2.50 cm thick. If the refractive index of water is 1.3330 and of glass is 1.6240, find the overall optical path.

1.10 A beam of light passes through 285.60 cm of water of index 1.3330, then through 15.40 cm of glass of index 1.6360, and finally through 174.20 cm of oil of index 1.3870.
Find to three significant figures *(a)* each of the separate optical paths and *(b)* the total optical path. *Ans. (a)* 380.7, 25.19, and 241.6 cm, *(b)* 647 cm

1.11 A ray of light in air is incident on the polished surface of a block of glass at an angle of 10°. *(a)* If the refractive index of the glass is 1.5258, find the angle of refraction to four significant figures. *(b)* Assuming the sines of the angles in Snell's law can be Replaced by the angles themselves, what would be the angle of refraction? *(c)* Find
the percentage error.

1.12 Find the answers to Prob. 1.11, if the angle of incidence is 45.0° and the refractive index is 1.4265.
1.13 A ray of light in air is incident at an angle of 54.0° on the smooth surface of a piece of glass. *(a)* If the refractive index is 1.5152, find the angle of refraction to four significant figures. *(b)* Find the angle of refraction graphically. (See Fig. P1.13).
Ans. (a) 32.272°, (b) 32.3°

Chapter 2 Superposition of Waves

The principle of superposition:

When two or more waves cross at a point, the displacement at that point is equal to the sum of the displacements of the individual waves. The individual wave displacements may be positive or negative. This is known as the principle of superposition and was first clearly stated by Young in 1802.

The principle is therefore applicable with great precision to light, and we can use it in investigating the disturbance in regions where two or more light waves are superimposed.

1. Superposition of Simple Harmonic Motions Along the Same direction.

Considering first the effect of superimposing two sine waves y_1 and y_2 traveling in the same direction and same frequency with different amplitudes (a_1 and a_2). We can write the separate waves as follows:

$$y_1 = a_1 \sin(wt - \alpha_1) \text{ .... (1)}$$

$$y_2 = a_2 \sin(wt - \alpha_2)$$

Here, (w) the same for both waves have same frequency.
According to the principle of superposition, the resultant wave Y is the sum of y_1 and y_2, and we have

$$Y = y_1 + y_2 = a_1 \sin(wt - \alpha_1) + a_2 \sin(wt - \alpha_2) \text{ - - - - - - -2}$$

Using the identity of sin (a + b) = sin a cos b + sin b cos a

$$Y = a_1 \sin wt . \cos \alpha_1 - a_1 \cos wt . \sin \alpha_1 + a_2 \sin wt . \cos \alpha_2 - a_2 \cos wt . \sin \alpha_2$$

$$Y = (a_1 \cos \alpha_1 + a_2 \cos \alpha_2) \sin wt - (a_1 \sin \alpha_1 + a_2 \sin \alpha_2) \cos wt \text{ .... 3}$$

Because of the amplitudes consider a constant, so we are setting:

$$\boldsymbol{a_1}\mathbf{cos}\,\boldsymbol{\alpha_1} + \boldsymbol{a_2}\mathbf{cos}\,\boldsymbol{\alpha_2} = A\cos\theta \quad \ldots\ldots\ldots\ldots.\ 4$$
$$\boldsymbol{a_1}\mathbf{sin}\,\boldsymbol{\alpha_1} + \boldsymbol{a_2}\mathbf{sin}\,\boldsymbol{\alpha_2} = A\sin\theta$$

Squaring and adding Equation (4), we will get

$$(A\cos\theta)^2 + (A\sin\theta)^2 = (\boldsymbol{a_1}\cos\boldsymbol{\alpha_1} + \boldsymbol{a_2}\boldsymbol{cos}\,\boldsymbol{\alpha_2})^2 + (\boldsymbol{a_1}\sin\boldsymbol{\alpha_1} + \boldsymbol{a_2}\boldsymbol{sin}\,\boldsymbol{\alpha_2})^2$$

$$A^2(cos^2\theta + sin^2\theta) = \boldsymbol{a_1}^2\boldsymbol{cos}^2\boldsymbol{a_1} + 2\,(\boldsymbol{a_1}\cos\boldsymbol{\alpha_1} + \boldsymbol{a_2}\boldsymbol{cos}\,\boldsymbol{\alpha_2}) + \boldsymbol{a_2}^2\,\boldsymbol{cos}^2\boldsymbol{\alpha_2} + \boldsymbol{a_1}^2\boldsymbol{sin}^2\boldsymbol{a_1} + 2\,(\boldsymbol{a_1}\sin\boldsymbol{\alpha_1} + \boldsymbol{a_2}\boldsymbol{sin}\,\boldsymbol{\alpha_2}) + \boldsymbol{a_2}^2\,\boldsymbol{sin}^2\boldsymbol{\alpha_2}$$

$$A^2 = \boldsymbol{a_1}^2\boldsymbol{cos}^2\boldsymbol{a_1} + 2\,\boldsymbol{a_1}\cos\boldsymbol{\alpha_1} + 2\boldsymbol{a_2}\boldsymbol{cos}\,\boldsymbol{\alpha_2} + \boldsymbol{a_2}^2\boldsymbol{cos}^2\boldsymbol{\alpha_2} + a_1{}^2sin^2a_1 + 2\,a_1\sin\alpha_1 + 2a_2 sin\,\alpha_2 + \boldsymbol{a_2}^2\,sin^2\alpha_2$$

$$A^2 = \boldsymbol{a_1}^2\,(\boldsymbol{cos}^2\boldsymbol{a_1} + sin^2a_1) + \boldsymbol{a_2}^2(\boldsymbol{cos}^2\boldsymbol{\alpha_2} + sin^2\alpha_2) + 2\,\boldsymbol{a_1}(\cos\boldsymbol{\alpha_1} + \boldsymbol{cos}\,\boldsymbol{\alpha_2}) + 2\,\boldsymbol{a_2}\,(\boldsymbol{sin}\,\boldsymbol{\alpha_1} + \boldsymbol{sin}\,\boldsymbol{\alpha_2})$$

Using the identity of cos (a - b) = cos a cos b + sin a sin b.

$$A^2 = \boldsymbol{a_1}^2 + \boldsymbol{a_2}^2 + 2\,\boldsymbol{a_1}\boldsymbol{a_2}\,\boldsymbol{cos}\,(\boldsymbol{\alpha_1} - \boldsymbol{\alpha_2}) \ldots\ldots\ldots\ldots.5$$

From equation 4, Dividing the lower equation by the upper one, we obtain:

$$\tan\theta = \frac{\boldsymbol{a_1}\mathbf{sin}\ \boldsymbol{\alpha_1} + \boldsymbol{a_2}\mathbf{sin}\ \boldsymbol{\alpha_2}}{\boldsymbol{a_1}\mathbf{cos}\ \boldsymbol{\alpha_1} + \boldsymbol{a_2}\mathbf{cos}\ \boldsymbol{\alpha_2}} \ldots\ldots\ldots\ldots.6$$

Equations (5) and (6) show that values of ***A*** and ***θ*** exist which satisfy Equation (4), and we can rewrite Equation (3), substituting the right-hand of Equation (4). This gives

$$Y = A\cos\theta\ \sin wt - A\sin\theta\ s\,wt$$

$$Y = A\sin[\![(wt]\!] - \theta) \ldots\ldots..7$$

Equation (7) is the same as either of our original equations for the separate wave but contains a new amplitude A and a new phase constant ***θ.***

Therefore, the sum of two (S.H.M) of the same frequency and along the same line is also a (S.H.M) of the same frequency. The amplitude and phase constant of the resultant motion can easily be calculated from those of the component motions by using equations (5) and (6), respectively. The addition of three or more S.H.M of the same frequency will likewise give rise to a resultant motion of the same type, since the motions can be added successively, each time giving an equation of the form of Equation (7).

Intensity

The resultant amplitude A depends on:

- ✓ the amplitudes $\mathbf{a_1}$ & $\mathbf{a_2}$ of the component motions.
- ✓ their difference of phase $\boldsymbol{\delta} = (\boldsymbol{\alpha_1} - \boldsymbol{\alpha_2})$.

Based on equation 5 and in the case where $\mathbf{a_1 = a_2}$,

$$I \approx A^2 = \mathbf{2a^2} + \mathbf{2a^2} \cos(\boldsymbol{\alpha_1} - \boldsymbol{\alpha_2})$$

$$I \approx A^2 = \mathbf{2a^2}[1 + \cos\boldsymbol{\delta}]$$

Using the identity 1+ cos θ =2 cos^2 θ /2

Then,
$$I \approx A^2 = \mathbf{2a^2}\left(\frac{2\cos\ \delta}{2}\right)$$

- ❖ $I \approx A^2 = \mathbf{4a^2\, cos^2}\ \frac{\delta}{2}$ **hase difference δ = 0 , 2π , 4 π, 6π, 8π, 10π ... (Even number of π) , this gives 4a^2 . it means the 4 times the intensity of either wave .**
- ❖ **If the phase difference δ = π , 3 π, 5 π, 7π, 9π... (Odd number of π) , this gives the intensity of zero.**
- ❖ **For intermediate values, the intensity varies between these limits according to the square of the cosine.**

2. Superposition of Simple Harmonic Motions Along the Same direction and amplitude but with one a distance ahead of the other.

let us find the resultant wave produced by two waves of equal frequency and amplitude traveling in the same direction +x, but with one a distance Δ ahead of the other. The equations of the two waves can be write them as follow:

$$y_1 = a \sin(\omega t - kx) \qquad (1)$$

$$y_2 = a \sin[\omega t - k(x + \Delta)] \qquad (2)$$

According to the principle of superposition, the resultant wave Y is the sum of y_1 and y_2, and we have:

$$y = y_1 + y_2 = a\{\sin(\omega t - kx) + \sin[\omega t - k(x + \Delta)]\}$$

Applying the trigonometric formula

$$\sin A + \sin B = 2 \sin \tfrac{1}{2}(A + B) \cos \tfrac{1}{2}(A - B)$$

we find
$$y = 2a \cos \frac{k\Delta}{2} \sin\left[\omega t - k\left(x + \frac{\Delta}{2}\right)\right] \qquad (3)$$

This corresponds to a new wave of the same frequency but with the amplitude equal to:

$$\boxed{2a \cos(k\Delta/2) = 2a \cos(\pi\Delta/\lambda).}$$

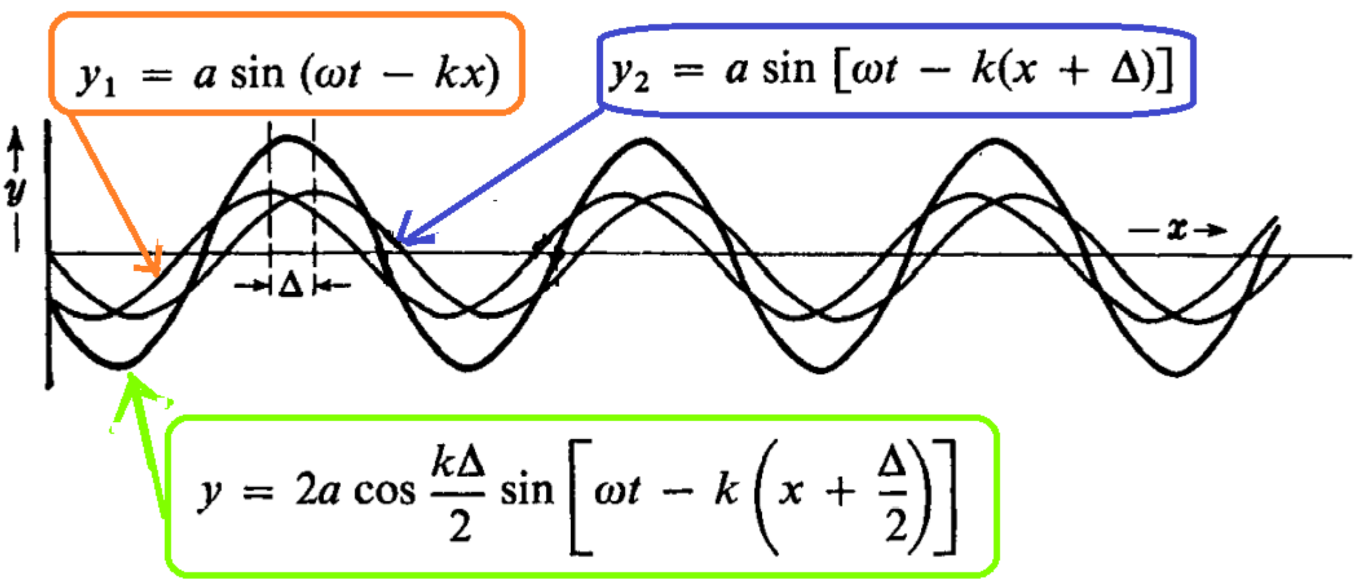


Figure 1 Superposition of two wave having same frequency and amplitude traveling in the same direction +x, but with one a distance Δ ahead of the other.

3. Superposition of Simple Harmonic Motions with Same amplitude and frequency but with opposite direction.

Two waves of the same frequency and amplitude travelling in opposite directions. This is usually achieved by using a travelling wave and its reflection, which will ensure that the frequency is exactly the same.

Two such waves can be represented by the equations

$y_1 = A\sin(wt - kx)$ .... **(1)**
$y_2 = A\sin(wt + kx)$ .... **(2)**
By adding the above equations, as follow:

$Y(x,t) = A\,[\sin(wt + kx) + \sin(wt + kx)]$ .... **(3)**

by trig identities as shown:

$$\sin\alpha + \sin\beta = 2\sin\left(\frac{\alpha}{2} + \frac{\beta}{2}\right)\cos\left(\frac{\alpha}{2} - \frac{\beta}{2}\right)$$

Here lets

$$\alpha = (\omega t - kx)$$
$$\beta = (\omega t + kx)$$

Therefore, we will get:

$$y(x,t) = 2A\sin\left(\frac{\omega t - \cancel{kx}}{2} + \frac{\omega t + \cancel{kx}}{2}\right)\cos\left(\frac{\cancel{\omega t} - kx}{2} - \frac{\cancel{\omega t} + kx}{2}\right)$$

$Y(x,t) = 2A\cos(-kx)\sin wt$ .... **(4)**

Equation 4 represents the standing waves.

For any value of *x* we have simple harmonic motion, whose amplitude varies with x between the limits 2a and zero.

When $kx = 0, \pi, 2\pi, 3\pi, \ldots\ldots n\pi$, the amplitude will be 2a.

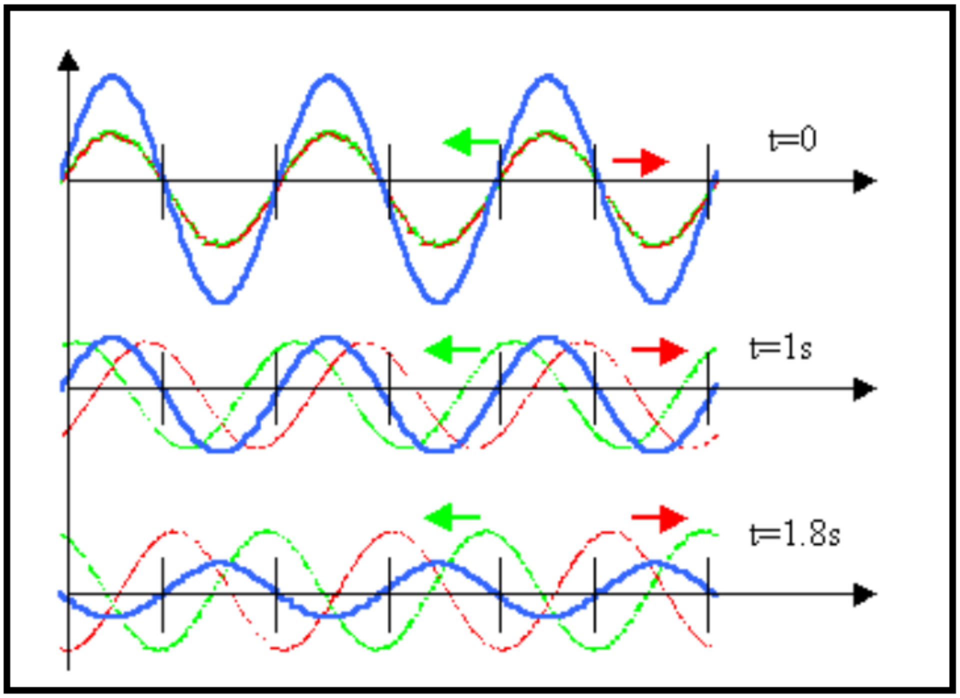
These points are the *Nodes* of the standing wave pattern.

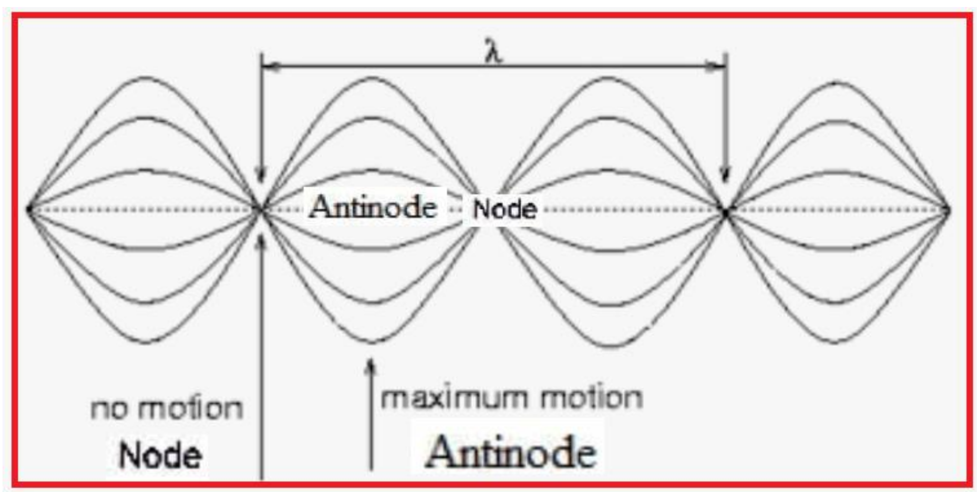
When $kx = \pi/2, 3\pi/2, 5\pi/2, \ldots\ldots (2n+1)\pi/2$, the amplitude will be Zero.

These points are called the *Antinodes* of the standing wave pattern

The latter positions correspond to the nodes and are separated by a distance

$\Delta x = \pi/k = \lambda/2$.





What is Group Velocity?

The Group Velocity of a Wave is defined as the Velocity at which an entire envelope of Waves moves through a medium. A most common Example, in this case, can be that of throwing stones in a water body which causes multiple Waves on the surface of water.

On throwing the stone, a ripple is created around the point where the stone drops. The ripple is formed of small Wavelets which propagate away from the dropping point in multiple directions. Here, a Wavelet having the shortest Wavelength propagates faster than others that have higher wavelength according to equation:

$$V = \lambda f$$

What is Phase Velocity?

The Velocity of each single wave in the group dependent on their time period , wavelength, and frequency when it propagates in specific media based on the equation:

$$V = \lambda f \ = \lambda \frac{1}{T}$$

What is the Relation Between Phase Velocity and Group Velocity?

It is important to establish a relation between the group velocity and wave velocity, and this can easily be done by considering the groups formed by superimposing two waves of slightly different wavelength, Therefore, let have two waves having different wavelength

The two waves have equal amplitudes but slightly different wavelengths, $\boldsymbol{\lambda}$ and λ^{-}, and slightly different velocities, v and v^{-}.: why?

$\boldsymbol{y_1 = a\sin(\omega t - kx)}$ .... **(1)**
$\boldsymbol{y_2} = \boldsymbol{a}\sin(\omega^{-} t - k^{-}\boldsymbol{x})$ .... **(2)**
According to the principle of superposition, the resultant wave Y is the sum of y_1 and y_2, and we have:
$\boldsymbol{Y = y_1 + y_2 = a\sin(\omega t - kx)} + \boldsymbol{a}\sin(\omega^{-}\boldsymbol{t} - k^{-}\boldsymbol{x})$ ---------(3)

by trig identities as shown:

$$\sin\alpha + \sin\beta = 2\sin\left(\frac{\alpha}{2} + \frac{\beta}{2}\right)\cos\left(\frac{\alpha}{2} - \frac{\beta}{2}\right)$$

Here lets

$$\alpha = (\omega t - kx)$$
$$\beta = (\bar{\omega} t - \bar{k}x)$$

$$\boldsymbol{Y = 2a\sin[\![\left(\frac{\omega t - kx}{2}\right]\!] + \frac{\omega^{-} t - k^{-} x}{2}\Big)\cos\left(\frac{\omega t - kx}{2} - \frac{\omega^{-} t - k^{-} x}{2}\right)}$$

$$\boldsymbol{Y = 2a\sin[\![\left(\frac{\omega+\omega^-}{2}]\!]\,t - \frac{k+k^-}{2}x\right)\cos\left(\frac{\omega-\omega^-}{2}t - \frac{k-k^-}{2}x\right)\ldots\ldots\ldots(4)}$$

$$\boldsymbol{Y = 2a\sin[\![(\omega)]\!]t - kx)\cos\left(\frac{\Delta\omega}{2}t - \frac{\Delta k}{2}x\right)\ldots\ldots\ldots(5)}$$

The wave velocity or more specific term it called phase velocity which is sometimes used. That it is identical with the quantity $\boldsymbol{v}$ in our previous equations is shown by evaluating the rate of change of the x coordinate under the condition that the phase remain constant. When the form of the phase in Eq. (1 and 2, or even 5) is used, the latter requirement becomes:

$$\boldsymbol{(\omega t - kx) = constant}$$

The wave velocity $\frac{dx}{dt} = \frac{\omega}{t}$ ......(6)

Here, $\omega = 2\pi f,\ k = \frac{2\pi}{\lambda}$

Prove $\omega = kv$?

The individual waves, having the average of the two k's, correspond to variations of the sine factor in Eq. (5), and according to Eq. (6), their phase velocity is the quotient of the multipliers of t and x. Therefore, the phase velocity $v\ is$:

$$\boldsymbol{\mathbf{v} = [\![\left(\frac{\omega+\omega^-}{k+k^-}]\!]\right) \approx \frac{\omega}{k}}$$

That is, the velocity is essentially that of either of the component waves, since these velocities are very nearly the same.

The envelope of modulation (wave packet), indicated by the broken curves shown in Fig. below is given by the cosine factor. This has a much smaller propagation number, equal to the difference of the separate ones, and a correspondingly greater wavelength.

From the amplitude of wave packet (part below):

$$\boldsymbol{\cos\left(\frac{\Delta\omega}{2}t - \frac{\Delta k}{2}x\right)}$$

$$\boldsymbol{\frac{\Delta\omega}{2}t - \frac{\Delta k}{2}x = constant}$$

$$\frac{dx}{dt}=\frac{\frac{\Delta\omega}{2}}{\frac{\Delta k}{2}}=\frac{\Delta\omega}{\Delta k}=\frac{d\omega}{dk}$$

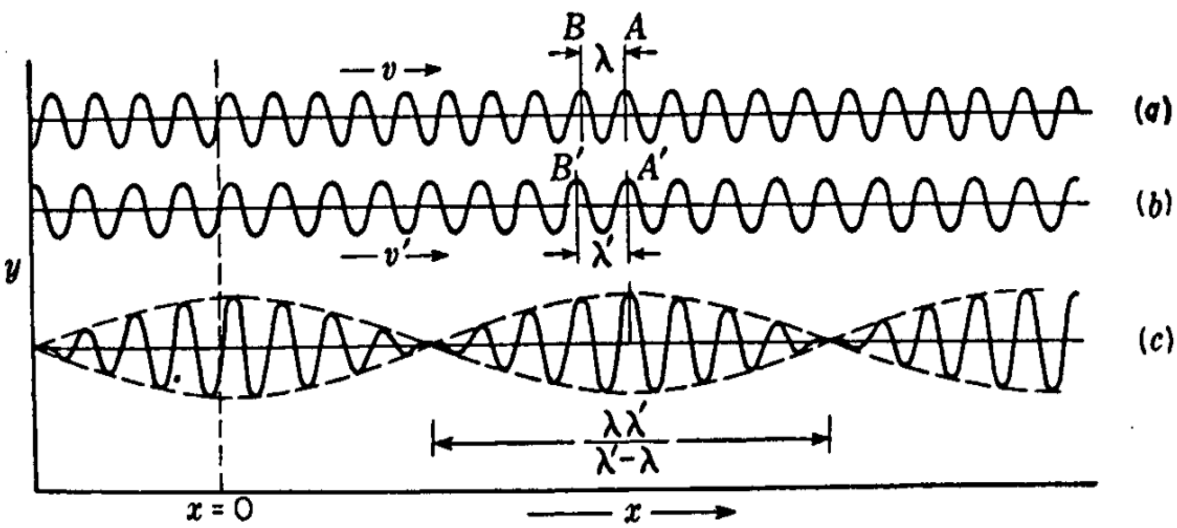
Hence the groups velocity (u) is $u=\frac{d\omega}{dk}$

Since no limit has been set on the smallness of the differences, they may be treated as infinitesimals and the approximate equality becomes exact. Then, since $\omega = kv$ we find for the relation between the group velocity **u** and the wave velocity $\boldsymbol{v}$ and If the variable is changed to $\boldsymbol{\lambda}$, through k = 2π/$\boldsymbol{\lambda}$, one obtains the useful form as below:

$$\mathbf{u}=\frac{d\omega}{dk}=v+k\frac{dv}{dk}=v-\lambda\frac{dv}{d\lambda}$$

Prove these equations?

It should be emphasized that $\boldsymbol{\lambda}$ here represents the actual wavelength in the medium. For light, this will not in most problems be the ordinary wavelength in air. Why?



Groups and group velocity of two waves of slightly different wavelength and frequency.

In non-dispersion media the group velocity u equal to phase velocity v.

It means $u = v$ prove that?

Show that $v_p v_g = c^2$

We know that

$$V_p = v\lambda \quad \text{...(1)}$$

$$\lambda = \frac{h}{mv} \quad \text{...(2)}$$

represents De-broglie wavelength

To obtain the value of frequency we equate the expression $E = hv$ with the reletivistic formula for total energy given by

$$E = mc^2 \quad \text{or} \quad v = \frac{mc^2}{h} \quad \text{...(3)}$$

Substituting the value of equation (ii) and (iii) in equation (i)

$$V_p = v\lambda = \frac{mc^2}{h} \frac{h}{mv} = \frac{c^2}{v} \quad \text{...(4)}$$

but $v = V_g$

so form equation 4 we have

$$V_p V_g = c^2$$

Thus the product of phase velocity (V_p) and group velocity (V_g) is equal to the square of the velocity of light.

4-ADDITION OF SIMPLE HARMONIC MOTIONS AT RIGHT ANGLES

Consider the effect when two sine waves of the same frequency but having displacements in two perpendicular directions act simultaneously at a point. Choosing the directions as y and z, we may express the two component motions as

$$y = a_1 \sin(\omega t - \alpha_1) \qquad \text{and} \qquad z = a_2 \sin(\omega t - \alpha_2) \tag{1}$$

These are to be added, according to the principle of superposition, to find the path of the resultant motion. One does this by eliminating t from the two equations, obtaining

$$\frac{y}{a_1} = \sin \omega t \cos \alpha_1 - \cos \omega t \sin \alpha_1 \tag{2}$$

$$\frac{z}{a_2} = \sin \omega t \cos \alpha_2 - \cos \omega t \sin \alpha_2 \tag{3}$$

MultiplyingEq.(2)by $\sin \alpha_2$ and Eq. (3) by $\sin \alpha_1$ and subtracting the first equation from the second gives

$$-\frac{y}{a_1} \sin \alpha_2 + \frac{z}{a_2} \sin \alpha_1 = \sin \omega t(\cos \alpha_2 \sin \alpha_1 - \cos \alpha_1 \sin \alpha_2) \tag{4}$$

Similarly, multiplying Eq (2) by $\cos \alpha_2$ and Eq. (3) by $\cos \alpha_1$, and subtracting the second from the first, we obtain

$$\frac{y}{a_1} \cos \alpha_2 - \frac{z}{a_2} \cos \alpha_1 = \cos \omega t(\cos \alpha_2 \sin \alpha_1 - \cos \alpha_1 \sin \alpha_2) \tag{5}$$

We can now eliminate t from Eqs.(4) and (5) by squaring and adding these equations. This gives

$$\sin^2(\alpha_1 - \alpha_2) = \frac{y^2}{{a_1}^2} + \frac{z^2}{{a_2}^2} - \frac{2yz}{a_1 a_2} \cos(\alpha_1 - \alpha_2) \tag{6}$$

When $\delta = \pi/2, 3\pi/2, 5\pi/2, \ldots$, as can readily be seen from Eq.(6) In this case

$$\boxed{\frac{y^2}{{a_1}^2} + \frac{z^2}{{a_2}^2} = 1}$$

which is the equation of an ellipse with semiaxes a_1 and a_2, coinciding with the y and z axes, respectively.

When $\delta = 0, 2\pi, 4\pi, \ldots,$ In this case we have

$$\boxed{y = \frac{a_1}{a_2} z}$$

which is representing a straight line passing through the origin, with a slope a_1/a_2.

If $\delta = \pi, 3\pi, 5\pi, \ldots,$

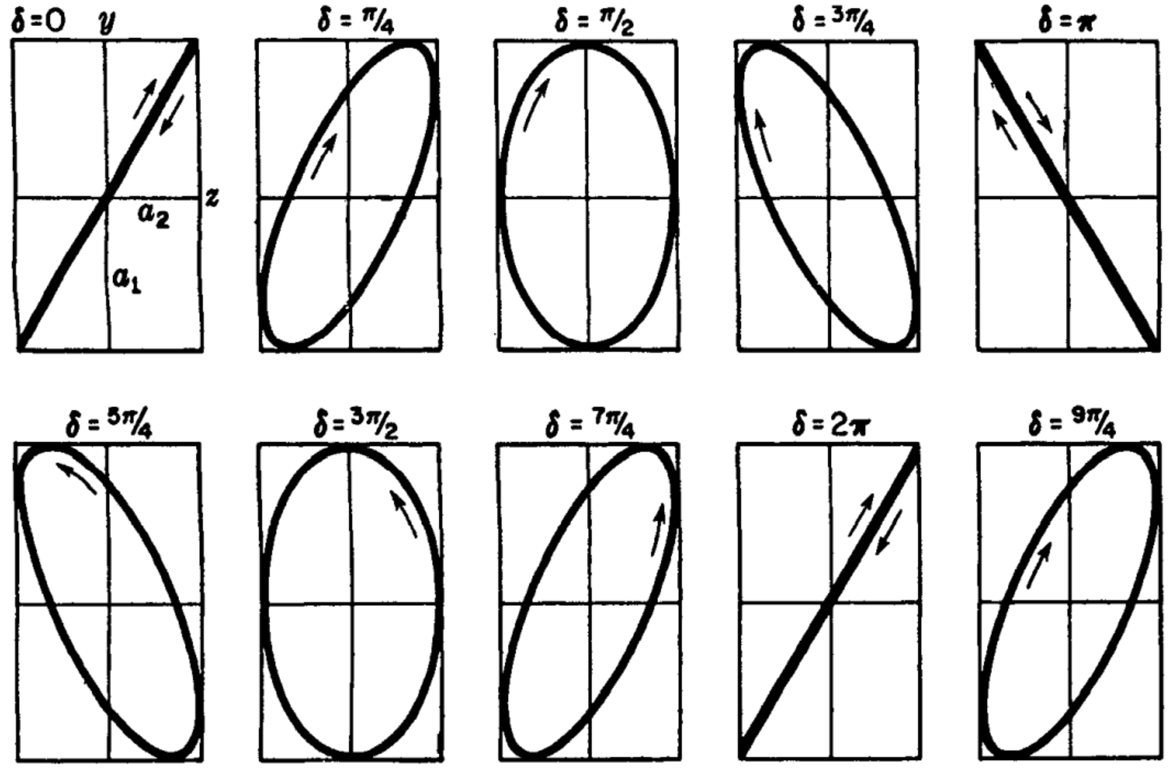
$$\boxed{y = -\frac{a_1}{a_2} z}$$

a straight line with the same slope, but of opposite sign.

A special case occurs when the amplitudes a_1 and a_2 of the two waves are equal and the phase difference (δ) is an odd multiple of $\pi/2$.

The vibration form is then a circle,

When the direction of rotation is clockwise $(\delta = \pi/2, 5\pi/2, \ldots)$
while if the rotation is counterclockwise $(\delta = 3\pi/2, 7\pi/2, \ldots)$



Composition at right angles of two simple harmonic motions of the same frequency but different phase.

PROBLEMS

12.1 Two waves traveling together along the same line are given by $Y1 = 5 \sin(wt+ \pi/2)$ and $Y2 = 7 \sin (wt+ \pi/3)$. Find (a) the resultant amplitude, (b) the initial phase angle of the resultant, and (c) the resultant equation of motion.

Ans. (a) 11.60, (b) 72.4°, (c) $Y = 11.60 \sin (wt + 72.4°)$

2. Two waves traveling together along the same line are represented by $Y1 = 25 \sin (wt+ \pi/4)$ and $Y2 = 15 \sin (wt+ \pi/6)$ Find (a) the resultant amplitude, (b) the initial phase angle of the resultant, and (c) the resultant equation for the sum of the two motions.

3.Two waves having amplitudes of 5 and 8 units and equal frequencies come together at a point in space. If they meet with a phase difference of $5 \pi /8$ rad, find the resultant intensity relative to the sum of the two separate intensities.

4.Two sources vibrating according to the equations $Yl = 4 \sin 2\pi t$ and $Y2 = 3 \sin 2\pi t$ send out waves in all directions with a velocity of 2.40 m/s. Find the equation of motion of a .particle 5 m from the first source and 3 m from the second. Note: $w= 2\pi$ rad/s.

Ans. $\boldsymbol{Y = 6.08 \sin(2\pi t - 25.3°)}$

5. Standing waves are produced by the superposition of two waves,

YI = 7 sin 2π (t / T - 2x / π) and Y2 = 7 sin 2π (t /T + 2x/π) traveling in opposite directions. Find (a) the amplitude, (b) the wavelength $\boldsymbol{\lambda}$, (c) the length of one loop, (d) the velocity of the waves, and (e) the period.

6. Prove that for water waves controlled by gravity the group velocity equals half the wave velocity.=

7. The phase velocity of waves in a certain medium is represented by $v = C_1 + C_2\lambda$ where the C_S are constants. What is the value of the group velocity? ***Ans.*** $\boldsymbol{u = C_1}$

Chapter 3

INTERFERENCE OF TWO BEAMS OF LIGHT

In this chapter we will discuss the Interference of Light represented in the following topic:

- ✓ **Coherent and Incoherent light sources.**
- ✓ **Huygens' Principle**
- ✓ **Interference of light.**
- ✓ **Interference by Division of Wave Front including:**
  1. Young’s Double-Slit Experiment,
  2. Fresnel's Biprism,
  3. Fresnel's Mirror
  4. Lloyd’s Mirror

- ✓ **Interference by Division of Amplitude including:**
  1. Michelson Interferometer.
  2. Tweeman & Green Interferometer.
  3. Jamin Interferometer
  4. Mach-Zehnder Interferometer,
  5. Raylegh's interferometer.

- ✓ **Interferometer by a Multiple Reflection:**
  1. Newton's Rings.
  2. Two Parallel Thin Film
- ✓ **Resolving power: meaning and application.**
- ✓ **Application of interference including:**
  1. Measure refractive Index.
  2. Measure film thickness

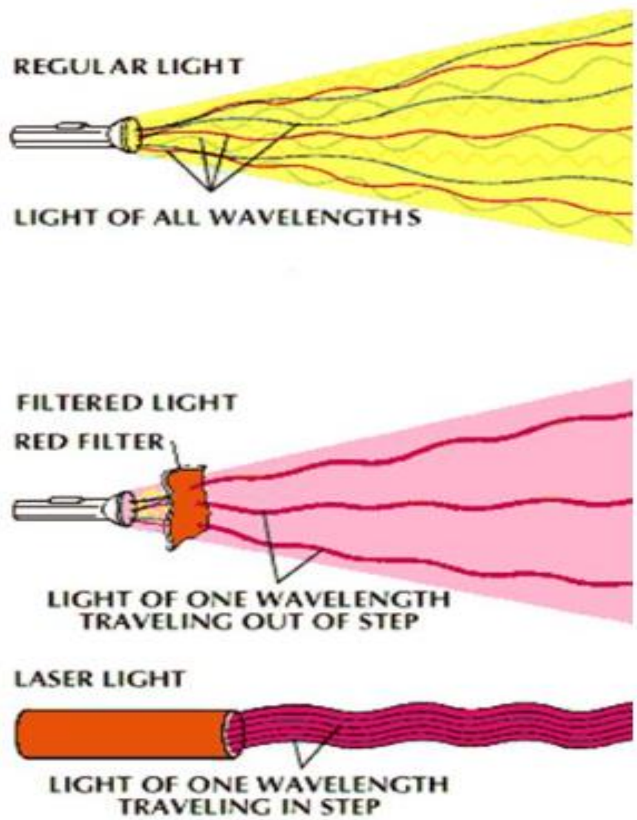
Coherent and Incoherent light sources.

https://www.toppr.com/ask/question/light-of-wavelength-65times-107-meters-is-made-incident-on-two-slits-1mm-apart-the/

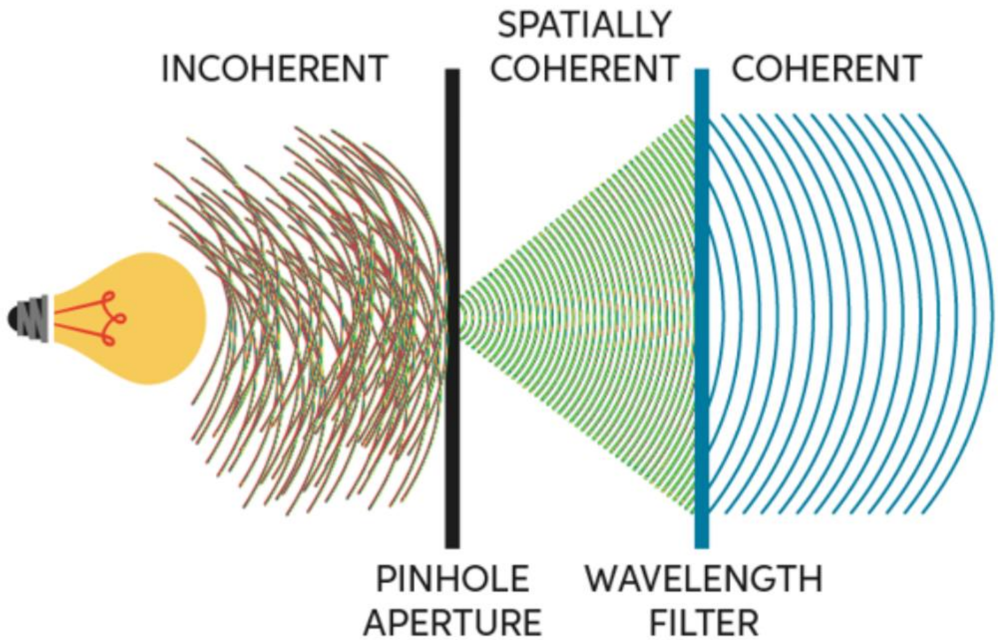
Conventional sources of light are incoherent in nature. The waves of any energy are incoherent whence the phase difference is constant between energy particles. As an example, both fluorescent tubes as well as tungsten lamps are incoherent light sources. Additionally, incoherent or non-coherent also has amplitude, fluctuating randomly in time and space.

Two light source waves with a constant phase difference, same amplitude, frequency and waveforms are claimed as perfectly coherent in nature.

As pre-mentioned, incoherent sources have random fluctuations in their parameters. This could be explained as the transitions in between the existing energy levels configured within an atom are entirely random. Thus, there is no control or prediction as in, when an atom might emit energy through radiation.

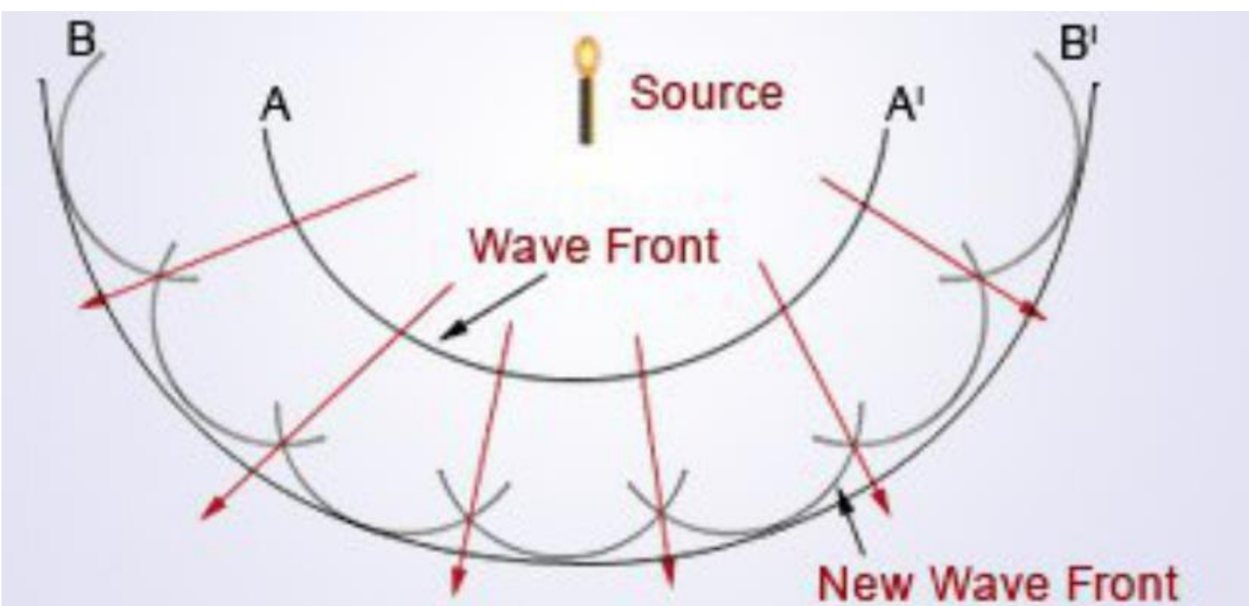


The light waves from the LASER beams are coherent, monochromatic, collimated and parallel in nature. However, any normal light source could be made coherent, if it is made smaller thus, reducing the number of atoms that would emit the quanta. This, however, would reduce the intensity. Researchers have an idea of using the atoms and molecules for the resonant source or structure but the intermittent emission from one electron is small enough to be available as power. The image shows the conversion from an incoherent to a coherent source.



Huygen's principle

Huygen's principle states that every point on the wavefront may be considered a source of secondary spherical wavelets which spread out in the forward direction at the speed of light. The new wavefront is the tangential surface to all these secondary wavelets.



Advantages and Disadvantages of Huygens Principle

Advantages:

- Huygens concept proved the reflection and refraction of light.
- The concepts like diffraction of light, as well as interference of light, were proved by Huygens.

Disadvantage:

- Concepts like emission of light, absorption of light and polarization of light were not explained by Huygens principle.
- Huygens principle failed to explain the photoelectric effect.

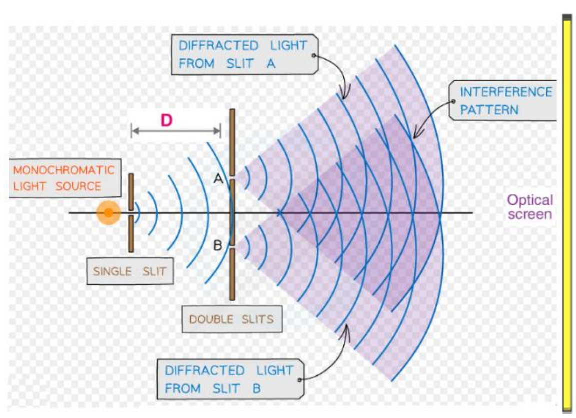
Interference by Division of Wave Front .

Under this category, the coherent sources are obtained by dividing the wavefront, originating from a common source, by employing mirrors, biprisms or lenses. This class of interference requires essentially a point source or a narrow-slit source. The instruments used to obtain interference by division of wavefront are the Fresnel biprism, Fresnel mirrors, Lloyd's mirror, lasers, etc.

1.Young's Double-Slit Experiment.

What is Young's Double Slit Experiment?

Young's double-slit experiment uses two coherent sources of light placed at a small distance apart. Usually, only a few orders of magnitude greater than the wavelength of light are used. Young's double-slit experiment helped in understanding the wave theory of light, which is explained with the help of a diagram. As shown, a screen or photodetector is placed at a large distance 'D' away from the slits. The original Young's double-slit experiment used diffracted light from a single source passed into two more slits to be used as coherent



sources. Lasers are commonly used as coherent sources in the modern-day experiments.

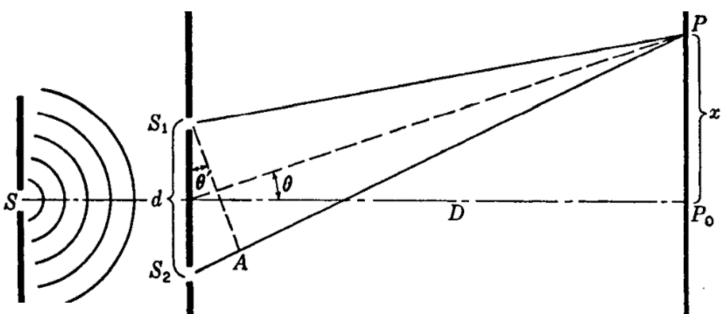
- When a monochromatic light source is placed behind a single slit, the light is diffracted producing two light sources at the double slits A and B.
- Since both light sources originate from the same primary source, they are **coherent** and will therefore create an observable interference pattern
- Both diffracted light from the double slits creates an interference pattern made up of bright and dark fringes.
- The wavelength of the light can be calculated from the interference pattern and experiment set up. These are related using the double-slit equation.

1.1 Interference Fringes from a Double Source.

We shall now derive an equation for the intensity at any point P on the screen Figure below and investigate the spacing of the interference fringes. Two waves arrive at P, having traversed different distances S_2P and S_1P.

Hence, they are superimposed with it phase difference given by:

$$\delta = \frac{2\pi}{\lambda}\Delta = \frac{2\pi}{\lambda}(S_2P - S_1P) \qquad (1)$$



It is assumed that the waves start out from SI and S2 in the same phase, because these slits were taken to be equidistant from the source slit S. Furthermore, the amplitudes are practically the same if (as is usually the case) SI and S2 are of equal width and very close together. The problem of finding the resultant intensity at P therefore reduces to that discussed in (chapter 2 – Section-1), where we considered the addition of two simple harmonic motions of the same frequency and amplitude, but of phase difference **δ**. The intensity was given by Equation below: see (chapter 2 – Section-1)

$$I \approx A^2 = 4a^2 \cos^2 \frac{\delta}{2} \qquad (2)$$

s the amplitude of the separate waves and **A** that of their resultant amplitude.

It now remains to evaluate the phase difference in terms of the distance **x** on the screen from the central point $\mathbf{P_0}$, the separation **d** of the two slits, and the distance **D** from the slits to the screen.

The corresponding path difference is the distance S_2A in Figure above, where the dashed line S_IA has been drawn to make S_I and **A** equidistant from **P**. As Young's experiment is usually performed, **D is some thousand times larger than d or x.**

Hence the angles **(θ)** and **(θ')** are very small and practically equal. Under these conditions, S_1AS_2 may be regarded as a right triangle, and the path difference becomes

$$d \sin\theta' \approx d \sin\theta$$

To the same approximation, we may set the sine of the angle equal to the tangent, so that

$$\sin\theta \approx x/D.$$

With these assumptions, we obtain:

$$\boxed{\Delta = d\sin\theta = d\frac{x}{D} \qquad (3)}$$

Equation (3) is the value of the path difference, and we will substitute it in Equation (1) to obtain the phase difference **δ** . Now from Eq. (2) .

As we proved in chapter 2 – Section-1, if the phase difference δ = 0 , 2π , 4 π ……. (Even number of π), this gives the intensity of $4a^2$.

That mean:

$$\frac{Xd}{D} = 0, \lambda, 2\lambda, 3\lambda, 4\lambda \dots\dots\dots = m\lambda$$

$$x = m\,\lambda\,\frac{D}{d} \dots\dots\dots\dots(4) \quad \textit{Bright fringes}$$

constructive interference

As we proved in chapter 2 – Section-1, if the phase difference δ = π , 3 π, 5 π, ... (Odd number of π) , this gives the intensity of zero.

That mean:

$$\frac{Xd}{D} = \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2}, \frac{7\lambda}{2} \dots \dots \dots .. = \left(m + \frac{1}{2}\right)\lambda$$

$$x = \left(m + \frac{1}{2}\right)\lambda \frac{D}{d} \dots \dots .. (5) \quad Dark\ fringes$$

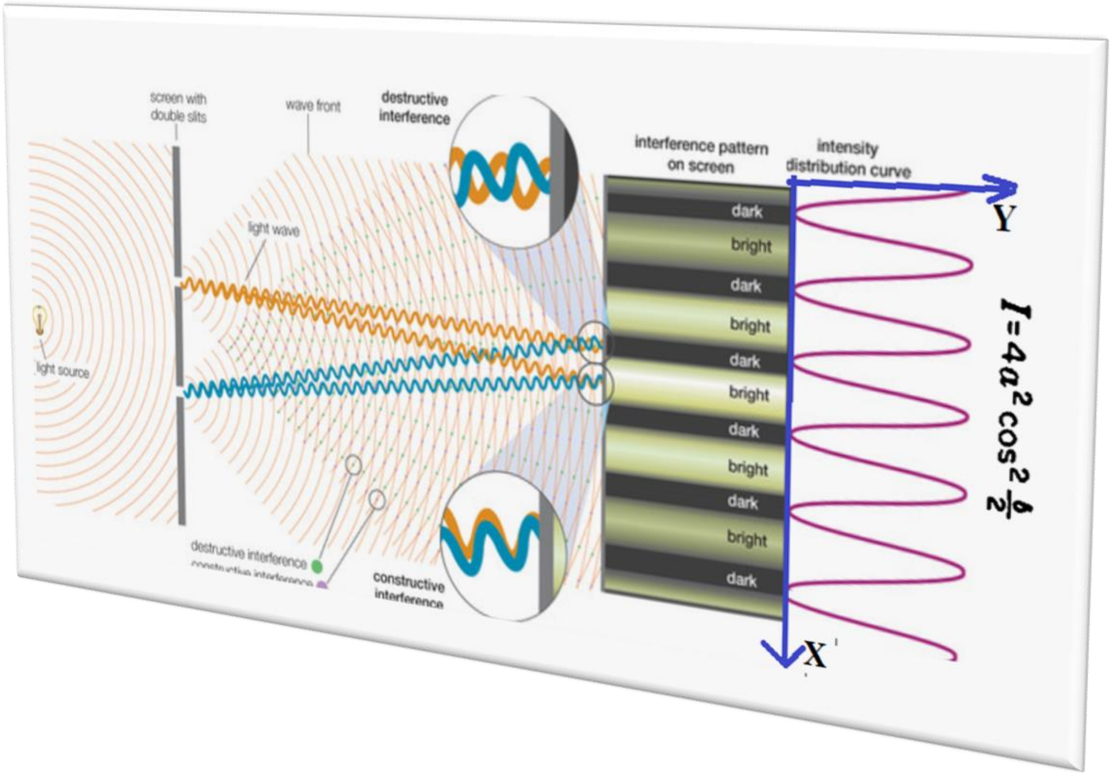
Destructive interference

The whole number m, which characterizes a particular bright fringe, is called the order of interference. Thus, the fringes with m = 0, 1,2,3,4 ... are called the zero, first, second, etc., orders.

1.2 INTENSITY DISTRIBUTION, IN THE FRINGE SYSTEM.

As shown in Figure below the intensity in the interference pattern varies between $4a^2$ and zero.

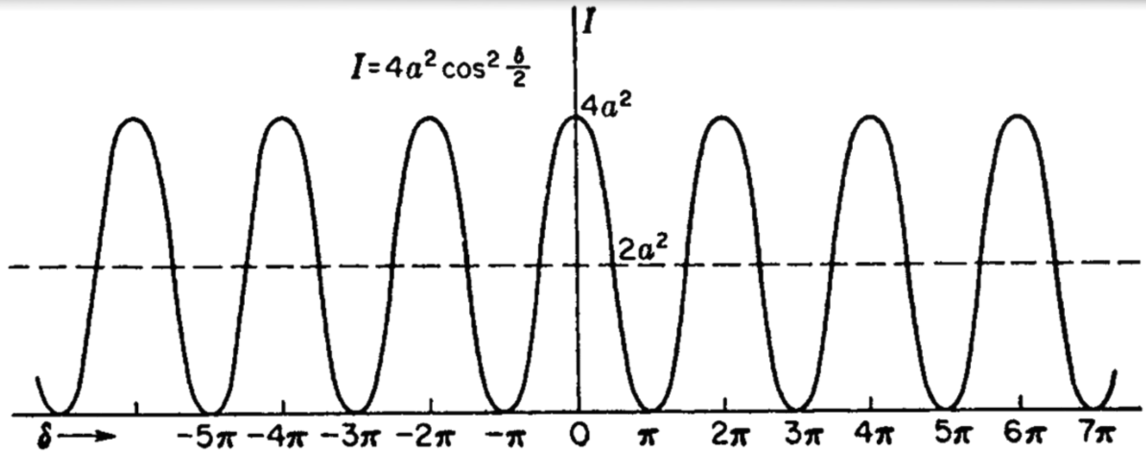
If the two beams of light arrive at a point on the screen exactly in same phase, we will get constrictive interference. If the two beams of light arrive at a point on the screen exactly out of phase, we will get destructive interference.



One may well ask what becomes of the energy of the two beams?

since the law of conservation of energy tells us that energy cannot be destroyed. The answer to this question is that the energy which apparently disappears at the minima is still present at the maxima, where the intensity is greater than would be produced by the two beams acting separately.

In other words, the energy is not destroyed but merely redistributed in the interference phenomena. The average intensity on the screen is exactly that which would exist in the absence of interference.



Intensity distribution for the interference fringes from two waves of the same frequency.

Example 1: A pair of screens are placed 13.7m apart. A third order fringe is seen on the screen 2.50cm from the central fringe. If the slits were cut 0.0960 cm apart, **determine** the wavelength of this light. Roughly what colour is it?

Example 2 Suppose you pass light from a He-Ne laser through two slits separated by 0.0100 mm and find that the third bright line on a screen is formed at an angle of 10.95 degrees relative to the incident beam. What is the wavelength of light?

PROBLEMS

1. Young's experiment is performed with orange light from a krypton arc. If the fringes are measured with a micrometer eyepiece at a distance 100 cm from the double slit, it is found that 25 of them occupy a distance of 12.87 mm between centers. Find the distance between the centers of the two slits.

2. A double slit with a separation of 0.250 mm between centers is illuminated with green light from a cadmium-arc lamp. How far behind the slits must one go to measure the fringe separation and find it to be 0.80 mm between centers?

3. When a thin film of transparent plastic is placed over one of the slits in Young's double-slit experiment, the central bright fringe, of the white-light fringe system, is displaced by 4.50 fringes. The refractive index of the material is 1.480, and the effective wavelength of the light is 5500 A. (a) By how much does the film increase the optical path? (b) What is the thickness of the film? (c) What would probably be observed if a piece of the material 1.0 mm thick were used? (d) Why?

2. Interfernce by Fresnel's Biprism

A schematic diagram of the biprism experiment is shown in Figure 2.1. The thin double prism P refracts the light from the slit sources S into two overlapping beams **ae** and be.

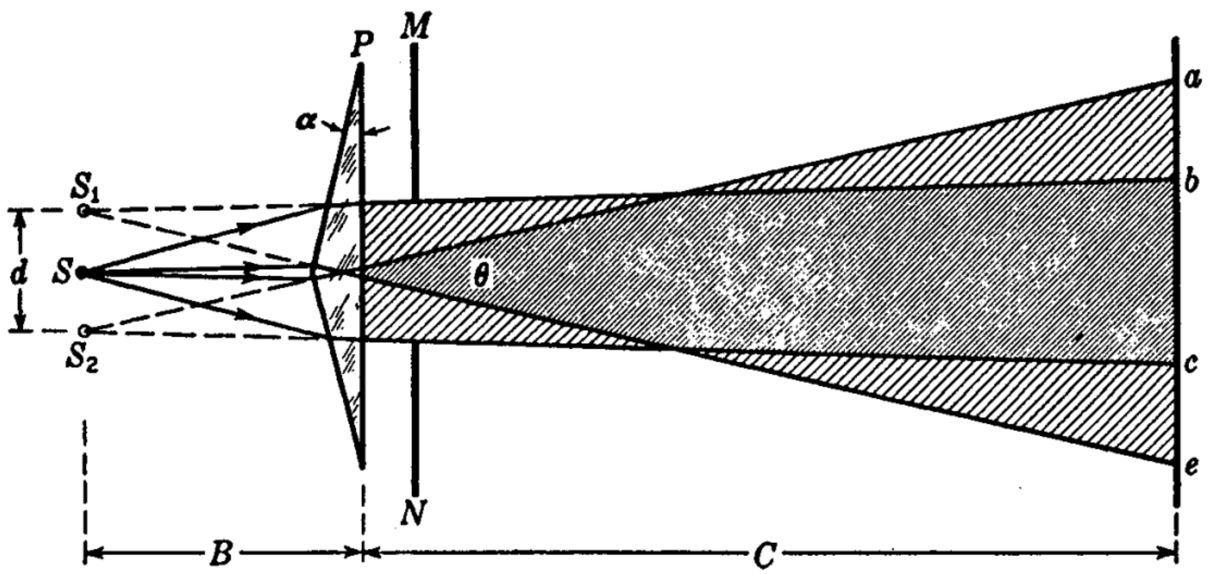


Figure 2.1 Diagram of Fresnel's biprism experiment.

If screens M and N are placed as shown in the Figure 2.1, interference fringes are observed only in the region be. When the screen ae is replaced by a photographic plate, a picture like the Figure 2.2-A is obtained. The closely spaced fringes in the center of the photograph are due to interference, while the wide fringes at the edge of the pattern are due to diffraction as shown Figure 2.2 B.

Just as in Young's double-slit experiment, the wavelength of light can be determined from measurements of the interference fringes produced by the biprism. Calling B and C the distances of the source and screen, respectively.

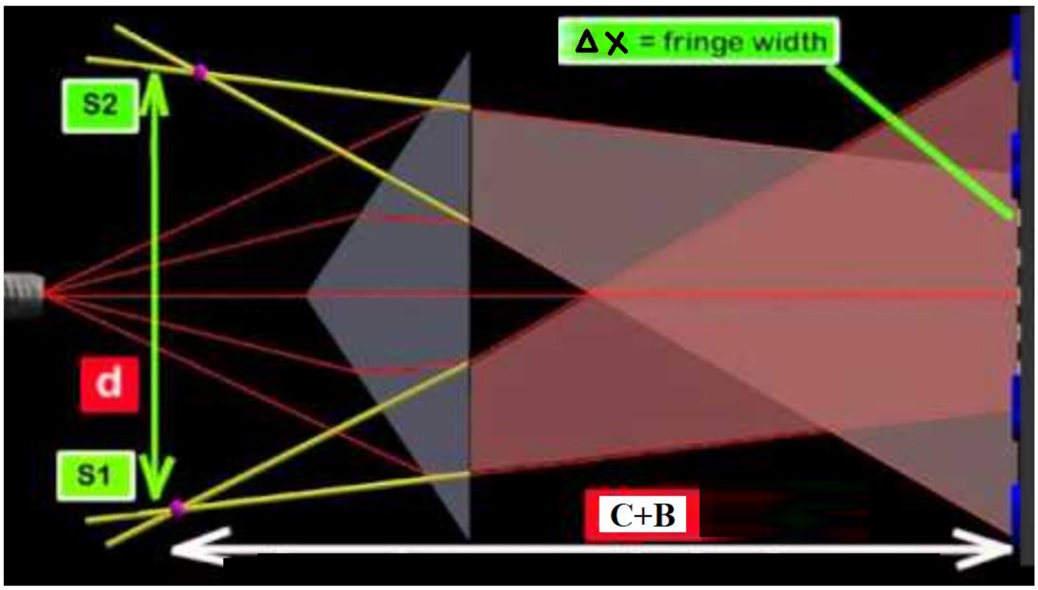
from the prism P: d is the distance between the virtual images S_1 and S_2 , and Δx the distance between the successive fringes on the screen, the wavelength of the light is given from equation (4) as:

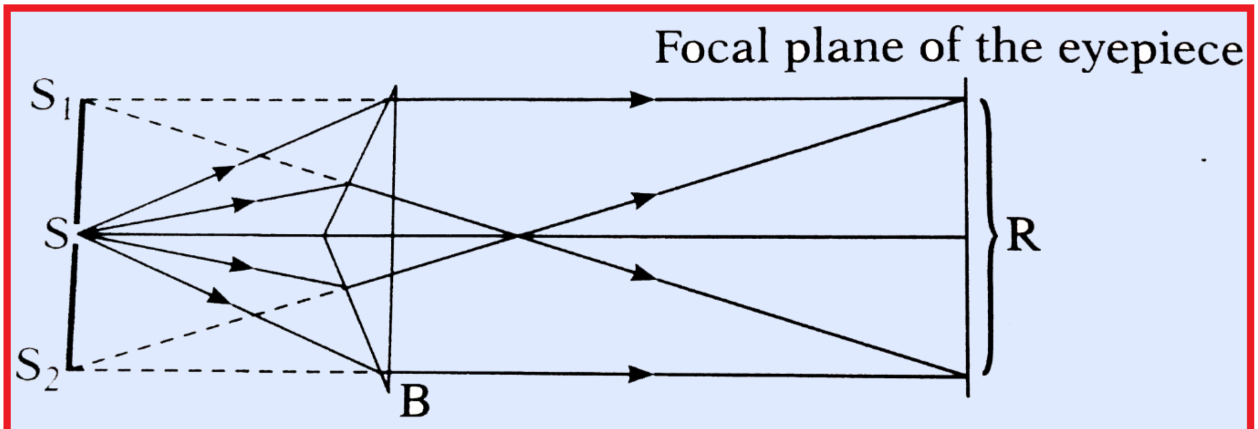
$$\lambda = \frac{\Delta x\ d}{C + B}$$

Thus, the virtual images S_1 and S_2 act like the two slit sources in Young's experiment.

To find d, the linear separation of the virtual sources, one can measure their angular separation θ on a spectrometer and assume, to sufficient accuracy, that d = Bθ.

If the parallel light from the collimator covers both halves of the biprism, two images of the slit are produced and the angle θ between these is easily measured with the telescope.





S : Source (a slit illuminated by monochromatic light),
B : Biprism, S_1, S_2 : Virtual sources,
R : Region of interference

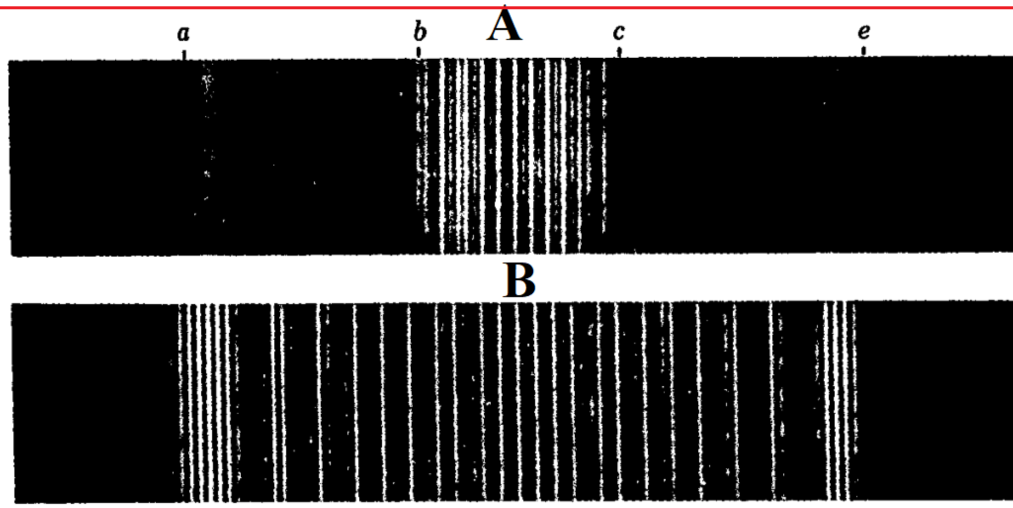
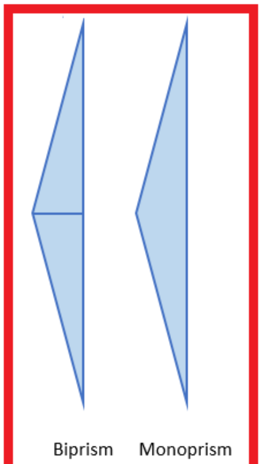


Figure 2.2 Interference and diffraction fringes produced in the Fresnel biprism experiment.

Question?

Why is the Fresnel Biprism made of two separate prisms joined together at their bases? Why not use a monoprism, a single prism having the same overall geometric and optical properties as that of the combination as shown in the following diagram?

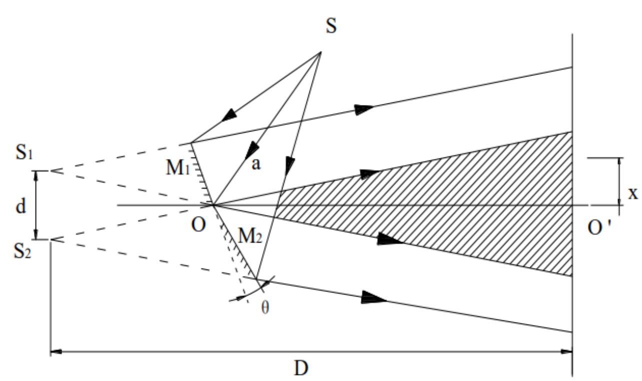


OTHER APPARATUS DEPENDING ON DIVISION OF THE WAVE FRONT INCLUDING:

A- Fresnel's Mirror

Fresnel's Mirrors have a structure as shown in Figure . Two plane mirrors M_1 and M_2 are orientated with a very small variable angle. Light from point source S is incident on the two mirrors, and the reflection form two virtual images S_1, S_2 of light source S, which act as coherent sources .

If $SO = a$, then $S_1O = S_2O = a$. The distance between S_1 and S_2 is d



Schematic of Fresnel's double mirror interference

$$d = 2a\sin\theta \qquad (1)$$

where θ is the angle between the mirrors. As in Young's experiment, we get the formulae:

$$d\frac{x}{D} = \pm(2k+1)\frac{\lambda}{2} \qquad (2)$$

(Dark interference fringes),

$$d\frac{x}{D} = \pm k\lambda \qquad (3)$$

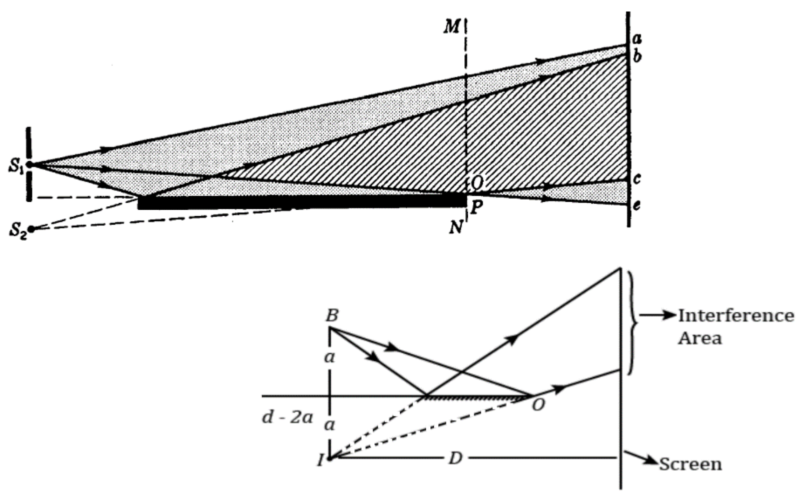
(Bright interference fringes),

$$\lambda = \frac{d}{D}\Delta x = \frac{2a\sin\theta}{a\cos\theta + OO'}\Delta x \approx \frac{2a\theta}{a + OO''}\Delta x \qquad (4)$$

B- **Lloyd's Mirror**

The purpose behind describing such a mirror was to provide more evidence to illustrate the wave nature of light. Produces interference between the light reflected in one long mirror and the light coming directly from the source without reflection. In this arrangement, known as Lloyd's mirror, the quantitative relations. Similar to other cases, the slit and its virtual image constituting the double source. As can be seen in the figure below.

From the figure we can see that in this arrangement a monochromatic light source illuminates a slit. The light from the slit is partly incident on a mirror and the rest of it reaches directly on the screen. The incident light gets reflected from the mirror and reaches the screen. If we produce back the reflected light rays, they appear to diverge from a virtual source. Hence, and behave as coherent sources and we get interference patterns in the overlapping region. O is the geometrical centre and the distance between and is taken as d. Hence, we can write $OS_1=OS_2$. The central point O receives only the direct light and therefore no interference pattern is seen here. The path difference at this point is zero and the bright central fringe is not visible here. Hence the central fringe is dark. We know that whenever there is a dark fringe it means destructive interference takes place at that point. Destructive interference means that the crest of one wave overlaps with the trough of another wave and there is a phase difference of 180 degree. This mirror is used to produce a two-source interference pattern. A laser can be used as a monochromatic light source to illuminate the slit . By varying the separation between the laser and the mirror we can make changes in the interference pattern on the screen. Here the incident light travels with a constant phase difference throughout the experiment. This means that the phase difference of arises due to the reflected ray. The reflected ray travels with a phase difference of than the incident ray.



Lloyd's mirror.

Condition for Maxima and minima

We know that the condition for maxima and minima defends on the path difference.

- *Condition for maxima*

Path difference=(2*m*+1) *λ/2*   Where m=0,1,2,3…...

- Condition for minima

$Path\ difference = m\,\lambda$ Where m=0,1,2,3…….

The conditions for maxima and minima in case of Lloyd's mirror is just the opposite of Fresnel's biprism.

Applications

1. Interference lithography- UV Photolithography and nanopatterning are the two most common applications of Lloyd's mirror. Lloyd's mirror has lots of advantages over double-slit interferometers.
2. It has many more applications like in Test pattern generation.
3. Optical measurement etc.

2-Interference by Division of Amplitude including:

The Michelson interferometer is an important example of this second class.

2.1 Michelson Interferometer. (Construction and works)

The arrangement of the ***Michelson Interferometer system*** is shown schematically in Figure 1 below.

The optical system consist of two highly polished plane mirrors M_1 and M_2 and two plane-parallel plates of glass G_1 and G_2. Sometimes the rear side of the plate G_1 is lightly silvered (shown by the heavy line in the figure) so that the light coming from the source S is divided into (1) a reflected and (2) a transmitted beam of equal intensity. In this process the amplitudes have divide as two sources.

The light reflected normally from mirror M_1 passes through G_1 a third time and reaches the eye as shown. The light reflected from the mirror M_2 passes back through G_2 for the second time, is reflected from the surface of G_1 and into the eye (Telescope).

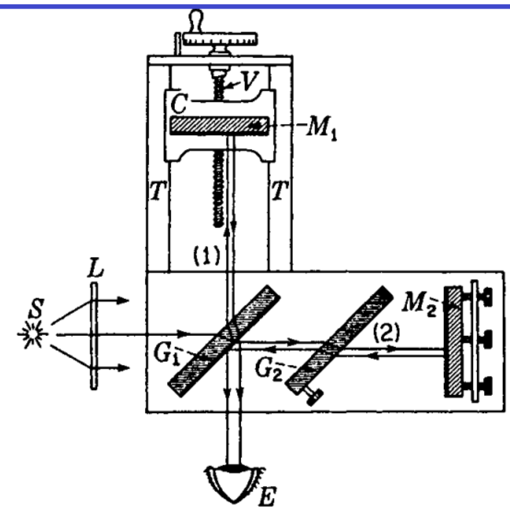


FIGURE 1 Diagram of the Michelson interferometer.

The purpose of the plate G_2 , called the ***compensating plate***, is to make the path in glass of the two rays equal.

To obtain the fringes, there are some conditions should be available:

- ✓ Using a monochromatic light source or laser.
- ✓ The light must originate from an extended source. ((A point source or a slit source, as used before with Young experiment, will not produce the desired system of fringes in this case. We will discuss this reason later)).
- ✓ The distances of M_1 and M_2 to the back surface of half reflected plate G_1 are the same by moving M_1.
- ✓ The mirror M_2 should adjust to be perpendicular to M_1.
- ✓ The plate G_1 be at 45° with incident light.

Important note An **extended source** suitable for use with a Michelson interferometer may be obtained in anyone of several ways. *A sodium flame* or a *mercury are*, if large enough, may be used without *a glass screen or a lens L* as shown in Figure 1. If the source is small, a ground-glass screen or a lens at L will extend the field of view.

The mirror M_1 is mounted on a carriage C and can be moved along the well-machined ways or tracks T. This slow and accurately controlled motion is accomplished by means of the screw V, which is calibrated to show the exact distance the mirror has been moved. To obtain fringes, the mirrors M_1 and M_2 are made exactly perpendicular to each other by means of screws shown on mirror M_2 .

Under these conditions, the interference pattern is a series of bright and dark parallel fringes as described in Example figure (2-b). The tilting screws on M_2 are turned until one pair of images falls exactly on the other figure (2-b), the interference fringes should appear. When they first appear, the fringes will not be clear unless the eye is focused on the mirror M_1 so the observer should look constantly at this mirror while searching for the fringes.

As M_1 is moved see figure 3, the fringe pattern shifts. For example, if a dark fringe appears in the field of view (corresponding to destructive interference) and M_1 is then moved a distance $\lambda/4$ toward M_2, the path difference changes by $\lambda/2$ see figure (2-c).

As M_1 is moved an additional distance λ /4 toward M, the bright fringe becomes a dark fringe. Thus, the fringe pattern shifts by one-half fringe each time M_1 is moved a distance λ /4. The wavelength of light is then measured.

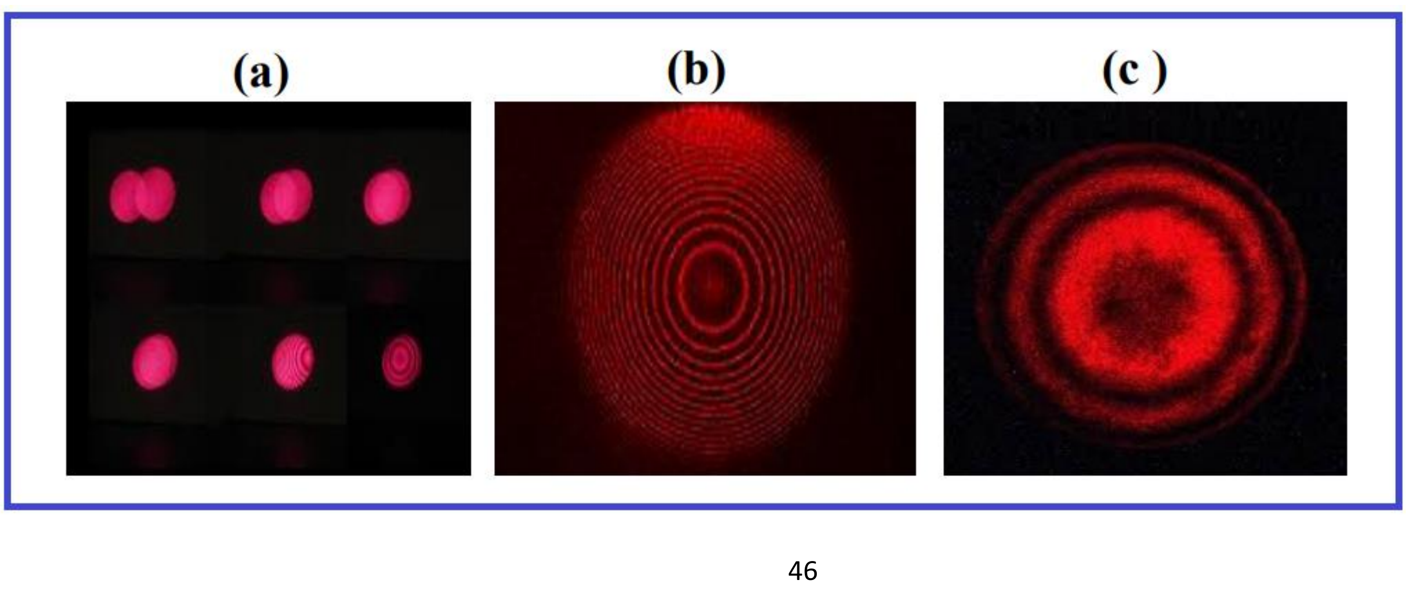


Figure (2) : a) A pair of images, b) arrangement two image and Circular fringes are produced with laser light when the mirrors M1 and M2 are exactly perpendicular to each other.(c) As M1 is moved, the fringe pattern shifts. For example, if a dark fringe appears in the field of view as forward or backward.

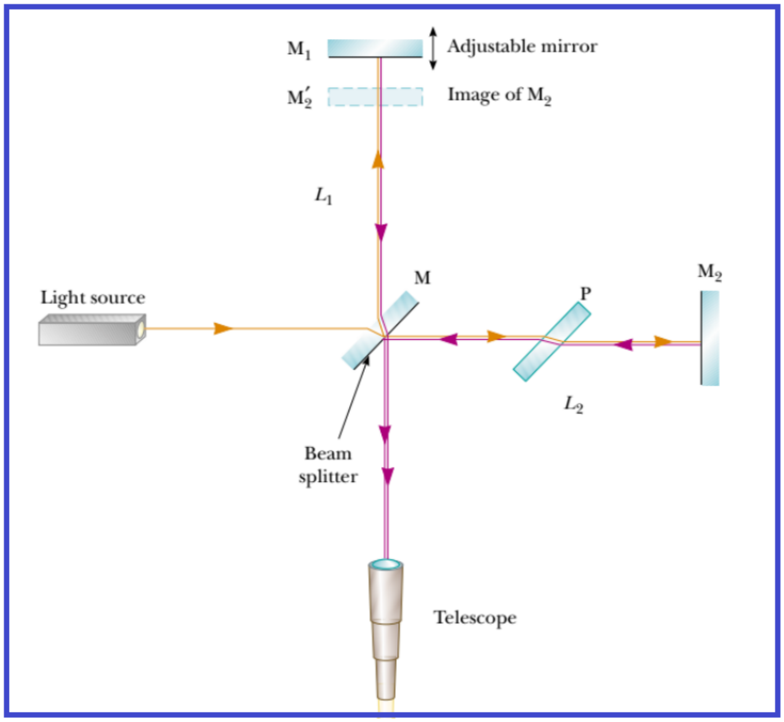


Figure (3) Adjustment of Michelson Interferometer.

Optical path

The two waves will interfere constructively or destructively as per the following conditions of Path difference Δ:

$$\Delta = \frac{2m\lambda}{2} = m\lambda \quad \text{- - - - (1)}$$ **For Bright Frings (constructive interference)**

$$\Delta = \frac{(2m+1)}{2}\lambda \quad \text{- - - - - (2)}$$ **For Dark Frings (distractive interference)**

The fringe order (m) is integer number (m=±1, ±2, ±3, ±5. . . . . ± m)

2.2 Types of fringes

2.2 .1 CIRCULAR FRINGES (fringes of equal inclination)

Circular fringes are produced with monochromatic light when the mirrors M1 and M2 are exactly perpendicular to each other. Once the conditions above are satisfied, the circular fringes can be produces. To understand how the fringes generated see the diagram of Figure 4. Figure (4) form of circular fringes (Concentric circular fringes) as known as fringes of equal inclination.

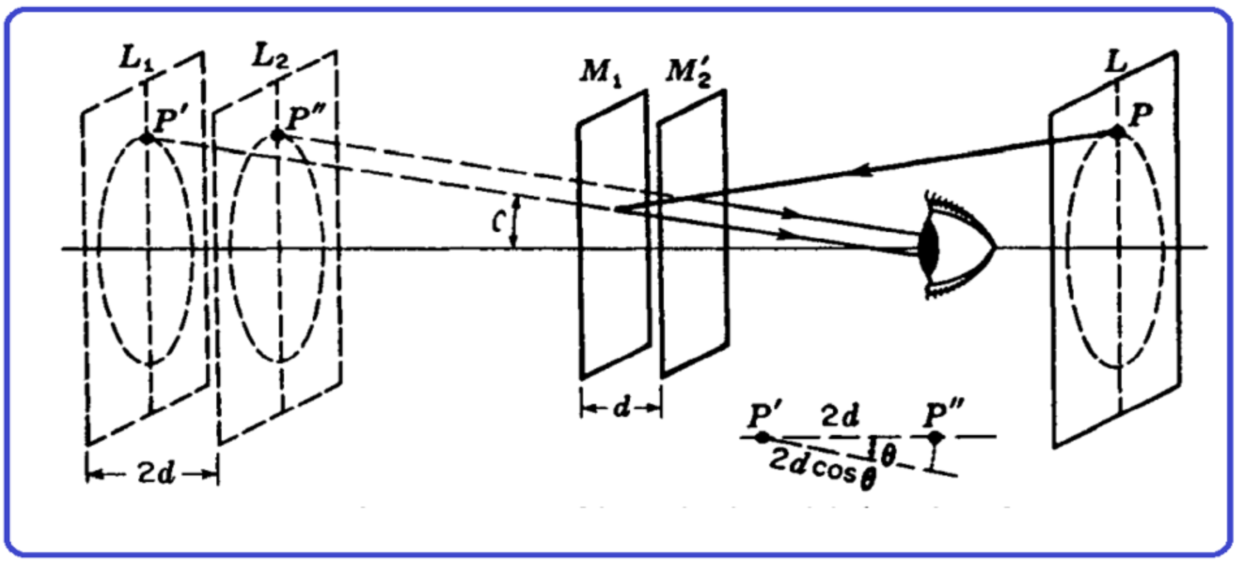


Figure (4) form of circular fringes (Concentric circular fringes)

Here the real mirror M_2 can be replaced by its virtual image $\acute{M}2$ formed by reflection in G_1• $\acute{M}2$ is then parallel to M_1. Due to several reflections in the real interferometer, we may now think of the extended source as being at L, behind the observer, and as forming two virtual images L_1 and L_2 in M_1 and $\acute{M}2$. These virtual sources are coherent in that the phases of corresponding points in the two are exactly the same at all instants. If d is the separation space between M_1 and $\acute{M}2$ the virtual sources will be separated by 2d. When d is exactly an integral number of half wavelengths, i.e., the path difference 2d equal to an integral number of whole wavelengths, all rays of light reflected normal to the mirrors will be in phase while the rays of light reflected at an angle will in general not be in phase. The path difference between the two rays coming to the eye from corresponding points P' and P'' is **2d cos θ**. The angle **θ** is necessarily the same for the two rays when M_1 is parallel to $\acute{M}2$ so that the rays are parallel. Hence the rays will reinforce each other to produce maxima for those angles that satisfying the relation:

$$2d\cos\theta = m\lambda \ ---3$$ **For Bright Frings (constructive interference)**

Since for a given **m**, $\boldsymbol{\lambda}$, and **d,** the angle **θ** is constant, the maxima will lie in the form of circles about the foot of the perpendicular from the eye to the mirrors.

The intensity distribution across the rings follows Equation (*),

$$I \approx A^2 = 4a\,cos^2\,\frac{\delta}{2}(*)$$

Where the phase difference is given by:

$$\delta = \frac{2\pi}{\lambda}\,2d\cos\theta \ ----3$$

Fringes of this kind, where parallel beams are brought to interference with a phase difference determined by the angle of inclination θ, are often referred to as fringes of equal inclination figure 6-a.

2.2 .2 Curved fringes (fringes of equal thickness):

When M1 and virtual image M2' are inclined to each other, the film enclosed is wedge shaped. Then curved fringes can be observed as shown in Figure 5 or Figure 6-b . These are also known as fringes of equal thickness.

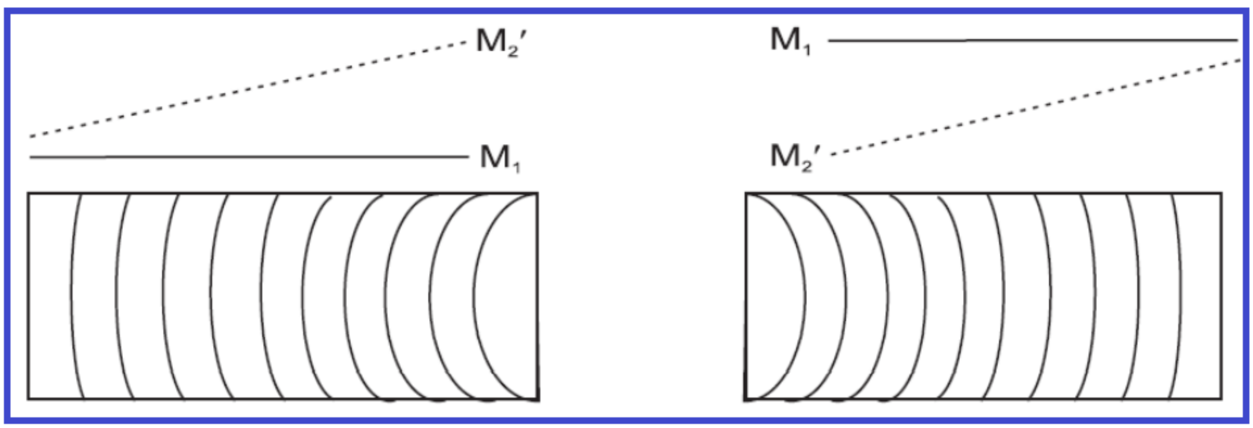


Figure 5: form fringes of equal thickness

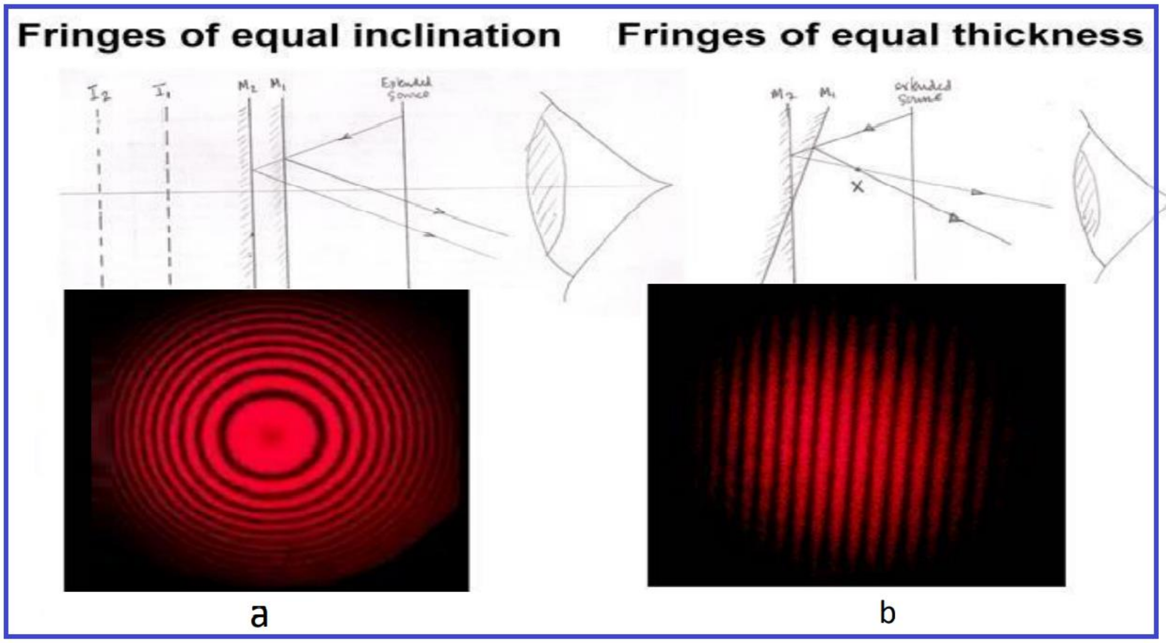


Figure 6: (a) Circular Fringes (fringes of equal inclination). (b) fringes of equal thickness

2.2.3 Straight line fringes:

When M1 and virtual image M2' intersect, straight line fringes are obtained around the point of intersection (see Figure 7).

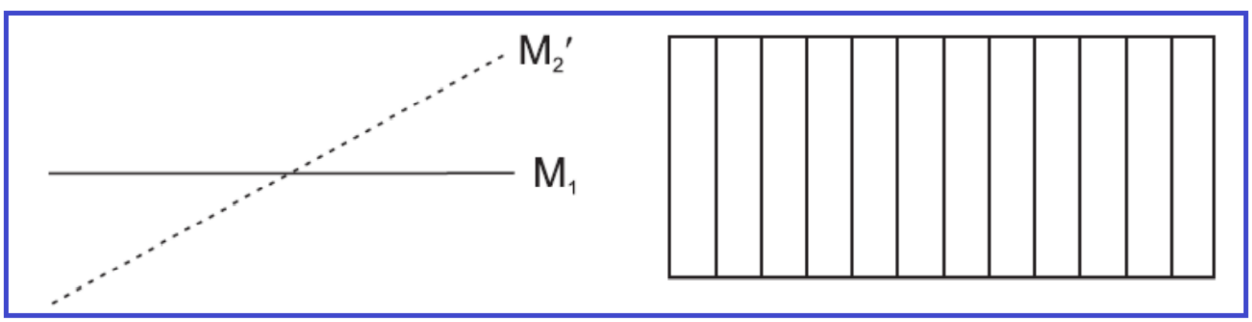


Figure 7: shape of Straight-line fringes

2.2.4 White light fringes:

If white light is used, the central fringe will be dark and other will be colored. The fringes are observed only when the path difference is small. These fringes are important because they are used to locate the position of zero path difference.

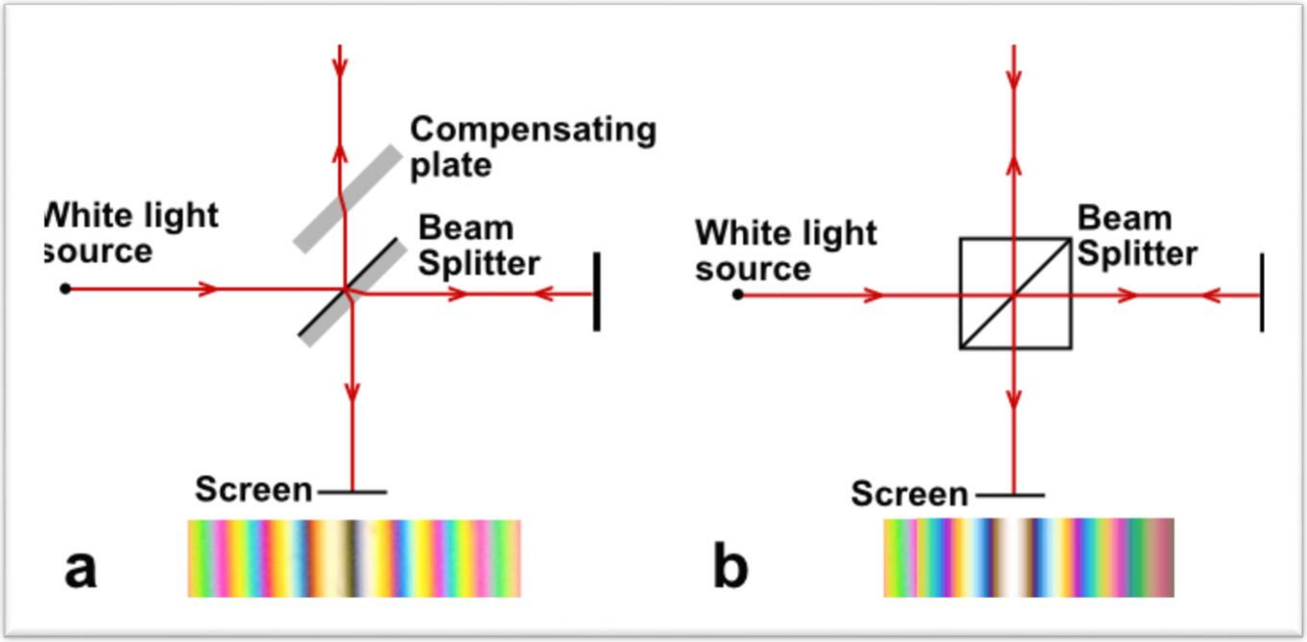


Figure 8:Micheslon' interferometer using white light: a)with compensating plate and beam splitter b) with beam splitter only.

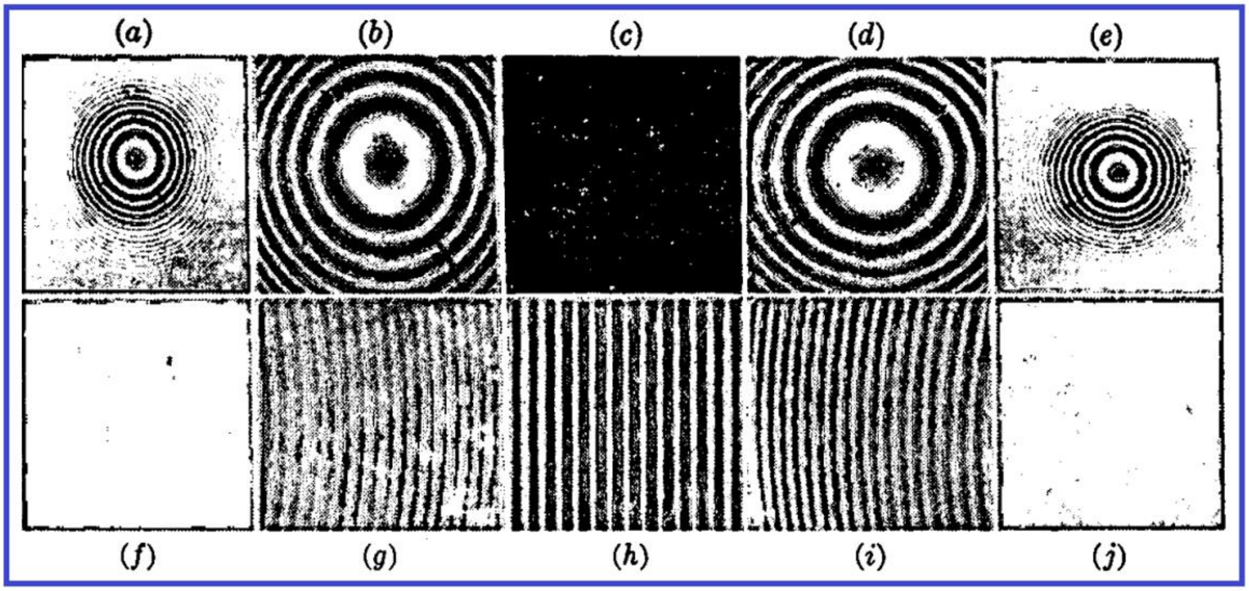


Figure 9: Appearance of the various types of fringes observed in the Michelson interferometer. Upper row, circular fringes. Lower row, localized fringes. Path difference increases outward, in both directions, from the center.

Example: Assume your laser has a wavelength of 560 nm, and your mirrors start at equal distances from the beam splitter. You shift your movable mirror by 1.5876 mm.

a) What's the path difference your two mirrors now?

b) It the center of the image on your detector dark, or bright?

Solution:

Steps 1) The unit is nm , convert to nm 1587.6 nm

2) The light beam must travel to the movable mirror and back,

So multiple the distance that the mirror shifted by 2 2x1587.6= 3175.2 nm

Therefore, the path difference is 3175.2 nm.

This number. an odd integer multiple of ($\lambda/2$) (in which case it would cause destructive interference and result in darkness in the center), or an integer multiple of λ (which would cause constructive interference and a result in a bright center)

Step 2:

3175.2 nm/560 nm =5.67≈ 6

5.67 is an integer, so it's constructive interference resulting in a bright center on the image at the detector.

Note: Limiting path difference as determined by the length of wave packets.

2.3 VISIBILITY OF THE FRINGES

There are three principal types of measurement that can be made with the interferometer:
(I) width and fine structure of spectrum lines,
(2) lengths or displacements in terms of wavelengths of light,
(3) refractive indices In Michelson's interferometer,
the intensity is given by:

The intensity distribution across the rings given by:

$\boldsymbol{I = 4I_0\, cos^2\, \frac{\delta}{2}}$ere the phase difference is given by:

$$\boldsymbol{\delta = \frac{2\pi}{\lambda}\ 2d\ \cos\theta}$$

Where d: distance between M1 & M2'.the intensity is max. When **δ** is an integral multiple of . The intensity is zero when is an odd multiple of $\boldsymbol{\pi}$.

Michelson's interferometer measured the visibility, defined as

$\boldsymbol{V = \frac{I_{max}-I_{min}}{I_{max}+\ I_{min}}}$re I_{max} and I_{min} are the intensities at the maxima and minima of the fringe pattern. The more slowly V decreases with increasing path difference, the sharper the line.

Example : which case will the visibility be 1? What does a visibility of one mean?
I_{max}= 4I, and I_{min} = zero.

$$V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}}$$

$$V = \frac{4I - 0}{4I + 0} = 1$$

A visibility of 1 means that fringes are most visible (i.e. .this is the highest possible contrast).

2.4 Uses of Michelson's Interferometer

1. Determination of wavelength of monochromatic light.

$$2d_1 = m_1\lambda$$

$$2d_2 = m_2\lambda$$

$$\Delta d = d_2 - d_1 = (m_2 - m_1)\frac{\lambda}{2}$$

$$where: \ N = (m_2 - m_1)$$

$$\boldsymbol{\lambda = \frac{2(d_2 - d_1)}{(m_2 - m_1)}} \text{ ------ } 6$$

Where d2 -d1: number of fringes counting the center of the field of view. (Conversely, if x is increased, the fringe pattern will expand.) N: number of fringes.

Michelson tested the lines from various sources and concluded that a certain red line in the spectrum of cadmium was the most satisfactory .The standard meter in Paris with the wavelengths of intense red, green, And blue lines of

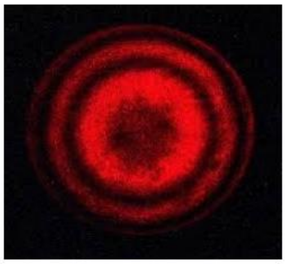
cadmium by Michelson and Benoit

Red line λ= 6438.4722 Å or λ = 643.84 nm

Green line λ= 5085.8240 Å or λ = 508.5824 nm

Blue line λ = 4799.9107 Å or λ= 479.9991 nm

Example: A red laser (He-Ne) of wavelength 632.8 nm is used in Michelson's interferometer. While kept the mirror M1 is fixed and M2 is moved. The fringes were found to move past a fixed cross-hair in the observer .find the distance of the mirror M2 that moved for single fringe to move past the origin line.



Solution:

$$\boldsymbol{\Delta d} = \boldsymbol{d_{2}} - \boldsymbol{d_1} = (\mathbf{m_2} - \mathbf{m_1}) \cdot \frac{\boldsymbol{\lambda}}{\mathbf{2}}$$

$$2(\Delta d) = m\lambda$$

$$\Delta d = m\frac{\lambda}{2} = 1x\frac{632.8nm}{2} = 316.4\,nm = 0.3146\,\mu m$$

2- Determination of different in wavelength between two neighbor lines or two waves.

Let the source of light emit close wavelength $\boldsymbol{\lambda}$ and such as sodium lamp, the apparatus is adjusted to form circular rings. the arrangement of the position of mirror M1 is moved and reached when a bright fringes of one set falls on the bright fringe of the other fringes are again distinct.

$$\boldsymbol{2d = m_1\,\lambda_1 = m_2\,\lambda_2} \text{ ----------}\mathit{7}$$

If condition $\lambda 1 > \lambda_2$, since $m_2 = m_1 + 1$, subst. in eq. 7

$$2d = m_1\lambda_1 = (m_1+1)\,\lambda_2$$

$$m_1 = \frac{\lambda_2}{\lambda_1 - \lambda_2}$$

$$\therefore 2d = \frac{\lambda_1\lambda_2}{\lambda_1 - \lambda_2}$$

$$\lambda_1 - \lambda_2 = \frac{\lambda_1\lambda_2}{2d}$$

If used **λ** as a mean wavelength of two wavelength λ_1 and λ_2, the small difference between them is:

$$\Delta\lambda = \frac{\lambda_{mean}^2}{2x} \text{ ----------} 8$$

3-determination of the refractive index of gases:

The path difference introduce between the two interfering beam is 2(n-1) L

Where n: Refractive index of gas

L: length of the tube. If m fringes cross the center of the field of view thus

$$\mathbf{2(n-1)L = m\lambda}$$

$$\boldsymbol{n = \frac{m\lambda}{2L} + 1} \;\ldots\ldots 9$$

Example / In an experiment for determine the refractive index of gas using Michelson interferometer a shift of 140 fringes is observed. When all the gas is removed from the tube. If the wavelength of light used is 5460Å and the length of the tube is 20 cm, calculate the refractive index of the gas?

Solution

$$n = \frac{m\lambda}{2L} + 1$$

$$n = \frac{140x5460x10^{-10}m}{2x0.2} + 1 = 1.00019$$

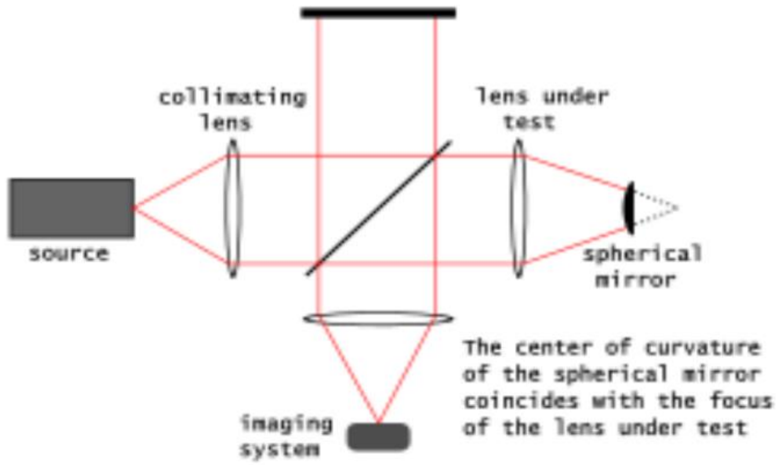
Index of refractive by interference Methods

If a thickness t of a substance having an index of refraction n is introduced into the path of one of the interfering beams in the interferometer, the optical path in this beam is increased because light travels more slowly in the substance and consequently has a shorter wavelength. The optical path is now nt through the medium, whereas it was practically t through the corresponding thickness of air (n = 1). Thus, the increase in optical path due to insertion of the substance is (n - l)t.t This will introduce (n - l)t). Extra waves in the path of one beam; so, if we call the number of fringes by which the fringe system is displaced when the substance is placed in the beam, we have

Interferometry

Instrument used by the principle of interference of light is called interferometer this instrument designed used to determine the refractive index of each solid ,liquid and solutions transparent substances .

2.2 Twyman & Green Interferometer.



Twyman-Green Interferometer.

A Twyman–Green interferometer is a variant of the Michelson interferometer principally used to test optical components. It was introduced in 1916 by Frank Twyman and Arthur Green.

Fig. 1 illustrates a Twyman–Green interferometer set up to test a lens. Light from a laser is expanded by a diverging lens (not shown), then is collimated into a parallel beam. A convex spherical mirror is positioned so that its center of curvature coincides with the focus of the lens being tested. The emergent beam is recorded by an imaging system for analysis.

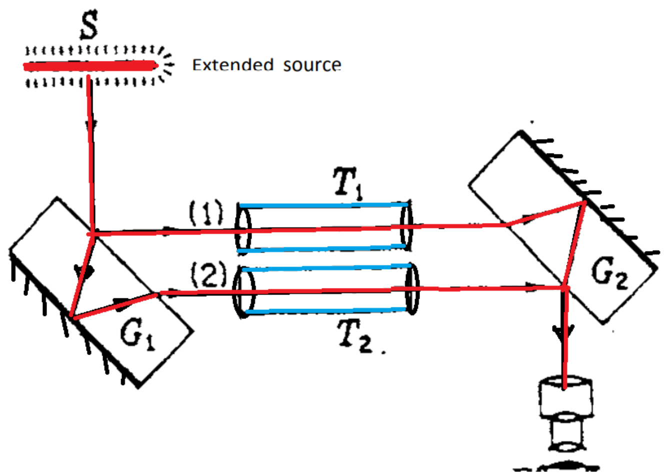
The fixed mirror in the Michelson interferometer is rotatable in the Twyman–Green interferometer, and while the light source is usually an extended source (although it can also be a laser) in a Michelson interferometer, the light source is always a point-like source in the Twyman– Green interferometer. The rotation of one mirror results in straight fringes appearing in the interference pattern, a fringing which is used to test the quality of optical components by observing changes in the fringe pattern when the component is placed in one arm of the interferometer.

Notes ***A homodyne interferometer uses a single-frequency laser source, whereas a heterodyne interferometer uses a laser source with two close frequencies.***

2.3 Jamin's interferometer

The Jamin interferometer is a type of interferometer. It was developed in 1856 by the French physicist Jules Jamin. The interferometer consists of two mirrors, made of the thickest glass possible. The beam reflection from the first surface of the mirror acts as a beam splitter. The incident light is split into two rays, parallel to each other and displaced by an amount depending on the thickness of the mirror. The rays are recombined at the second mirror and ultimately imaged onto a screen. a transparent pressure chamber can be positioned in the instrument. The phase shift due to changes in pressure is quite easy to measure.

If a phase-shifting element is added to one arm of the interferometer, then the displacement it causes can be determined by simply counting the interference fringes, i.e., the minima.



The Jamin interferometer.

The Jamin interferometer allows very exact measurements of the refractive index of gases; a transparent pressure chamber can be positioned in the instrument. The phase shift due to changes in pressure is quite easy to measure.

G1&G2 is two thickness parallel plates with one mirror face. S is broader source of monochromatic light. Two Similar evacuated tubes T1 and T2 of equal length are placed in the two parallel beams. To calculate its refractive index (n) Gas is slowly admitted to tube T2.

If the number of fringes **m** crossing the field is counted while the gas reaches the desired pressure and temperature, the value of **n** can be found by applying the optical path is now **(n. t)** Through the medium, whereas it was practically **t** through the corresponding thickness of air (**n = 1**). Thus. the increase in optical path due to insertion of the substance is **(n - 1) t.**

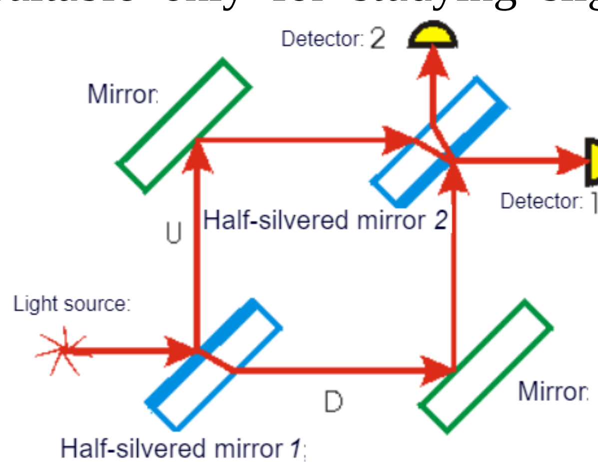
The number of fringes by which the fringe system is displaced when the Substance is placed in the beam, we have

$$(n - 1)t = \Delta m \, . \, \lambda$$

In principle a measurement of , t, and λ thus gives a determination of n.

2.4 Mach–Zehnder interferometer

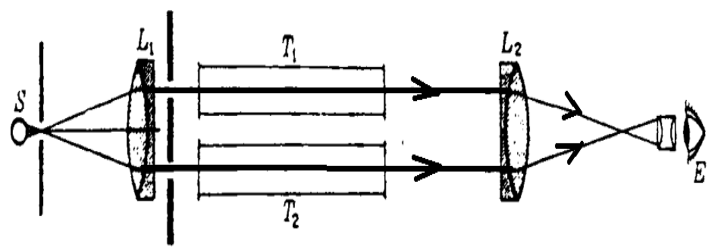
The Mach-Zehnder interferometer is suitable only for studying slight changes of refractive index. Which like Michelson' interferometer except that it have a large separation in beams path.



2.5 Rayleigh's interferometer.

It is a type of interferometer which employs two beams of light from a single source. The two beams are recombined after traversing two optical paths, and the interference pattern after recombination allows the determination of the difference in path lengths.

In Rayleigh's refractometer monochromatic light from a linear source S is made parallel by a lens L_1, L_2 are two converging lenses and split into two beams by a fairly wide double slit. After passing through two exactly similar tubes T_1, T_2 then, the beams are brought to interfere by the lens L_2. This form of refractometer is often used to measure slight differences in refractive index of liquids and solutions.



Rayleigh's refractometer

Interference Involving Multiple Reflections.

Some of the most beautiful effects of interference result from the multiple reflection of light between the two surfaces of a thin film of transparent material. These effects require no special apparatus for their production or observation and are familiar to anyone who has noticed the colors shown by thin films of oil on water, by soap bubbles, or by cracks in a piece of glass.

1.Reflection from a Plane-Parallel Film.

Let a ray of light from a source S be incident on the surface of such a film at A (Fig. 1). Part of this will be reflected as ray 1 and part refracted in the direction AF. Upon arrival at F, part of the latter will be reflected to B and part refracted toward H. At B the ray FB will be again divided. A continuation of this process yields two sets of parallel rays, one on each side of the film. In each of these sets, of course, the intensity decreases rapidly from one ray to the next. If the set of parallel reflected rays is now collected by a lens and focused at the point P, each ray will have traveled a different distance, and the phase relations may be such as to produce destructive or constructive interference at that point. It is such interference that produces the colors of thin films when they are viewed by the naked eye. In such a case L is the lens of the eye, and P lies on the retina.

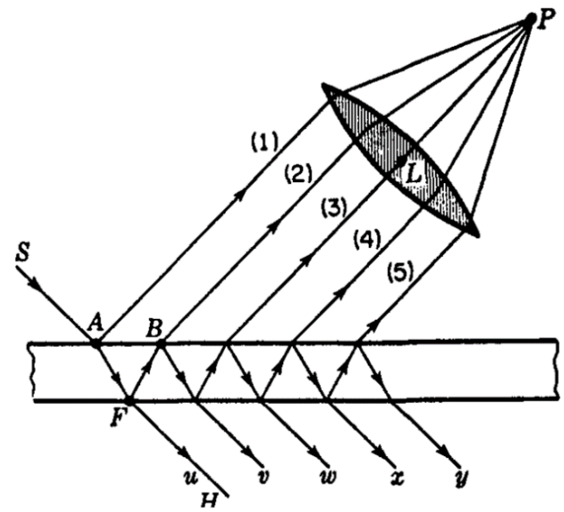


FIGURE 1
Multiple reflections in a plane-parallel film.

To find the phase difference between these rays, we must first evaluate the difference in the optical path traversed by a pair of successive rays, such as rays 1 and 2.

In Fig.2 let d be the thickness of the film, n its index of refraction, λ the wavelength of the light, and $\emptyset$ and $\acute{\emptyset}$ the angles of incidence and refraction.

If BD is perpendicular to ray 1, the optical paths from D and B to the focus of the lens will be equal. Starting at A, ray 2 has the path AFB in the film and ray 1 the path AD in air. The difference in these optical paths is given by:

$$\Delta = n(AFB) - AD$$

If BF is extended to intersect the perpendicular line AE at G, AF = GF because of the equality of the angles of incidence and reflection at the lower surface. Thus, we have

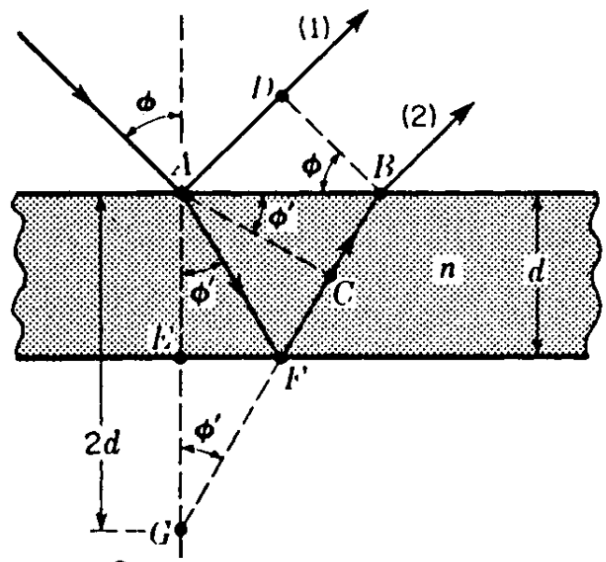


FIGURE 2
Optical-path difference between two consecutive rays in multiple reflection

$$\Delta = n(GB) - AD = n(GC + CB) - AD$$

Now AC is drawn perpendicular to FB, so the broken lines AC and DB represent two successive positions of a wave front reflected from the lower surface. The optical paths must be the same by any ray drawn between two wave fronts, so we may write. $n(CB) = AD$

The path difference then reduces to

$$\Delta = n(GC) = n(2d\cos\acute{\emptyset}) \ldots\ldots 1$$

If this path difference is a whole number of wavelengths, we might expect rays 1 and 2 to arrive at the focus of the lens in phase with each other and produce a maximum of intensity. However, we must take account of the

fact that ray 1 undergoes a phase change of 1t at reflection, while ray 2 does not, since it is internally reflected. The condition

$$\mathbf{2nd\ cos\acute{\emptyset} = m\lambda \ldots\ldots\ (2)} \qquad \textbf{Minima condition}$$

As before, m = 0, 1, 2, ... is the order of interference.

$$\cos\acute{\emptyset} = 1\ when\ the\ angle\ is\ \mathbf{zero}\ degree\ \ or\ Norml\ incident$$

On the other hand, if conditions are such that:

$$\mathbf{2nd\ cos\acute{\emptyset} = \left(m + \frac{1}{2}\right)\lambda \ldots (3)} \qquad \textbf{Maxima condition}$$

Ray 2 will be in phase with 1, but 3, 5, 7, . .. will be out of phase with 2, 4, 6, ... Since 2 is more intense than 3, 4 more intense than 5, etc., these pairs cannot cancel each other, and since the stronger series combines with 1, the strongest of all, there will be a maximum of intensity.

Now, the Interference In the Transmitted light that emerging from the lower side of the film, can also be brought together with a lens, and caused to interfere.

However, there are no phase changes at reflection for any of the rays, and the relations are such that Eq. (2) now becomes the condition for maxima and Eq. (3) the condition for minima.

$$\mathbf{2nd\ cos\acute{\emptyset} = m\lambda \ldots\ldots\ (\acute{2})} \qquad \textbf{Maxima}$$

$$\mathbf{2nd\ cos\acute{\emptyset} = \left(m + \frac{1}{2}\right)\lambda \ldots (\acute{3})} \qquad \textbf{Minima}$$

The phase difference between successive rays in the transmitted set or between all but the first pair in the reflected set, which by :

$$\boldsymbol{\delta = K\Delta = \frac{4\pi}{\lambda} nd \cos \acute{\phi}}$$

If we now, consider the soap film and normal incidence (i.e. $\cos \acute{\phi} = 1$) it is easy to see why thick films reflect light but do not show colored effects and appear transparent and why very thin films appear 'black' i.e. do not reflect light. The idea of the next set of calculation is to see which wavelengths will 'fit' into the film and so give a maximum. In other words, to find out which colors will be seen. Consider a soap film of refractive index 1.33 illuminated by white light.

Example 1/

A soap bubble 250 nm thick is illuminated by white light. The index of refraction of the soap film is 1.36, Which colors are *not* seen in the reflected light? Which colors appear strong in the reflected light? What color does the soap film appear at normal incidence?

Solution: For destructive interference, we must have . $\mathbf{nd} = \frac{\mathbf{m\lambda}}{\mathbf{2}}$

Thus, the wavelengths that are *not* reflected satisfy $\lambda_m = \frac{\mathbf{2nd}}{m}$ where m=1,2,3…. . It follows that.

$$\lambda_1 = \frac{(2)\,(1.36)\,(250 \times 10^{-9})}{(1)} = 680\,\text{nm},$$

$$\lambda_2 = \frac{(2)\,(1.36)\,(250 \times 10^{-9})}{(2)} = 340\,\text{nm}.$$

These are the only wavelengths close to the visible region of the electromagnetic spectrum for which destructive interference occurs. In fact, 680 nm lies right in the middle of the red region of the spectrum, whilst 340 nm lies in the ultraviolet region (and is, therefore, invisible to the human eye). It follows that the only non-reflected color is *red*.

For constructive interference, we must have:

$\mathbf{nd} = (\mathbf{m} + \mathbf{1/2})\frac{\boldsymbol{\lambda}}{\mathbf{2}}$. Thus, the wavelengths that are *strongly reflected* satisfy:

$$\boldsymbol{\lambda_m} = \frac{\boldsymbol{2nd}}{\left(\mathbf{m}+\frac{\mathbf{1}}{\mathbf{2}}\right)}$$

where m=0,1,2,3…. . It follows that.

$$\lambda_1 = \frac{(2)(1.36)(250 \times 10^{-9})}{(1/2)} = 1360\,\text{nm},$$

$$\lambda_2 = \frac{(2)(1.36)(250 \times 10^{-9})}{(3/2)} = 453\,\text{nm},$$

$$\lambda_3 = \frac{(2)(1.36)(250 \times 10^{-9})}{(5/2)} = 272\,\text{nm}.$$

A wavelength of 272 nm lies in the ultraviolet region whereas 1360 nm lies in the infrared. Clearly, both wavelengths correspond to light which is invisible to the human eye. The only strong reflection occurs at 453 nm, which corresponds to the *blue-violet* region of the spectrum. The reflected light is weak in the red region of the spectrum and strong in the blue-violet region. The soap film will, therefore, possess a pronounced *blue* color.

Example 2/

A film of oil 0.0005 mm thick and of refractive index 1.42 lies on a pool of water. Which colour will be missing from the spectrum when a point on the film is viewed at 40^0 to the vertical?

Solution

Destructive interference occurs: $m\lambda = 2nt\cos r$
$m\lambda = 2 \times 1.42 \times 5 \times 10^{-7} \times \cos r$

Therefore: $m\lambda = 2 \times 1.42 \times 5 \times 10^{-7} \times 0.89$

m = 1 1.26×10^{-6} m (infrared)
m = 2 6.33×10^{-7} m (orange)
m = 3 4.22×10^{-7} m (ultraviolet).

Thus the only colour missing from the visible part of the spectrum will be an orange line of wavelength 6.33×10^{-7} m.

Example 3/

A thin film of water ($n = 4/3$) is 3100 A°, If it is illuminated by white light at normal incidence, the color of film in the reflected light will be :

A Blue B Black C Yellow D Red

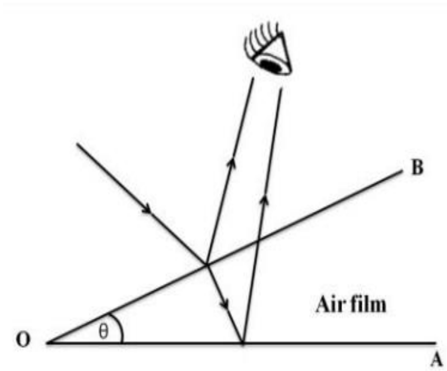
Example 4/

What is minimum thickness (in nm) of a soap film (n=1.3) that results in constructive interference in reflected light if the film is illuminated with light whose wavelength in free space is 620nm.

1.2 Variable Thickness (wedge-shaped film).

A thin film having zero thickness at one end and progressively increasing thickness at other end is called a wedge shaped film.

Consider two plain surface OA and OB inclined at an angle θ and enclosing a wedge-shaped film. The thickness of the air film is increasing from O to A as shown in figure 3. If a parallel beam of monochromatic light is allowed to fall on the upper surface and the surface is viewed by reflected light, then alternate dark and bright fringes becomes visible.



Figue.3

For more details we can see the figure 4 below.

When the light is incident on the wedge from above, it is gets partly reflected from the glass-to-air boundary at the top of the air film. Part of the light is transmitted through the air film and gets reflected party at the air-to-glass boundary, as show in figure (4). The two rays BC and DE, thus reflected from the top and bottom of the air fi1m, are coherent as they are derived from the same ray AB through division of amplitude. The ray is close enough if the thickness of the film is of the order o f a wavelength of light. For small film thickness the rays interference producing darkness or brightness fringes due to the phase difference. The thickness of the glass plates is large compared with wavelength of the incident light. Hence, the observed interference effects are entirely due to the wedge-shaped air film along the length of the film due to variation of the film thickness.

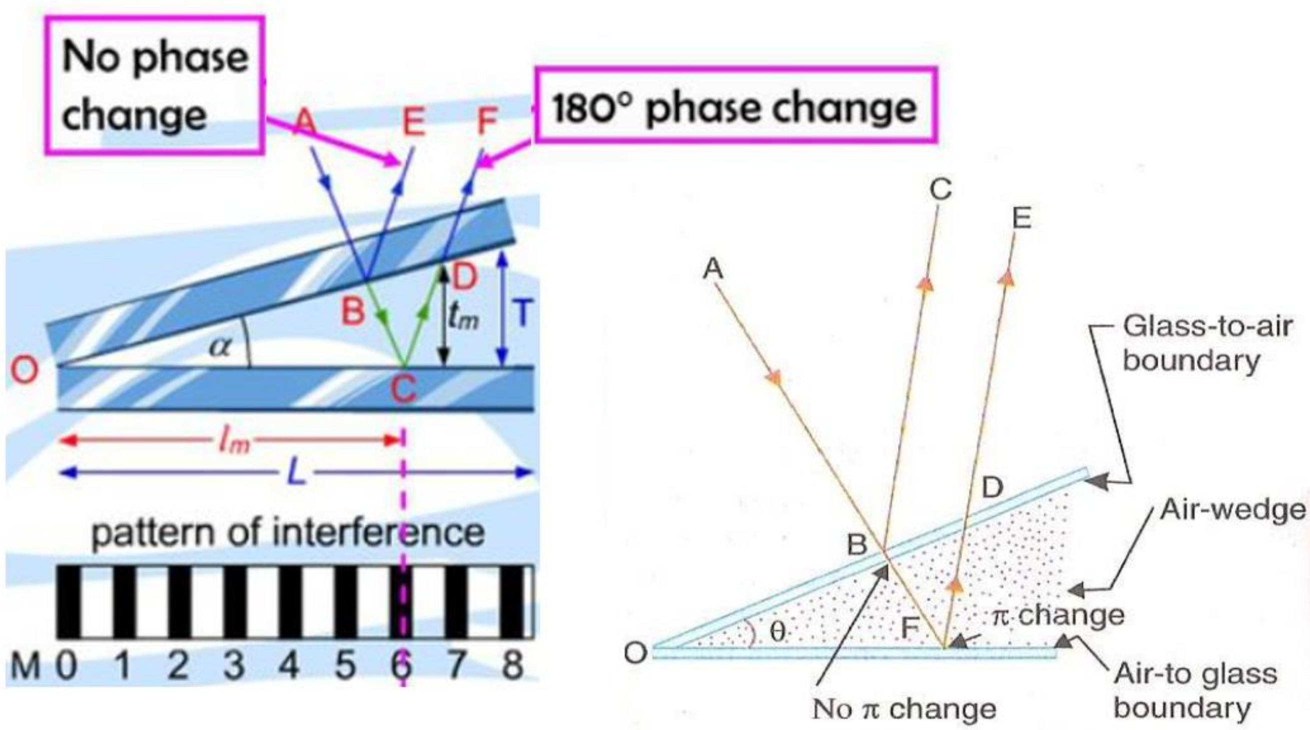


figure 4

Because ray DE travels more distance than BC. Also ray DE undergoes a phase change of half wavelength (π Change) occurs at the air to glass boundary due to reflection . The optical phase difference between the two rays BC and DE is given by :

$$\Delta = 2ndcos\ \theta + \lambda/2$$

Condition for constructive interference (or maxima or brightness)

If the bath deafferents is an integral multiple of λ **(mλ)**, then the waves interfere constructively.

$$\Delta = 2ndcos\ \theta + \frac{\lambda}{2} = m\lambda$$

$$2ndcos\ \theta = m\lambda - \frac{\lambda}{2} = (2m-1)\lambda/2$$

$$\mathbf{2\ n\ d\ cos\ \theta = (2m-1)\lambda/2}$$

- ✓ **Condition for destructive interference (or minima or darkness)**

If the bath deafferents is an odd multiple of λ/2. It means the bath deafferents is **(2m+1)λ/2** , then the waves interfere constructively.

$$\Delta = 2ndcos\ \theta + \frac{\lambda}{2} = (2\mathbf{m}+\mathbf{1})\boldsymbol{\lambda}/\mathbf{2}$$

$$2ndcos\ \theta = (2\mathbf{m}+\mathbf{1})\boldsymbol{\lambda}/\mathbf{2} - \boldsymbol{\lambda}/\mathbf{2}$$

$$2ndcos\ \theta = m\boldsymbol{\lambda} + \boldsymbol{\lambda}/\mathbf{2} - \boldsymbol{\lambda}/\mathbf{2}$$

$$2ndcos\ \theta = m\boldsymbol{\lambda}$$

The interference pattern in wedge shaped film consists of alternate dark and bright bands which are parallel to each other, and they are equally spaced.

Fringe at the apex is dark: At the apex, the two glass slides are in contact with each other. The thickness of the air film at the contact edge is negligible ($d \approx 0$). The optical path difference there becomes

$$\Delta = 2nd - \frac{\lambda}{2} = 0 - \frac{\lambda}{2} = -\frac{\lambda}{2}$$

It implies that a path difference of $\lambda/2$ or a phase difference of π occurs between reflected waves at the edge. The two waves interference destructively. Therefore, the fringe at the apex is always dark.

Referring to the figure 5 down (next page), let us say that a dark fringe occurs at A where the relation $2n\, d \cos\theta = m\lambda$ is satisfied . If there is a normal incident, the term $\cos\theta = 1$. Then we get:

$$\mathbf{2nd = m\lambda}$$

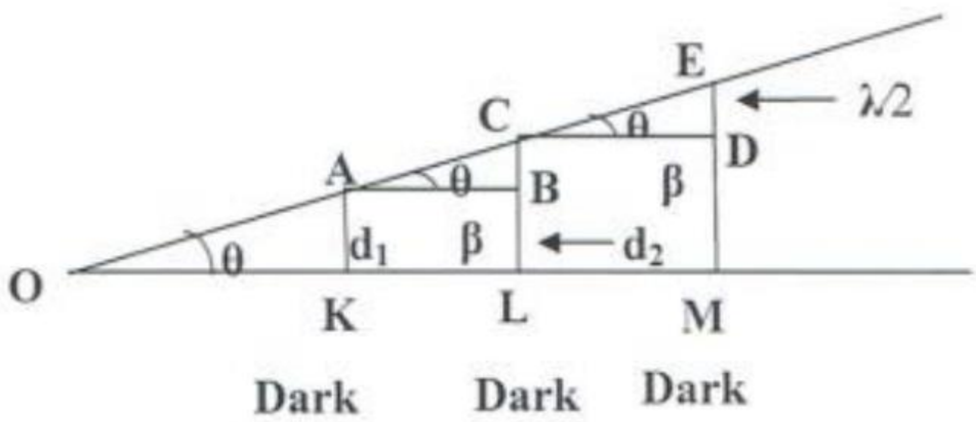
Now, let the thickness of air film at A is denoted by d_1, then,

$$\mathbf{2nd_1 = m\lambda} \quad \text{.....(1)}$$

The next dark fringe occurs at C where the thickness of air film at C is denoted by d_2. Here the d_2=CL and the relation that sitasfied for this next dark fringe is

$2nd_2=(m_{+1})\lambda$ …….(2)

Subtracting Eq.1 from Eq.2 , we get



$2n(d_2 - d_1)=\lambda$ ………….(3)

But $(d_2 - d_1)$=BC

$2n\ BC = \lambda$

$BC = \lambda /2n$

From Δ ABC , $\tan\theta$ =BC/AB

$AB \tan\theta = \lambda /2n$

AB is the separation of two successive dark fringes, and it also equal to the separation of two successive bright fringes. Therefore, it called fringe width β , where AB= β

$\beta = \lambda / 2n \tan \theta$ ....(4)

For small angle θ , $\tan \theta = \theta$

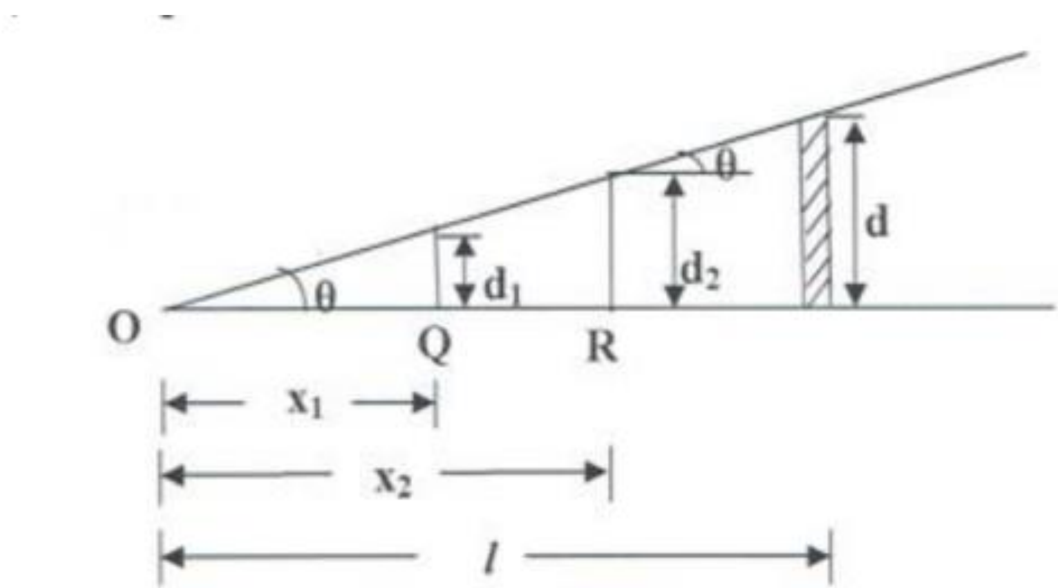
$\beta = \lambda / 2n\theta$ ....(5)

As the equation on the right side of the above equation are all constant, β is constant for a given wedge angle. According to equation From eq.(5), increase the angle makes the fringes move closer.

Determination of the Wedge Angle

The wedge angle Θ can be experimentally determined with help of travelling microscope. Using the microscope the positions of dark fringes at two distant point Q and R are noted figure . Let the distance OQ be x_1, and OR be x_2 .Let the thickness of the wedge be d_1 at Q and d_2 at R.

figure 5



The dark fringe at Q is given by

$$2nd_1 = m\lambda$$

But as θ is very small, we can write

$$d_1 = x_1 \tan\theta \cong x_1\theta$$

$$\therefore 2nx_1\theta = m\lambda \qquad (1)$$

We can write similarly for dark fringe at R as

$$2nx_2\theta = (m + N)\lambda \qquad (2)$$

Where N is the number of dark fringes lying between the positions Q and R. subtracting equation (1) from equation (2), we get

$$2n(x_2 - x_1)\theta = N\lambda$$

$$\therefore \theta = \frac{N\lambda}{2n(x_2 - x_1)} \qquad (3)$$

In case of air n=1 and the above relation reduces to

$$\theta = \frac{N\lambda}{2(x_2 - x_1)} \qquad (3)$$

Determination of the Thickness of the Spacer

The thickness of the spacer used to form the wedge shaped air film between the glass slides can be determined from above measurements. If (t) is the thickness of the spacer (foil or wire) used, we can write from figure 5 that

$$t = l \tan\theta \cong l\theta \qquad (6)$$

where l is the length of the air wedge. Using the equation (6) into equation (2), we obtain

$$t = \frac{lN\lambda}{2(x_2 - x_1)} \qquad (7)$$

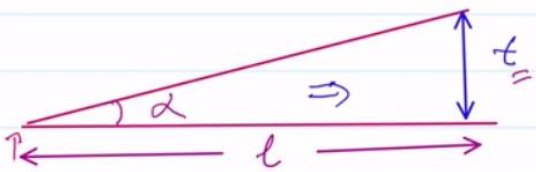
Problem and solution

Two plane glass plates 10.0 cm long, touching at one end, are separated at the other end by a strip of paper 0.1 mm thick. When the plates are illuminated by monochromatic light, the average distance between consecutive dark fringes is 0.020 cm. Calculate the wavelength of the light.

$l = 10\ cm$

$t = 0.1\ mm = 0.01\ cm$

$\beta = 0.02\ cm$



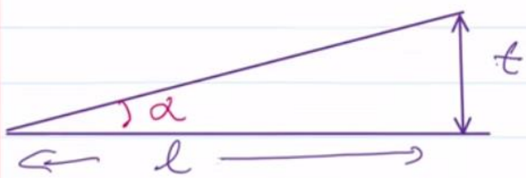
$\tan\alpha = \frac{t}{l}$ but if α is very small, $\tan\alpha \simeq \alpha$

$$\boxed{\alpha = \frac{t}{l}}$$

$$\beta = \frac{\lambda}{2\mu\alpha} = \frac{\lambda l}{2\mu t} = \frac{\lambda l}{2t} \quad \text{for air film}$$

$$\lambda = \frac{\beta \times 2t}{l} = \frac{0.02\ cm \times 2 \times 0.01\ cm}{10\ cm} = 4 \times 10^{-5}\ cm$$

A thin wedge of air is formed between a sheet of glass 5 cm long and a horizontal glass plate. One end of the sheet of glass is in contact with a glass plate. The other end is supported by a thin metal film 0.05 mm thick. The horizontal plate is illuminated from above with light 589 nm. How many dark interference fringes are observed per cm in the reflected light?



$l = 5$ cm

$t = 0.05$ mm $= 0.005$ cm

$\lambda = 589$ nm $= 5.89 \times 10^{-5}$ cm

Fringes (dark) / cm

$\tan\alpha = \frac{t}{l}$ or $\alpha = \frac{t}{l}$ if α is very small

$$\therefore \beta = \frac{\lambda}{2\mu\alpha} = \frac{\lambda l}{2\mu t} = \frac{5.89 \times 10^{-5} \times 5}{2 \times 1 \times .005}$$

$$\beta = 2.945 \times 10^{-2} \text{ cm}$$

Number of dark fringes / cm

$$l = n\beta \quad \text{or} \quad n = \frac{l}{\beta} = \frac{1 \text{ cm}}{2.945 \times 10^{-2} \text{ cm}}$$

$$n = 33.95 \cong 34$$

Two glass plates 12 cm long touch at one end, and are separated by a wire 0.048 mm in diameter at the other. How many bright fringes will be observed over the 12cm distance in the light of wavelength 6800 Å, reflected normally from the plates?

$l = 12\ cm$

$d = 0.048\ mm = 0.0048\ cm$

$\lambda = 6800\ A^{\circ} = 6.8 \times 10^{-5}\ cm$

$\tan\alpha = \frac{d}{l}$ or $\alpha = \frac{d}{l}$ if α is very small

$$\beta = \frac{\lambda}{2\mu\alpha} = \frac{\lambda l}{2\mu d} = \frac{6.8\times10^{-5} \times 12\ cm}{2\times1\times0.0048}$$

$\beta = 0.085\ cm$

$l = n \times \beta$

$$n = \frac{l}{\beta} = \frac{12}{0.085} = 141.17 \simeq 141 \text{ Fringes}$$

A foil is enclosed between two glass plates at one end to form a wedge shaped air film. When the film is viewed in mercury green light of wavelength 5460Å along normal, 12 fringes are seen in 0.4cm width. Deduce the angle of the wedge, the thickness of the foil if plate length is 3.0 cm in all and also find fringe width if water is introduced in the wedge space.

$\lambda = 5460 A^0 = 5.46 \times 10^{-5}$ cm

12 fringes in 0.4 cm width

$\beta = \frac{0.4 \text{ cm}}{12} = 0.03334$ cm

$\alpha = ?$

$t = ?$ when $l = 3$ cm

$\beta = ?$ if water film $\mu = 1.33$

$\beta = \frac{\lambda}{2\mu\alpha} \therefore \alpha = \frac{\lambda}{2\mu\beta}$

$= \frac{5.46 \times 10^{-5} \text{ cm}}{2 \times 1 \times 0.03334}$

$\alpha = 8.174 \times 10^{-4}$ rad

$\tan\alpha = \frac{t}{l}$

or $\alpha = \frac{t}{l}$

$t = \alpha \times l = 8.174 \times 10^{-4} \times 3 = 2.45 \times 10^{-3}$ cm

$\beta = \frac{\lambda}{2\mu\alpha} = \frac{5.46 \times 10^{-5} \text{ cm}}{2 \times 1.33 \times 8.174 \times 10^{-4}} = 0.025$ cm

Light of wavelength 6000Å falls normally on a thin wedge shaped film of refractive index 1.4 forming fringes that are 2 mm apart. Find the angle of the wedge

$\lambda = 6000 A^{0} = 6 \times 10^{-5}\ cm$

$\mu = 1.4$

$\beta = 2mm = 0.2\ cm$

$$\beta = \frac{\lambda}{2\mu\alpha}$$

$$\alpha = \frac{\lambda}{2\mu\beta} = \frac{6 \times 10^{-5}\ cm}{2 \times 1.4 \times 0.2} = \underline{\underline{1.07 \times 10^{-4}\ rad.}}$$