C H A P T E R FOUR

(CAPACITANCE AND CAPACITORS)

Consider two conductor's carrying charges of equal magnitude but of opposite sign. Such a combination of two conductors is called a capacitor. The conductors are called *plates.*

*The capacitance *C* of a capacitor is the ratio of the magnitude of the charge on either conductor to the magnitude of the potential difference between them:

$$C = \frac{Q}{\Delta V}$$

Note that by definition *capacitance is always a positive quantity*

we see that capacitance has SI units of coulombs per volt. The SI unit of capacitance is the farad (F), which was named in honor of Michael Faraday:

$1\ \mathrm{F} = 1\ \mathrm{C/V}$

The farad is a very large unit of capacitance. In practice, typical devices have capacitances ranging from microfarads (10^{-6} F) to picofarads (10^{-12} F).

CALCULATING CAPACITANCE

We can calculate the capacitance of a pair of oppositely charged conductors, We assume a charge of magnitude *Q* , and we calculate the potential difference, We then use the expression $C=Q/\Delta V$to evaluate the capacitance

*We can calculate the capacitance of an isolated spherical conductor of radius *R* and charge *Q* if we assume that the second conductor making up the capacitor is a concentric hollow sphere of infinite radius. The electric potential of the sphere of radius *R* is simply kQ/R, and setting V=0 at infinity as usual, we have

$$C = \frac{Q}{\Delta V} = \frac{Q}{k_e Q/R} = \frac{R}{k_e} = 4\pi\epsilon_0 R$$

This expression shows that the capacitance of an isolated charged sphere is proportional to its radius and is independent of both the charge on the sphere and the potential difference.

*The capacitance of a pair of conductors depends on the geometry of the conductors. Let us illustrate this with three familiar geometries, namely, parallel plates, concentric cylinders, and concentric spheres. In these examples, we assume that the charged conductors are separated by a vacuum.

Parallel-Plate Capacitors

The value of the electric field between the plates is

$$E = \frac{\sigma}{\epsilon_0} = \frac{Q}{\epsilon_0 A}$$

Because the field between the plates is uniform, the magnitude of the potential difference between the plates equals *Ed*; therefore,

$$\Delta V = Ed = \frac{Qd}{\epsilon_0 A}$$

Therefore the capacitance is

$$C = \frac{Q}{\Delta V} = \frac{Q}{Qd/\epsilon_0 A}$$

$$C = \frac{\epsilon_0 A}{d} \quad \rightarrow \textbf{(4.1)}$$

The capacitance of a parallel-plate capacitor is proportional to the area of its plates and inversely proportional to the plate separation

EXAMPLE 4.1:- A parallel-plate capacitor has an area A =2.00 X10^{-4} m^2 and a plate separation d=1.00 mm. Find its capacitance.

Solution from Equation 4.1, we find that

$$C = \epsilon_0 \frac{A}{d} = (8.85 \times 10^{-12}\ \mathrm{C^2/N \cdot m^2})\left(\frac{2.00 \times 10^{-4}\ \mathrm{m^2}}{1.00 \times 10^{-3}\ \mathrm{m}}\right)$$

$$= 1.77 \times 10^{-12}\ \mathrm{F} = \boxed{1.77\ \mathrm{pF}}$$

Exercise **What is the capacitance for a plate separation of 3.00 mm?**

EXAMPLE 4.2:- A solid cylindrical conductor of radius *a* and charge *Q* is Coaxial with a cylindrical shell of negligible thickness, radius b> a and charge -*Q* (Fig. 4.1a). Find the capacitance of this cylindrical capacitor if its length is L.

Solution:- If we assume that L is much greater than *a* and *b*, we can Neglect end effects. In this case, the electric field is perpendicular to the long axis of the cylinders and is confined to the region between them (Fig. 4.1b). We must first calculate the potential difference between the two cylinders, which is given in general by:

$$V_b - V_a = -\int_a^b \mathbf{E} \cdot d\mathbf{s}$$

where E is the electric field in the region $a < r < b$ In Chapter 2, we showed using Gauss's law that the magnitude of the electric field of a cylindrical charge distribution having linear charge density λ is $E_r = 2k\lambda/r$.

$$V_b - V_a = -\int_a^b E_r\, dr = -2k_e\lambda \int_a^b \frac{dr}{r} = -2k_e\lambda \ln\left(\frac{b}{a}\right)$$

using the fact that we obtain λ=Q\L

$$C = \frac{Q}{\Delta V} = \frac{Q}{\frac{2k_e Q}{\ell}\ln\left(\frac{b}{a}\right)} = \boxed{\frac{\ell}{2k_e \ln\left(\frac{b}{a}\right)}}$$

we see that the capacitance per unit length of a combination of concentric cylindrical conductors is

$$\frac{C}{\ell} = \frac{1}{2k_e \ln\left(\frac{b}{a}\right)}$$

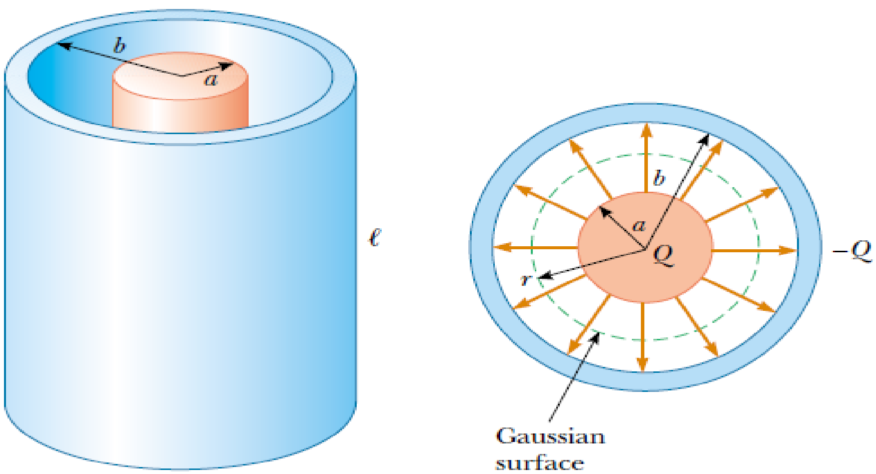

Figure 4.1 (a) A cylindrical capacitor consists of a solid cylindrical conductor of radius *a* and length L surrounded by a coaxial cylindrical shell of radius *b*. (b) End view. The dashed line represents the end of the cylindrical Gaussian surface of radius *r* and length L.

EXAMPLE 4.3:-A spherical capacitor consists of a spherical conducting shell of radius *b* and charge-*Q* concentric with a smaller conducting Sphere of radius *a* and charge *Q* (Fig. 4.2). Find the capacitance of this device.

Solution As we showed in Chapter 2, the field outside a spherically symmetric charge distribution is given by the expression $k\,Q\,/r^{2}$. In this case, this result applies to the field between the spheres ($a < r < b$). From Gauss's law we see that only the inner sphere contributes to this field. Thus, the potential difference between the spheres is

$$V_b - V_a = -\int_a^b E_r\,dr = -k_eQ\int_a^b \frac{dr}{r^2} = k_eQ\left[\frac{1}{r}\right]_a^b$$

$$= k_eQ\left(\frac{1}{b} - \frac{1}{a}\right)$$

The magnitude of the potential difference is

$$\Delta V = |V_b - V_a| = k_eQ\frac{(b-a)}{ab}$$

Substituting this value for ΔV into Equation 4.1, we obtain

$$C = \frac{Q}{\Delta V} = \boxed{\frac{ab}{k_e(b-a)}}$$

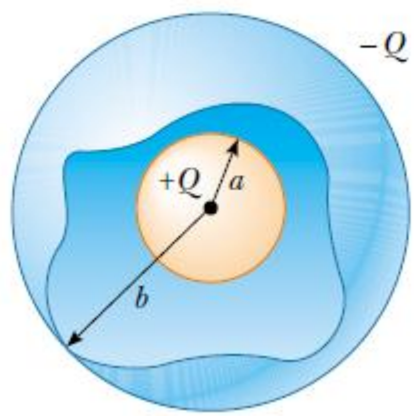

Figure 4.2 A spherical capacitor consists of an inner sphere of radius *a* surrounded by a concentric spherical shell of radius *b*. The electric field between the spheres is directed radially outward when the inner sphere is positively charged.

COMBINATIONS OF CAPACITORS:

-Parallel Combination

Two capacitors connected as shown in Figure 4.3a are known as a parallel combination of capacitors. **The individual potential differences across capacitors connected in parallel are all the same and are equal to the potential difference applied across the combination.**

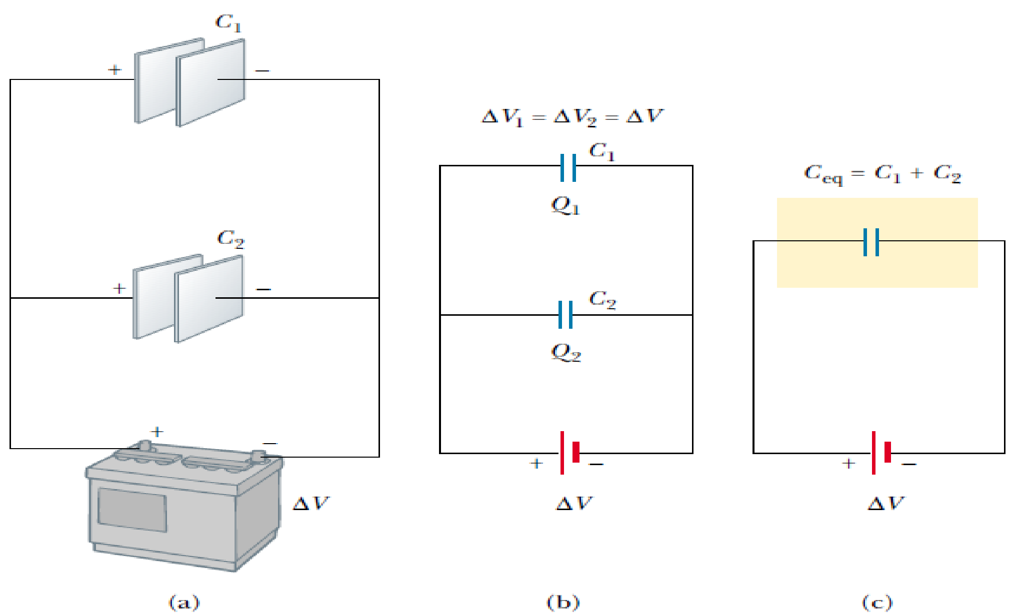

Figure 4.3 (a) A parallel combination of two capacitors in an electric circuit in which the potential difference across the battery terminals is ΔV. (b) The circuit diagram for the parallel combination. (c) The equivalent capacitance is $C_{eq} = C_1 + C_2$.

The *total charge Q* stored by the two capacitors is:

$$Q = Q_1 + Q_2 \qquad \textbf{(4.2)}$$

That is, **the total charge on capacitors connected in parallel is the sum of the charges on the individual capacitors**. Because the voltages across the capacitors are the same, the charges that they carry are

$$Q_1 = C_1 \Delta V \qquad Q_2 = C_2 \Delta V$$

Suppose that we wish to replace these two capacitors by one *equivalent capacitor* having a capacitance C_{eq} , as shown in Figure 4.3c. The effect this equivalent capacitor has on the circuit must be exactly the same as the effect of the combination of the two individual capacitors.

$$Q = C_{eq}\,\Delta V$$

Substituting these three relationships for charge into Equation 4.2, we have

$$C_{eq}\,\Delta V = C_1\,\Delta V + C_2\,\Delta V$$

$$C_{eq} = C_1 + C_2 \quad \text{(parallel combination)}$$

If we extend this treatment to three or more capacitors connected in parallel, we find the equivalent capacitance to be

$$C_{eq} = C_1 + C_2 + C_3 + \cdots \qquad \text{(parallel combination)}$$

Thus, the equivalent capacitance of a parallel combination of capacitors is Greater than any of the individual capacitances.

-Series Combination:-

Two capacitors connected as shown in Figure 4.4a are known as a *series combination* of capacitors.

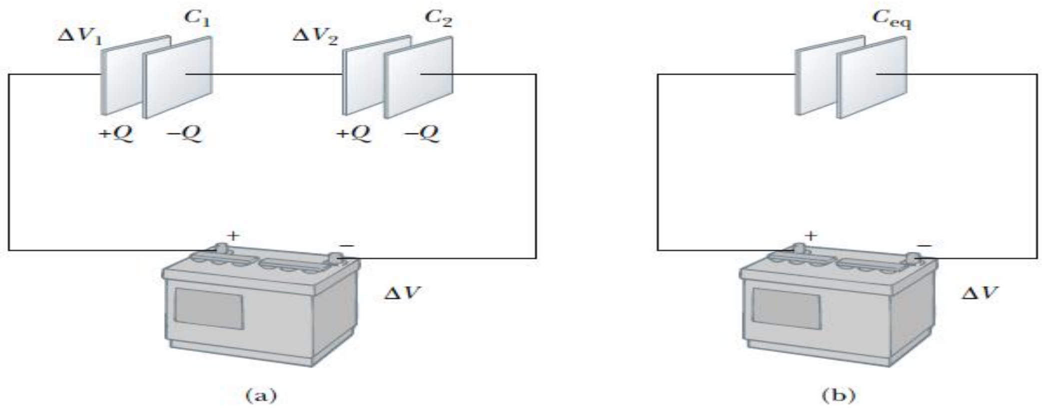

Figure 4.4 (a) A series combination of two capacitors. The charges on the two capacitors are the same. (b) The capacitors replaced by a single equivalent capacitor.

the charges on capacitors connected in series are the same.

From Figure 26.9a, we see that the voltage ΔV across the battery terminals is split between the two capacitors:

$$\Delta V = \Delta V_1 + \Delta V_2$$

where ΔV_1 and ΔV_2 are the potential differences across capacitors C_1 and C_2 , respectively. In general, **the total potential difference across any number of capacitors connected in series is the sum of the potential differences across the individual capacitors.**

$$\Delta V = \frac{Q}{C_{\text{eq}}}$$

Because we can apply the expression $Q = C\,\Delta V$ to each capacitor shown in Figure 4.4a, the potential difference across each is

$$\Delta V_1 = \frac{Q}{C_1} \qquad \Delta V_2 = \frac{Q}{C_2}$$

Therefore $\Delta \mathrm{V} = \frac{Q}{C_{eq}}$

$$\frac{Q}{C_{\text{eq}}} = \frac{Q}{C_1} + \frac{Q}{C_2}$$

Canceling Q , we arrive at the relationship

$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2}$$

When this analysis is applied to three or more capacitors connected in series, the relationship for the equivalent capacitance is

$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots \qquad \left(\begin{array}{l}\text{series}\\ \text{combination}\end{array}\right)$$

EXAMPLE 4.4:- Find the equivalent capacitance between *a* and *b* for the combination of capacitors shown in Figure 4.5a. All capacitances are in microfarads.

Solution:-The 1.0μF and 3.0μF capacitors are in parallel and combine according to the expression $C_{eq} = C_1 + C_2 = 4.0\ \mu F$. The 2.0μF and 6.0μF capacitors also are in parallel and have an equivalent capacitance of 8.0 μF. Thus, the upper branch in Figure 4.5b consists of two 4.0μF capacitors in series, which combine as follows:

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{4.0\ \mu\text{F}} + \frac{1}{4.0\ \mu\text{F}} = \frac{1}{2.0\ \mu\text{F}}$$

$$C_{eq} = \frac{1}{1/2.0\ \mu\text{F}} = 2.0\ \mu\text{F}$$

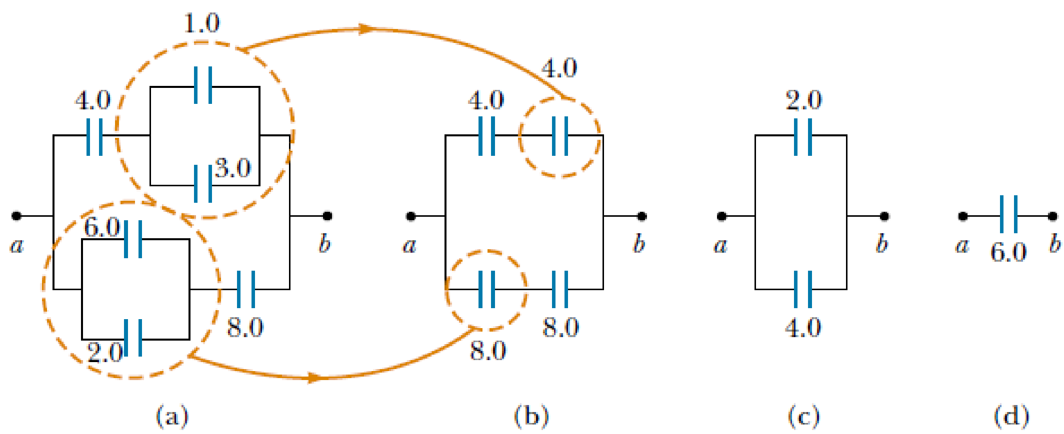

Figure 4.5 To find the equivalent capacitance of the capacitors in part (a), we reduce the various combinations in steps as indicated in parts (b), (c), and (d), using the series and parallel rules described in the text.

The lower branch in Figure 4.5b consists of two 8.0 μF capacitors in series, which combine to yield an equivalent capacitance of 4. μF. Finally, the 2.0 μF and 4.0 μF capacitors in Figure 26.10c are in parallel and thus have an equivalent capacitance of 6.0 μF.

Note:-

1-Energy stored in a charged capacitor is

$$U = \frac{Q^2}{2C} = \tfrac{1}{2}Q\,\Delta V = \tfrac{1}{2}C(\Delta V)^2$$

2-Energy stored in a parallel-plate capacitor is

$$U = \frac{1}{2}\,\frac{\epsilon_0 A}{d}\,(E^2 d^2) \;=\; \frac{1}{2}\,(\epsilon_0 A d)E^2$$

3- Energy density in an electric field is

$$u_E = \tfrac{1}{2}\epsilon_0 E^2$$