C H A P T E R THREE (Electric Potential)

POTENTIAL DIFFERENCE AND ELECTRIC POTENTIAL

The potential energy of the charge–field system is decreased by an amount **($dU = -q_0$E.ds).** For a finite displacement of the charge from a point *A* to a point *B*, the change in potential energy of the system $\Delta U = U_B - U_A$ is

$$\Delta U = -q_0 \int_A^B \mathbf{E} \cdot d\mathbf{s} \qquad \textbf{(3.1)}$$

Because the force q_0E is conservative, this line integral does not depend on the path taken from *A* to *B*.

The potential energy per unit charge U/q_0 is independent of the value of q_0 and has a unique value at every point in an electric field. This quantity U/q_0 is called the electric potential (or simply the potential) *V*. Thus, the electric potential at any point in an electric field is

$$V = U/q_0 \qquad \textbf{(3.2)}$$

The potential difference $\Delta V = V_B - V_A$ between any two points *A* and *B* in an electric field is defined as the change in potential energy of the system divided by the test charge q_0 :

$$\Delta V = \frac{\Delta U}{q_0} = -\int_A^B \mathbf{E} \cdot d\mathbf{s} \qquad \textbf{(3.3)}$$

the potential difference is proportional to the change in potential energy, and we see from Equation 3.3

$$\Delta U = q_0 \Delta V$$

-Electric potential is a scalar characteristic of an electric field, independent of the charges that may be placed in the field.

-The electric potential at an arbitrary point in an electric field equals the work required per unit charge to bring a positive test charge from infinity to that point.

Thus, if we take point *A* in Equation 3.3 to be at infinity, the electric potential at any point *P* is

$$V_p = -\int \mathbf{E} \cdot d\mathbf{s} \qquad (3.4)$$

In reality, V_p represents the potential difference ΔV between the point *P* and a point at infinity. Because electric potential is a measure of potential energy per unit charge, the SI unit of both electric potential and potential difference is joules per coulomb, which is defined as a volt (V):
That is, 1 J of work must be done to move a 1C charge through a potential difference of 1 V.

$$1\ \text{V} = 1\ \frac{\text{J}}{\text{C}}$$

Equation 3.3 shows that potential difference also has units of electric field times distance. From this, it follows that the SI unit of electric field (N/C) can also be expressed in volts per meter:

$$1\ \frac{\text{N}}{\text{C}} = 1\ \frac{\text{V}}{\text{m}}$$

Because 1 V =1 J/C and because the fundamental charge is approximately 1.6×10^{-19} C the electron volt is related to the joule as follows:

$$\mathbf{1\ eV = 1.60 \times 10^{-19}\ C.V = 1.60 \times 10^{-19}\ J}$$

POTENTIAL DIFFERENCES IN A UNIFORM ELECTRIC FIELD

Consider a uniform electric field directed along the negative *y* axis, as shown in Figure 3.1. Let us calculate the potential difference between two points *A* and *B* separated by a distance *d*, where *d* is measured parallel to the field lines. Equation 3.3 gives

$$V_B - V_A = \Delta V = -\int_A^B \mathbf{E} \cdot d\mathbf{s} = -\int_A^B E \cos 0^\circ\ ds = -\int_A^B E\ ds$$

Because *E* is constant, we can remove it from the integral sign; this gives

$$\Delta V = -E \int_A^B ds = -Ed$$

→***(3.6)*** **(Potential difference in a uniform electric field)**

The minus sign indicates that point *B* is at a lower electric potential than point *A*; that is $V_B < V_A$.

Electric field lines always point in the direction of decreasing electric potential, as shown in Figure 3.1a.

Now suppose that a test charge q_0 moves from *A* to *B*. We can calculate the change in its potential energy from Equations 3.3 and 3.6:

$$\Delta U = q_0 \Delta V = -\, q_0 E\, d \quad \rightarrow (3.7)$$

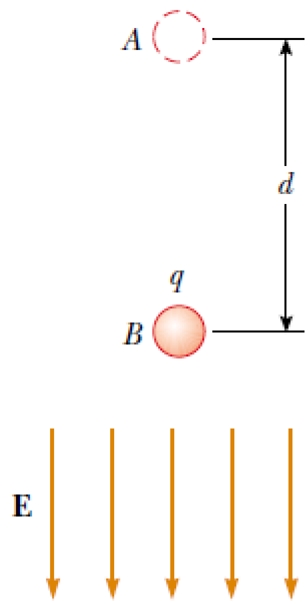

Figure 3.1 When the electric field E is directed downward, point *B* is at a lower electric potential than point *A*. Apositive test charge that moves from point *A* to point *B* loses electric potential energy.

Note

1- A positive charge loses electric potential energy when it moves in the direction of the electric field

2- As the charged particle gains kinetic energy, it loses an equal amount of potential energy.

3- A negative charge gains electric potential energy when it moves in the direction of the electric field

4- The name equipotential surface is given to any surface consisting of a continuous distribution of points having the same electric potential.

***EXAMPLE 3.1*:-** A 12-V battery is connected between two parallel plates, as shown in Figure 3.2. The separation between the plates is *d* = 0.30 cm, and we assume the electric field between the plates to be uniform. Find the magnitude of the electric field between the plates.

Solution: The magnitude of the electric field between the plates is, from Equation 3.6,

$$E= \frac{|VB - VA|}{d} = \frac{12V}{0.3x10^{-2}} = 4.0x10^3 \text{ V/m}$$

This configuration, which is shown in Figure 3.2 and called a *parallel-plate capacitor.*

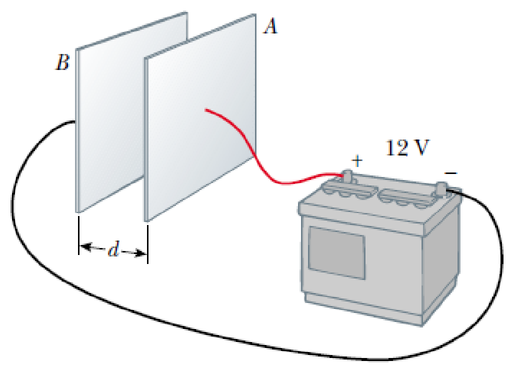

Figure 3.2 A 12-V battery connected to two parallel plates. The electric field between the plates has a magnitude given by the potential difference ΔV divided by the plate separation *d*.

***EXAMPLE 3.2*:-** A proton is released from rest in a uniform electric field that has a magnitude of 8.0 X10^4 V/m and is directed along the positive *x* axis (Fig. 3.3). The proton undergoes a displacement of 0.50 m in the direction of E. (a) Find the change in electric potential between points *A* and *B.*

Solution Because the proton (which, as you remember, carries a positive charge) moves in the direction of the field, we expect it to move to a position of lower electric potential. From Equation 3.6, we have

$$\Delta V = -Ed = -(8.0 \text{ x}10^4 \text{ V/m})(0.50 \text{ m})$$
$$= -4\text{x}10^4 \text{V}$$

(b) Find the change in potential energy of the proton for this displacement.

Solution:-

$$\Delta U = q_0 \Delta V = e\Delta V$$
$$= (1.6\text{x}10^{-19}\text{C})(-4\text{X}10^{4}\text{V})$$
$$= -6.4\text{X}10^{-15}\text{J}$$

The negative sign means the potential energy of the proton decreases as it moves in the direction of the electric field.

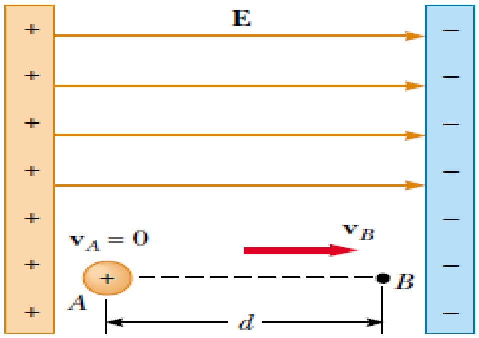

Figure 3.3 A proton accelerates from *A* to *B* in the direction of the electric field.

ELECTRIC POTENTIAL AND POTENTIAL ENERGY DUE TO POINT CHARGES

Consider an isolated positive point charge *q*. To find the electric potential at a point located a distance *r* from the charge

$$V_B - V_A = -\int \mathbf{E}.d\mathbf{s}$$

where *A* and *B* are the two arbitrary points .At any field point, the electric field due to the point charge is $E = k_e q\hat{r}/r^2$ where $\hat{r}$ is a unit vector directed from the charge toward the field point. The quantity E.ds can be expressed as

$$E.ds = k_e \frac{q}{r^2} \hat{r}.ds$$

Because the magnitude of $\hat{r}$ is 1, the dot product $\hat{r}.ds = ds\cos\theta$ where θ is the angle between $\hat{r}$ and *d*s. thus, $ds \cos\theta = dr$. That is, any displacement *d*s along the path from point *A* to point *B* produces a change *dr* in the magnitude of r. Making these substitutions, we find that hence, the expression for the potential difference becomes

$$V_B - V_A = -\int E_r\,dr = -k_e q\int_{r_A}^{r_B}\frac{dr}{r^2} = \frac{k_e q}{r}\Bigg]_{r_A}^{r_B}$$

$$V_B - V_A = k_e q\left[\frac{1}{r_B} - \frac{1}{r_A}\right] \qquad \rightarrow(3.10)$$

The integral of E.ds is *independent* of the path between points *A* and *B*—as it must be because the electric field of a point charge is conservative.

-The electric potential created by a point charge at any distance *r* from the charge is

$$V = k_e q/r \quad \rightarrow(3.11)$$

For a group of point charges, we can write the total electric potential at *P* in the form (Electric potential due to several point charges)

$$V = k_e\sum_i \frac{q_i}{r_i} \qquad \rightarrow(3.12)$$

where the potential is again taken to be zero at infinity and r_i is the distance from the point *P* to the charge q_i .

we can express the potential energy or Electric potential energy due to two charges as

$$U = k_e\frac{q_1 q_2}{r_{12}}$$

the total potential energy of the system of three charges shown in Figure 3.4 is

$$U = k_e\left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right) \qquad \rightarrow(3.13)$$

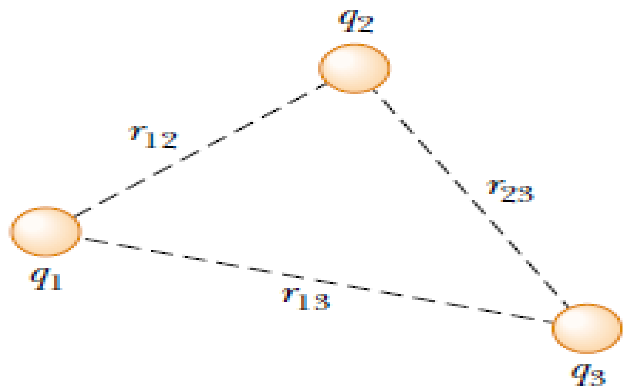

Figure 3.4 Three point charges are fixed at the positions shown. The potential energy of this system of charges is given by Equation 3.13.

***EXAMPLE 3.3*:-** A charge q_1=2.00 µC is located at the origin, and a charge q_2=-6.00 C is located at (0, 3.00) m, as shown in Figure 3.5a. (a) Find the total electric potential due to these charges at the point *P*, whose coordinates are (4.00, 0) m.

Solution For two charges, the sum in Equation 3.12 gives

$$V_P = k_e\left(\frac{q_1}{r_1} + \frac{q_2}{r_2}\right)$$

$$= 8.99 \times 10^9 \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\left(\frac{2.00 \times 10^{-6}\text{ C}}{4.00\text{ m}} + \frac{-6.00 \times 10^{-6}\text{ C}}{5.00\text{ m}}\right)$$

$$= -6.29 \times 10^3\text{ V}$$

(b) Find the change in potential energy of a 3.00µC charge as it moves from infinity to point *P* (Fig. 3.6b).

Solution When the charge is at infinity, U_i=0 , and when the charge is at *P*, U_f=q_3V_p; therefore,

$$\Delta U = q_3V_P - 0 = (3.00 \times 10^{-6}\text{ C})(-6.29 \times 10^3\text{ V})$$

$$= -18.9 \times 10^{-3}\text{ J}$$

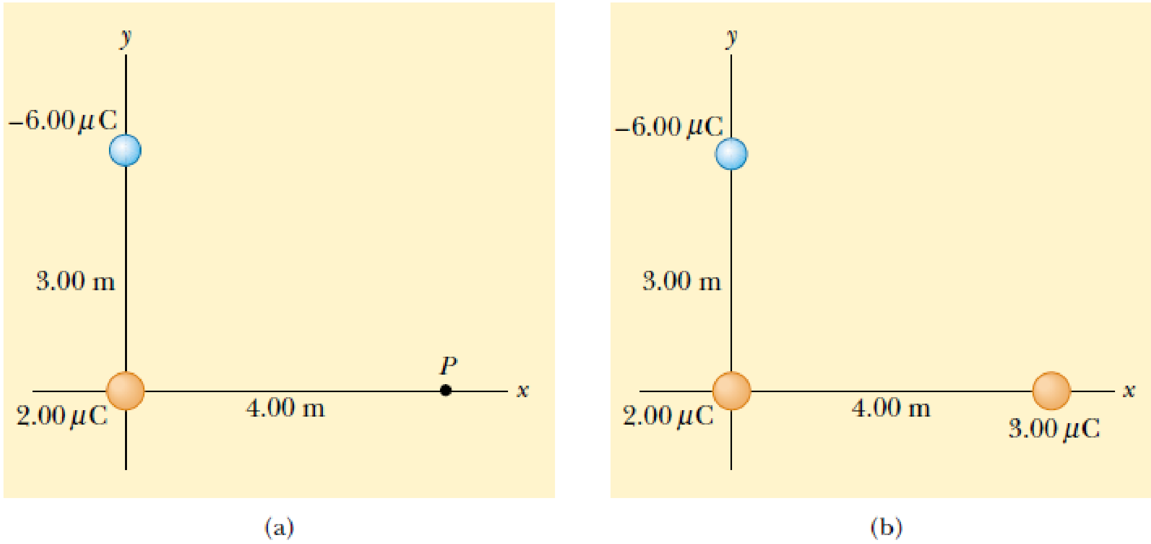

Figure 3.5 (a) The electric potential at *P* due to the two charges is the algebraic sum of the potentials due to the individual charges. (b) What is the potential energy of the three-charge system?

OBTAINING THE VALUE OF THE ELECTRIC FIELD FROM THE ELECTRIC POTENTIAL

From Equation 3.3 we can express the potential difference *dV* between two points a distance *ds* apart as

$$dV = -\mathbf{E}\cdot d\mathbf{s} \qquad \rightarrow(3.14)$$

If the electric field has only one component E_x , then E. *d*s = $E_x\, dx$
Therefore, Equation 3.14 becomes $dV = -E_x\, dx$ or

$$E_x = -\frac{dV}{dx} \qquad \text{\_ \_ } \mathbf{(3.15)}$$

If the charge distribution creating an electric field has spherical symmetry such that the volume charge density depends only on the radial distance *r*, then the electric field is radial. In this case, E. *d*s = $E_r\, dr$,and thus we can express *dV* in the form $dV = -E_r\, dr$,Therefore,

$$E_r = -\frac{dV}{dr}$$

_ _ **(3.16)**

(Note:- equipotential surfaces are perpendicular to field lines)
When a test charge undergoes a displacement ds along an equipotential surface, then dv=0 because the potential is constant along an equipotential surface. From Equation 3.14, then, dV= - E. ds ; thus, E must be perpendicular to the displacement along the equipotential surface .This shows that the equipotential surfaces must always be perpendicular to the electric field lines.

In general, the electric potential is a function of all three spatial coordinates. If *V*(*r*) is given in terms of the cartesian coordinates, the electric field components E_x , E_y , and E_z can readily be found from *V*(*x*, *y*, *z*) as the partial derivatives

$$E_x = -\frac{\partial V}{\partial x} \qquad E_y = -\frac{\partial V}{\partial y} \qquad E_z = -\frac{\partial V}{\partial z}$$

For example, if $V = 3x^2y + y^2 + yz$, then

$$\frac{\partial V}{\partial x} = \frac{\partial}{\partial x}(3x^2y + y^2 + yz) = \frac{\partial}{\partial x}(3x^2y) = 3y\frac{d}{dx}(x^2) = 6xy$$

EXAMPLE 3.4:- An electric dipole consists of two charges of equal magnitude and opposite sign separated by a distance 2*a*, as shown in Figure 3.6. The dipole is along the *x* axis and is centered at the origin. (a) Calculate the electric potential at point *P*.
Solution:- For point *P* in Figure 3.7

$$V = k_e\sum\frac{q_i}{r_i} = k_e\left(\frac{q}{x-a} - \frac{q}{x+a}\right) = \frac{2k_eqa}{x^2 - a^2}$$

(b) Calculate *V* and *Ex* at a point far from the dipole.
Solution If point *P* is far from the dipole, such that x>>a then a^2 can be neglected in the term x^2- a^2 and *V* becomes

$$V \approx \frac{2k_e qa}{x^2} \qquad (x \gg a)$$

Using Equation 3.15 and this result, we can calculate the electric field at a point far from the dipole:

$$E_x = -\frac{dV}{dx} = \frac{4k_e qa}{x^3} \qquad (x \gg a)$$

(c) Calculate V and E_x if point P is located anywhere between the two charges.

Solution:-

$$V = k_e \sum \frac{q_i}{r_i} = k_e \left(\frac{q}{a-x} - \frac{q}{x+a} \right) = -\frac{2k_e qx}{x^2 - a^2}$$

$$E_x = -\frac{dV}{dx} = -\frac{d}{dx}\left(-\frac{2k_e qx}{x^2 - a^2} \right) = 2k_e q \left(\frac{-x^2 - a^2}{(x^2 - a^2)^2} \right)$$

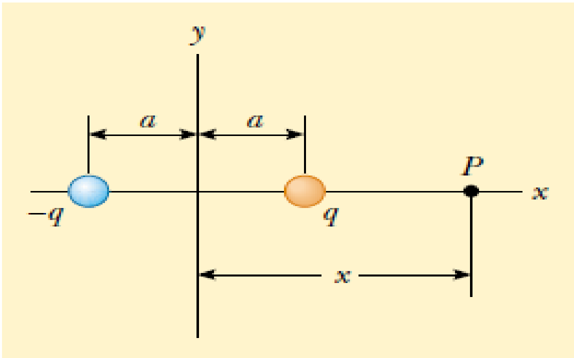

Figure 3.6 An electric dipole located on the x axis

ELECTRIC POTENTIAL DUE TO CONTINUOUS CHARGE DISTRIBUTIONS

The electric potential dV at some point P due to the charge element dq is

$$dV = k_e \frac{dq}{r}$$

where r is the distance from the charge element to point P. in general, a different distance from point P and because ke is constant, we can express V as

$$V = k_e \int \frac{dq}{r}$$

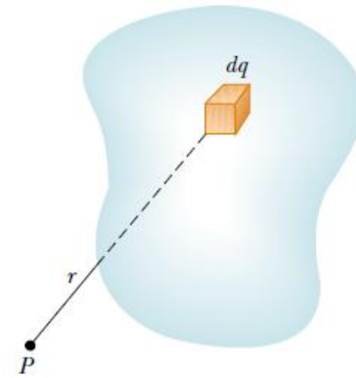

Figure 3.7 The electric potential at the point *P* due to a continuous charge distribution can be calculated by dividing the charged body into segments of charge *dq* and summing the electric potential contributions over all segments.

EXAMPLE 3.5:- (a) Find an expression for the electric potential at a point *P* located on the perpendicular central axis of a uniformly charged ring of radius *a* and total charge *Q*.

Solution Let us orient the ring so that its plane is perpendicular to an *x* axis and its center is at the origin. We can then take point *P* to be at a distance *x* from the center of the ring, as shown in Figure 3.8. The charge element *dq* is at a distance $\sqrt{x^2 + a^2}$ from point *P*. Hence, we can express *V* *as:*

$$V = k_e \int \frac{dq}{r} = k_e \int \frac{dq}{\sqrt{x^2 + a^2}}$$

Because each element *dq* is at the same distance from point *P*, we can remove $\sqrt{x^2 + a^2}$ from the integral, and *V* reduces to

$$V = \frac{k_e}{\sqrt{x^2 + a^2}} \int dq = \frac{k_e Q}{\sqrt{x^2 + a^2}} \qquad (3.17)$$

The only variable in this expression for *V* is *x*. This is not surprising because our calculation is valid only for points along the *x* axis, where *y* and *z* are both zero.

(b) Find an expression for the magnitude of the electric field at point *P*.

Solution From symmetry, we see that along the *x* axis E can have only an *x* component. Therefore, we can use Equation 3.15:

$$E_x = -\frac{dV}{dx} = -k_e Q \frac{d}{dx}(x^2 + a^2)^{-1/2}$$
$$= -k_e Q(-\tfrac{1}{2})(x^2 + a^2)^{-3/2}(2x)$$
$$= \frac{k_e Q x}{(x^2 + a^2)^{3/2}}$$

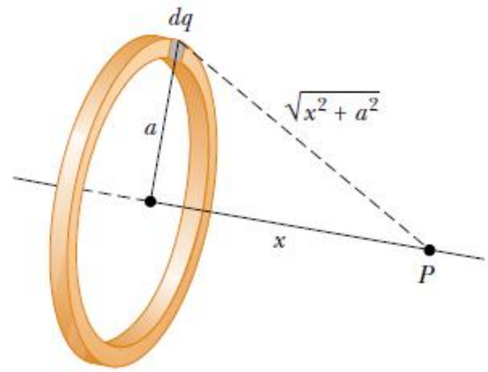

Figure 3.8 A uniformly charged ring of radius *a* lies in a plane perpendicular to the *x* axis. All segments *dq* of the ring are the same distance from any point *P* lying on the *x* axis

EXAMPLE 3.6:- Find (a) the electric potential and (b) the magnitude of the electric field along the perpendicular central axis of a uniformly charged disk of radius *a* and surface charge density σ.

Solution (a) Again, we choose the point *P* to be at a distance *x* from the center of the disk and take the plane of the disk to be perpendicular to the *x* axis. We can simplify the problem by dividing the disk into a series of charged rings. The electric potential of each ring is given by Equation 3.17. Consider one such ring of radius *r* and width *dr*, .The surface area of the ring is $dA = 2\pi r\, dr$;and $dq = \sigma dA = \sigma 2\pi r\, dr$

the potential at the point *P* due to this ring is

$$dV = \frac{k_e\, dq}{\sqrt{r^2 + x^2}} = \frac{k_e\, \sigma 2\pi r\, dr}{\sqrt{r^2 + x^2}}$$

To find the *total* electric potential at *P*, we sum over all rings making up the disk. That is, we integrate *dV* from *r* = 0 to *r* = *a*

$$V = \pi k_e \sigma \int_0^a \frac{2r\,dr}{\sqrt{r^2 + x^2}} = \pi k_e \sigma \int_0^a (r^2 + x^2)^{-1/2}\, 2r\,dr$$

This integral is of the form $u^n\,du$ and has the value $u^{n+1}/(n+1)$, where n=-1/2 and u=r^2+x^2 .This gives

$$V = 2\pi k_e \sigma[(x^2 + a^2)^{1/2} - x]$$

(b) As in Example 3.5, we can find the electric field at any axial point from

$$E_x = -\frac{dV}{dx} = 2\pi k_e \sigma\left(1 - \frac{x}{\sqrt{x^2 + a^2}}\right)$$

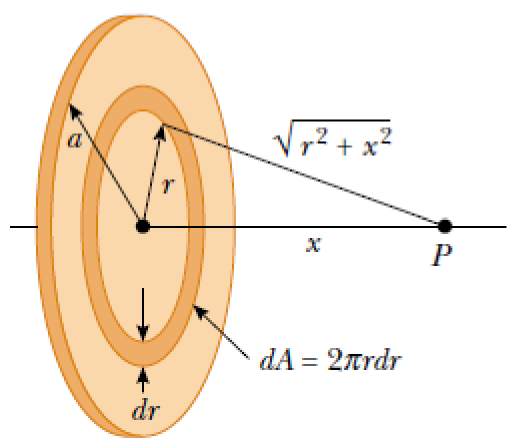

Figure 3.9 A uniformly charged disk of radius *a* lies in a plane perpendicular to the *x* axis. The calculation of the electric potential at any point *P* on the *x* axis is simplified by dividing the disk into many rings each of area 2π*r dr*.

EXAMPLE 3.7:- A rod of length L located along the *x* axis has a total charge *Q* and a uniform linear charge density λ= *Q*/L . Find the electric potential at a point *P* located on the *y* axis a distance *a* from the origin (Fig. 3.10).

Solution The length element *dx* has a charge *dq* = λ *dx*. Because this element is a distance r =$\sqrt{x^2 + a^2}$ from point *P*, we can express the potential at point *P* due to this element

$$dV = k_e \frac{dq}{r} = k_e \frac{\lambda\, dx}{\sqrt{x^2 + a^2}}$$

To obtain the total potential at *P*, we integrate this expression over the limits *x* = 0 to *x*=L . Noting that k_e and λ are constants, we find that

$$V = k_e \lambda \int_0^{\ell} \frac{dx}{\sqrt{x^2 + a^2}} = k_e \frac{Q}{\ell} \int_0^{\ell} \frac{dx}{\sqrt{x^2 + a^2}}$$

This integral has the following value

$$\int \frac{dx}{\sqrt{x^2 + a^2}} = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

Evaluating *V*, we find that

$$V = \frac{k_e Q}{\ell} \ln\left(\frac{\ell + \sqrt{\ell^2 + a^2}}{a}\right)$$

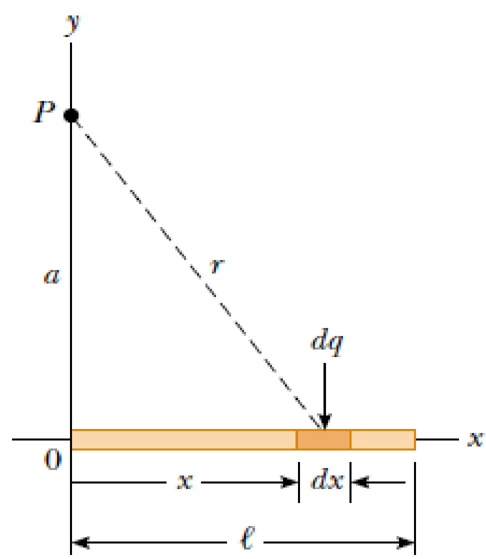

Figure 3.10 A uniform line charge of length located along the *x* axis. To calculate the electric potential at *P*, the line charge is divided into segments each of length *dx* and each carrying a charge
dq =λ *dx*.

EXAMPLE 3.7:- An insulating solid sphere of radius *R* has a uniform positive volume charge density and total charge *Q*. (a) Find the electric potential at a point outside the sphere, that is, for r >R.Take the potential to be zero at r =∞

Solution:- In Example 2.5, we found that the magnitude of the electric field outside a uniformly charged sphere of radius *R* is

$$E_r = k_e \frac{Q}{r^2} \qquad (\text{for } r > R)$$

where the field is directed radially outward when Q is positive. In this case, to obtain the electric potential at an exterior point, such as B in Figure 3.11, we use Equation 3.4 and the expression for E_r given above:

$$V_B = -\int_{\infty}^{r} E_r\, dr = -k_e Q \int_{\infty}^{r} \frac{dr}{r^2}$$

$$V_B = k_e \frac{Q}{r} \qquad (\text{for } r > R)$$

Because the potential must be continuous at $r = R$, we can use this expression to obtain the potential at the surface of the sphere. That is, the potential at a point such as C shown in Figure 3.12 is

$$V_C = k_e \frac{Q}{R} \qquad (\text{for } r = R)$$

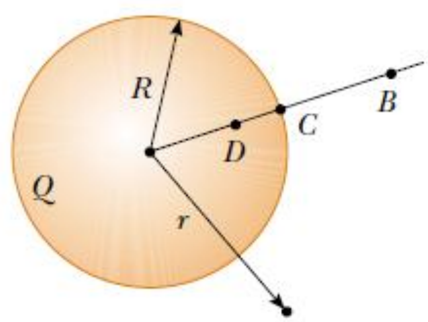

Figure 25.11A uniformly charged insulating sphere of radius R and total charge Q. The electric potentials at points B and C are equivalent to those produced by a point charge Q located at the center of the sphere

ELECTRIC POTENTIAL DUE TO A CHARGED CONDUCTOR

every point on the surface of a charged conductor in equilibrium is at the same electric potential. Consider two points A and B on the surface of a charged conductor, as shown in Figure 3.12. Along a surface path connecting these points, E is always perpendicular to the displacement ds; therefore Using this result and Equation 3.3, we conclude that the potential difference between A and B is necessarily zero:

$$V_B - V_A = -\int_A^B \mathbf{E} \cdot d\mathbf{s} = 0$$

This result applies to any two points on the surface. Therefore, *V* is constant everywhere on the surface of a charged conductor in equilibrium.

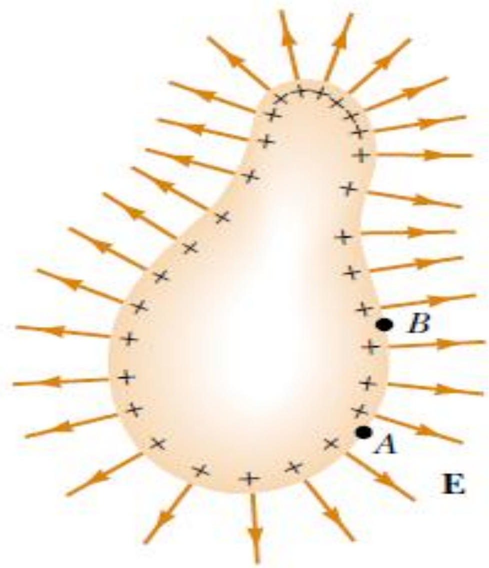

Figure 3.12 An arbitrarily shaped conductor carrying a positive charge.

Note:-

1-The surface of any charged conductor in electrostatic equilibrium is an equipotential surface. Furthermore, because the electric field is zero inside the conductor, we conclude from the relationship that the electric potential is constant everywhere inside the conductor and equal to its value at the surface.

2- the electric field is large near convex points having small radii of curvature and reaches very high values at sharp points.

SUMMARY

When a positive test charge q_0 is moved between points *A* and *B* in an electric field E, the change in the potential energy is

$$\Delta U = -q_0 \int_A^B \mathbf{E} \cdot d\mathbf{s}$$

The electric potential V=U/ q_0 is a scalar quantity and has units of joules per coulomb (J/C), where 1 J/C=1 V.
The potential difference ΔV between points *A* and *B* in an electric field E is defined as

$$\Delta V = \frac{\Delta U}{q_0} = -\int_A^B \mathbf{E} \cdot d\mathbf{s}$$

The potential difference between two points *A* and *B* in a uniform electric field E is

$$\Delta V = -Ed$$

where *d* is the magnitude of the displacement in the direction parallel to E.
An equipotential surface is one on which all points are at the same electric potential. Equipotential surfaces are perpendicular to electric field lines.
If we define V=0 at $r_A=\infty$ the electric potential due to a point charge at any distance *r* from the charge is

$$V = k_e \frac{q}{r}$$

We can obtain the electric potential associated with a group of point charges by summing the potentials due to the individual charges.
The potential energy associated with a pair of point charges separated by a distance r_{12} is

$$U = k_e \frac{q_1 q_2}{r_{12}}$$

This energy represents the work required to bring the charges from an infinite separation to the separation r_{12} . We obtain the potential energy of a distribution of point charges by summing terms like Equation 3.12 over all pairs of particles.

TABLE 25.1 Electric Potential Due to Various Charge Distributions

Charge Distribution	Electric Potential	Location
Uniformly charged ring of radius a	$V = k_e \dfrac{Q}{\sqrt{x^2 + a^2}}$	Along perpendicular central axis of ring, distance x from ring center
Uniformly charged disk of radius a	$V = 2\pi k_e \sigma[(x^2 + a^2)^{1/2} - x]$	Along perpendicular central axis of disk, distance x from disk center
Uniformly charged, *insulating* solid sphere of radius R and total charge Q	$V = k_e \dfrac{Q}{r}$	$r \geq R$
	$V = \dfrac{k_e Q}{2R}\left(3 - \dfrac{r^2}{R^2}\right)$	$r < R$
Isolated *conducting* sphere of radius R and total charge Q	$V = k_e \dfrac{Q}{r}$	$r > R$
	$V = k_e \dfrac{Q}{R}$	$r \leq R$

If we know the electric potential as a function of coordinates *x*, *y*, *z*, we can obtain the components of the electric field by taking the negative derivative of the electric potential with respect to the coordinates. For example, the *x* component of the electric field is

$$E_x = -\frac{dV}{dx}$$

The electric potential due to a continuous charge distribution is

$$V = k_e \int \frac{dq}{r}$$

Every point on the surface of a charged conductor in electrostatic equilibrium is at the same electric potential. The potential is constant everywhere inside the conductor and equal to its value at the surface.