C H A P T E R ONE (ELECTRIC FIELDS)

PROPERTIES OF ELECTRIC CHARGES:

The electric charge has the following important properties (according to Franklin's model of electricity):

•Two kinds of electric charges occur in nature, with the property that unlike charges attract one another and like charges repel one another.

• **Electric charge is conserved** (this means that, charge is not created in the process. The electric field state is due to a *transfer* of charge from one object to the other. One object gains some amount of negative charge while the other gains an equal amount of positive charge. For example, when a glass rod is rubbed with silk, the silk obtains a negative charge that is equal in magnitude to the positive charge on the glass rod.

• **Electric charge is quantized (**this means that the net charge in a closed region remains the same).

INSULATORS AND CONDUCTORS:

It is convenient to classify substances in terms of their ability to conduct electric charge:

1-Electrical conductors(materials in which electric charges move freely such as copper, aluminum, and silver) When such materials are charged in some small region, the charge readily distributes itself over the entire surface of the material.

2-Electrical insulators(materials in which electric charges cannot move freely such as glass, rubber, and wood)When such materials are charged by rubbing, only the area rubbed becomes charged, and the charge is unable to move to other regions of the material.

3- Semiconductors are a third class of materials, and their electrical properties are somewhere between those of insulators and those of conductors such as silicon and germanium.

(**Note**: When a conductor is connected to the Earth by means of a conducting wire or pipe, it is said to be grounded.)

Charging by induction:

To understand induction, consider a neutral (uncharged) conducting sphere insulated from ground, as shown in Fig.1a. When a negatively charged rubber rod is brought near the sphere, the region of the sphere nearest the rod obtains an excess of positive charge while the region farthest from the rod obtains an equal excess of negative charge, as shown in Fig.1b. (That is, electrons in the region nearest the rod migrate to the opposite side of the sphere. This occurs even if the rod never actually touches the sphere.) If the same experiment is performed with a conducting wire connected from the sphere to ground (Fig. 1c),some of the electrons in the conductor are so strongly repelled by the presence of the negative charge in the rod that they move out of the sphere through the ground wire and into the Earth. If the wire to ground is then removed (Fig.1d), the conducting sphere contains an excess of *induced* positive charge. When the rubber rod is removed from the vicinity of the sphere (Fig. 1e), this induced positive charge remains on the ungrounded sphere. Note that the charge remaining on the sphere is uniformly distributed over its surface because of the repulsive forces among the like charges. Also note that the rubber rod loses none of its negative charge during this process.

Charging an object by induction requires no contact with the body inducing the charge. This is in contrast to charging an object by rubbing (that is, by *conduction*), which does require contact between the two objects.A process similar to induction in conductors takes place in insulators.

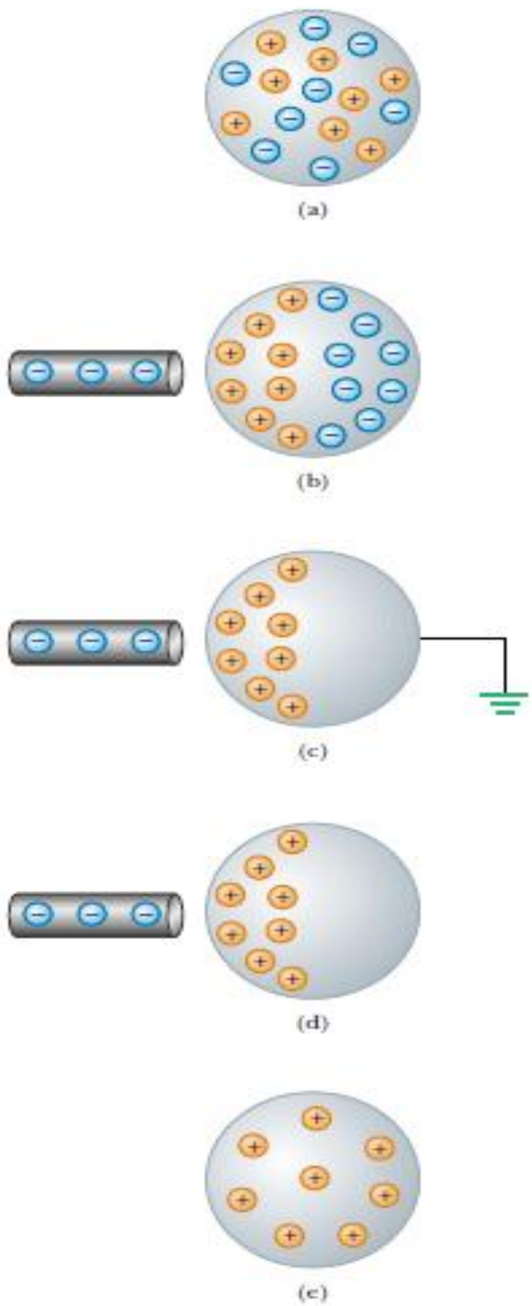

Figure 1 Charging a metallic object by *induction* (that is, the two objects never touch each other). (a) A neutral metallic sphere, with equal numbers of positive and negative charges.(b) The charge on the neutral sphere is redistributed when a charged rubber rod is placed near the sphere. (c) When the sphere is grounded, some of its electrons leave through the ground wire. (d) When the ground connection is removed, the sphere has excess positive charge that is nonuniformly distributed. (e) When the rod is removed, the excess positive charge becomes uniformly distributed over the surface of the sphere.

COULOMB'S LAW

Coulomb's experiments showed that the electric force between two stationary charged particles

• is inversely proportional to the square of the separation **r** between the particles and directed along the line joining them;

• is proportional to the product of the charges q_1 and q_2 on the two particles;

• is attractive if the charges are of opposite sign and repulsive if the charges have the same sign.

From these observations, we can express Coulomb's law as an equation giving the magnitude of the electric force (sometimes called the *Coulomb force*) between two point charges:

$$F = k\frac{|q_1||q_2|}{r^2}$$

Where **k** is a constant called the Coulomb constant.The value of the Coulomb constant depends on the choice of units. The SI unit of charge is the coulomb (C). The Coulomb constant **k** in SI units has the value:

$$k = 8.987\,5 \times 10^{9}\ \text{N.m}^2/\text{C}^2$$

For most purposes, the approximation $k = 9 \times 10^{9}$ **N.m²/C²** is sufficiently accurate

This constant is also written in the form:

$$k = \frac{1}{4\pi\varepsilon_o}$$

Where the constant ε_o is known as the *permittivity of free space* and has the value $8.85 \times 10^{-12}\ \text{C}^2/\text{N.m}^2$.

The smallest unit of charge known in nature is the charge on an electron or proton, which has an absolute value of

$$|e| = 1.6 \times 10^{-19}\ \text{C}$$

Table 1 Charge and Mass of the Electron, Proton, and Neutron

Particle	Charge (C)	Mass (kg)
Electron (e)	-1.6×10^{-19}	9.11×10^{-31}
Proton (p)	$+1.6 \times 10^{-19}$	$1.672\,5 \times 10^{-27}$
Neutron (n)	0	$1.674\,8 \times 10^{-27}$

EXAMPLE 1:-The electron and proton of a hydrogen atom are separated (on the average) by a distance of approximately 5.3 $\times 10^{-11}$ m. Find the magnitudes of the electric force and the gravitational force between the two particles.

Solution: From Coulomb's law, we find that the attractive electric force has the magnitude

$$F_e = k\frac{|q_1||q_2|}{r^2}$$

$F_e = (9 \times 10^9 \ N.m^2/C^2)\ (1.6 \times 10^{-19}\ C)^2/(5.3 \times 10^{-11}\ m)^2$

$F_e = 8.2 \times 10^{-8}\ N$

Using Newton's law of gravitation and Table 1 for the particle masses, we find that the gravitational force has the magnitude

$$F_g = \frac{G\, m_1\, m_2}{r^2}$$

$F_g = (6.7 \times 10^{-11} N.m^2/kg^2) \times (9.11 \times 10^{-31} kg)(1.67 \times 10^{-27} kg)/(5.3 \times 10^{-11})^2$

$F_g = 3.6 \times 10^{-47}\ N$

The ratio Thus, the gravitational force between charged atomic particles is negligible when compared with the electric force. Note the similarity of form of Newton's law of gravitation and Coulomb's law of electric forces.

* * *

The force is a vector quantity thus, the law expressed in vector form:

$$F_{12} = k\frac{|q_1||q_2|}{r^2}\,\hat{\mathbf{r}} \qquad (1)$$

Where $\hat{\mathbf{r}}$ is a unit vector directed from $\mathbf{q_1}$ to $\mathbf{q_2}$, as shown in Figure (2a)

from equation(1) , we see that if q_1 and q_2 have the same sign, as in Figure 2a, the product q_1q_2 is positive and the force is repulsive. If q_1 and q_2 are of opposite sign, as shown in Figure 2b, the product q_1q_2 is negative and the force is attractive. Noting the sign of the product q_1q_2 is an easy way of determining the direction of forces acting on the charges

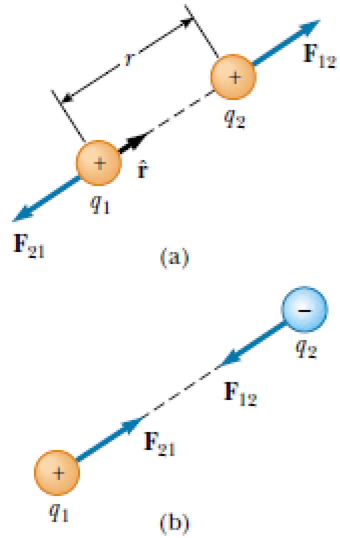

Figure 2*Two point charges separated by a distance r exert a force on each other that is given by Coulomb's law. The force F_{21} exerted by q_2 on q_1 is equal in magnitude and opposite in direction to the*

When more than two charges are present, the force between any pair of them is given by Equation (1). Therefore, the resultant force on any one of them equals the vector sum of the forces exerted by the various individual charges. For example, if four charges are present, then the resultant force exerted by particles2, 3, and 4 on particle 1 is:

$$\mathbf{F_1=F_{21}+F_{31}+F_{41}}$$

EXAMPLE 2:-Consider three point charges located at the corners of a right triangle as shown in Figure 3, where $q_1=q_3=5\mu c$,$q_2=-2\ \mu c$, and a=0.1m. Find the resultant force exerted on q_3 .

Solution : First, note the direction of the individual forces exerted by q_1 and q_2 on q_3 . The force F_{23} exerted by q_2 on q_3 is attractive because q_2 and q_3 have opposite signs. The force F_{13} exerted by q_1 on q_3 is repulsive because both charges are positive. The magnitude of F_{23} is:

$\mathbf{F_{23}= k\ |q_2\ ||q_3|\ /\ a^2}$

$F_{23}= (9\ x10^9 N.m^2/C^2)x(2.0\ x10^{-6}\ C)(5.0\ x10^{-6}\ C)/(0.10\ m)^2$

$F_{23}= 9.0N$

Note that because q_3 and q_2 have opposite signs, F_{23} is to the left, as shown in Figure 4.
The magnitude of the force exerted by q_1 on q_3 is:
$F_{13} = k\,|q_1|\,|q_3| / (\sqrt{2}a)^2$
$F_{13} = (9 \times 10^9\ N.m^2/C^2)(5.0 \times 10^{-6}\ C)(5.0 \times 10^{-6}\ C)/2(0.10\ m)^2$
$F_{13} = 11 N$

The force F_{13} is repulsive and makes an angle of 45° with the x axis. Therefore, the x and y components of F_{13} are equal, with magnitude given by :
$F_{13} \cos 45° = 7.9\ N$
The force F_{23} is in the negative x direction. Hence, the x and y components of the resultant force acting on q_3 are:

$F_{3x} = F_{13x} + F_{23} = 7.9\ N - 9.0\ N = -1.1\ N$
$F_{3y} = F_{13y} = 7.9\ N$

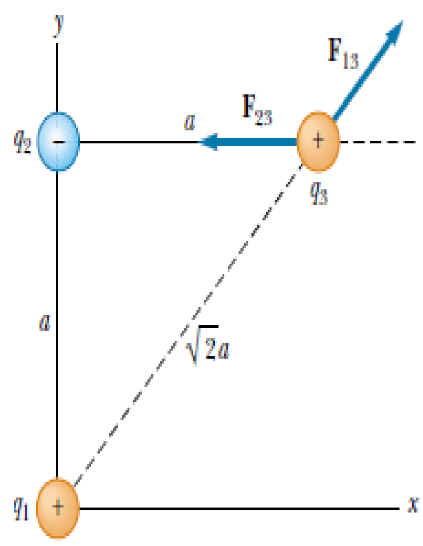

Figure 3: The force exerted by q_1 on q_3 is F_{13} The force exerted by q_2 on q_3 is F_{23} . The resultant force F_3 exerted on q_3 is the vector sum $F_{13} + F_{23}$.

THE ELECTRIC FIELD

the electric field **E** at a point in space is defined as the electric force $\mathbf{F_e}$ acting on a positive test charge $\mathbf{q_0}$ placed at that point divided by the magnitude of the test charge:

$$E = \frac{F}{q_o}$$

Note that **E** is the field produced by some charge external to the test charge it is not the field produced by the test charge itself. For example, every electron comes with its own electric field. The vector **E** has the SI units of newton per coulomb (**N/C**). To determine the direction of an electric field, consider a point charge q located a distance **r** from a test charge $\mathbf{q_0}$ located at a point **P**, as shown in Figure (5) **q** According to Coulomb's law, the force exerted by **q** on the test charge is

$$E = k\frac{q\,q_o}{q_o\,r^2}\ \hat{\mathbf{r}}$$

where $\hat{\mathbf{r}}$ is a unit vector directed from **q** toward $\mathbf{q_0}$. Because the electric field at **P**, the position of the test charge, is defined by $E \equiv F_e/q_0$,we find that at **P**, the electric field created by **q** is:

$$E = k\frac{q}{r^2}\ \hat{\mathbf{r}}$$

If is positive, as it is in Figure (5a), the electric field is directed radially out ward from it. If q is negative, as it is in Figure (5b), the field is directed toward it.

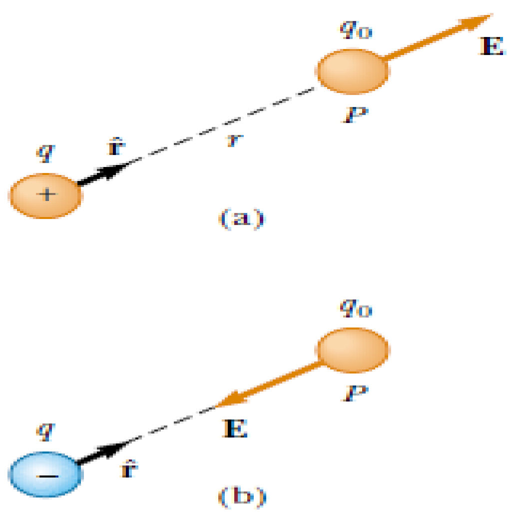

Figure 5: A test charge q_0 at point P is a distance r from a point charge q. (a) If q is positive, then the electric field at P points radially outward from q. (b) If q is negative, then the electric field at P points radially inward toward q.

- at any point **P**, the total electric field due to a group of charges equals the vector sum of the electric fields of the individual charges

$$\boldsymbol{E} = \boldsymbol{k}\sum_{i} \frac{\boldsymbol{q_i}}{\boldsymbol{r_i^2}}\, \boldsymbol{r_i}\hat{}$$

Where $\mathbf{r_i}$ is the distance from the ith charge $\mathbf{q_i}$ to the point P (the location of the test charge) and $\mathbf{r_i}\hat{}$ is a unit vector directed from $\mathbf{q_i}$ toward **P**.

EXAMPLE 3:-A charge q_1=7.0 μC is located at the origin, and a second Charge q_2=5.0 μC is located on the **x-**axis, 0.30 m from the origin (Fig.6). Find the electric field at the point **P**, which has coordinates (0, 0.40) m.

Solution First, let us find the magnitude of the electric field at P due to each charge. The fields E_1 due to the 7.0 μC charge and E_2 due to the -5.0 μC charge are shown in Figure(6)their magnitudes are:

$E_1 = k|q_1| / r_1^2 = (9 \times 10^9\ N.m^2/C^2)\ (7.0 \times 10^{-6}\ C) / (0.40\ m)^2$
$E_1 = 3.9 \times 10^5 N/C$

$E_2 = k|q_2| / r_2^2 = (9 \times 10^9\ N.m^2/C^2)\ (5.0 \times 10^{-6}\ C) / (0.50\ m)^2$
$E_2 = 1.8 \times 10^5 N/C$

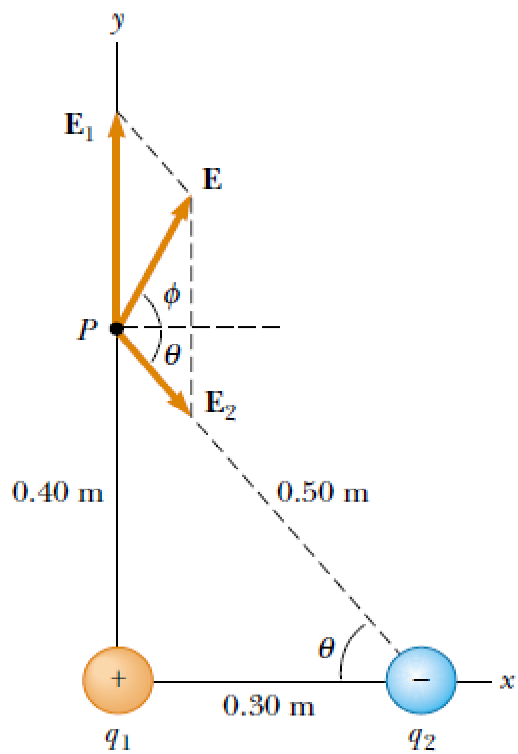

Figure 6 The total electric field E at P equals the vector sum E_1+E_2 where E_1 is the field due to the positive charge q_1 and E_2 is the field due to the negative charge q_2

EXAMPLE 4:- An **electric dipole** is defined as a positive charge **q** and a negative charge **-q** separated by some distance. For the dipole shown in Figure 7, find the electric field **E** at **P** due to the charges, where **P** is a distance **y>>a** from the origin.

Solution: At P, the fields $\mathbf{E_1}$ and $\mathbf{E_2}$ due to the two charges are equal in magnitude because **P** is equidistant from the charges. The total field is:

$\mathbf{E = E_1 + E_2}$, where

$\mathbf{E_1 = E_2 = k\,q/r^2 = k\,q\,/\,(y^2 + a^2)}$

The **y** components of $\mathbf{E_1}$ and $\mathbf{E_2}$ cancel each other, and the **x** components add because they are both in the positive **x** -direction. Therefore, **E** is parallel to the **x** axis and has a magnitude equal to $\mathbf{2E_1\cos\theta}$. From Fig. 7 we see that

$\mathbf{\cos\theta = a/r = a/(\,y^2 + a^2\,)^{1/2}}$ Therefore,

$\mathbf{E = 2E_1\cos\theta = 2\,k\,q\,a/\,(y^2 + a^2)\,(\,y^2 + a^2\,)^{1/2}}$

$\mathbf{E = 2\,k\,q\,a\,/\,(\,y^2 + a^2\,)^{3/2}}$

Because **y>>a** we can neglect a^2 and write

$\mathbf{E \approx 2\,k\,q\,a\,/\,y^3}$

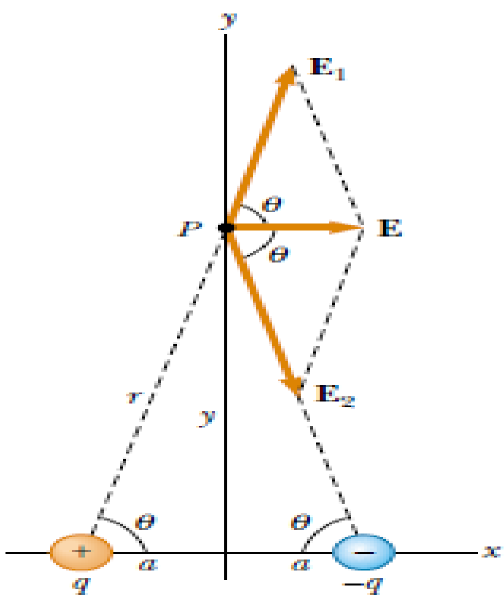

Figure 7 The total electric field E atP due to two charges of equal magnitude and opposite sign (an electric dipole) equals the vector sum $E_1 + E_2$. The field E_1 is due to the positive charge q, and E_2 is the field due to the negative charge -q.

ELECTRIC FIELD OF A CONTINUOUS CHARGE DISTRIBUTION

To evaluate the electric field created by a continuous charge distribution, we use the following procedure: First, we divide the charge distribution into small elements, each of which contains a small charge $\mathbf{dq}$, as shown in Figure 8.
The electric field at *P* due to one element carrying charge Δ*q* is

$$dE = k\frac{dq}{r^2}\,\mathbf{r}\hat{}$$

Where *r* is the distance from the element to point *P* and **r̂** is a unit vector directed from the charge element toward *P*. The total electric field at *P* due to all elements in the charge distribution is approximately

$$dE = k\sum_i \frac{dq_i}{r_i^2}$$

where the index **i** refers to the **i th** element in the distribution. Because the charge distribution is approximately continuous so:

$$E = k\int \frac{dq}{r^2}\,r\hat{} \quad \textbf{(Electric field of a continuous charge distribution)}$$

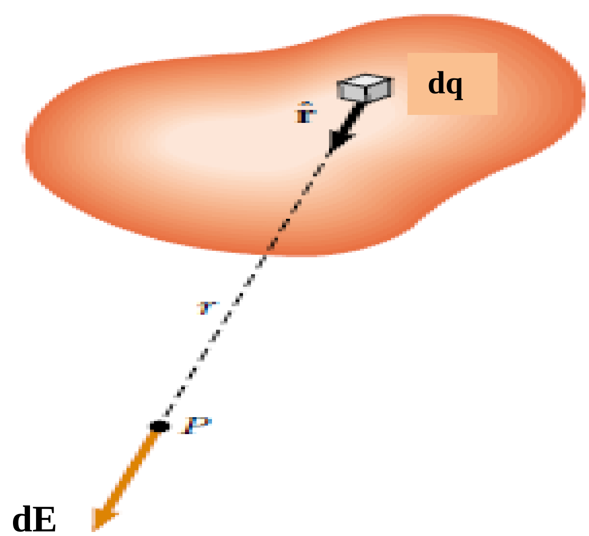

Figure 8: **The electric field at *P* due to a continuous charge distribution is the vector sum of the fields ΔE due to all the elements Δq of the charge distribution.**

Note

• If a charge *Q* is uniformly distributed throughout a volume *V*, the volume charge density ρ is defined by:

$$\rho = Q/V$$

where ρ has units of coulombs per cubic meter (C/m^3).

• If a charge *Q* is uniformly distributed on a surface of area *A*, the surface charge density σ is defined by:

$$\sigma = Q/A$$

where σ has units of coulombs per square meter (C/m^2).

• If a charge *Q* is uniformly distributed along a line of length L, the linear charge density λ is defined by:

$$\lambda = Q/L$$

where λ has units of coulombs per meter (C/m).

• If the charge is no uniformly distributed over a volume, surface, or line, we have to express the charge densities as

$\rho = dQ/dV$ $\quad\sigma = dQ/dA$ $\quad\lambda = dQ/dL$

where *dQ* is the amount of charge in a small volume, surface, or length element.

EXAMPLE 5:- A rod of length **L** has a uniform positive charge per unit Length **λ** and a total charge ***Q***. Calculate the electric field at a point ***P*** that is located along the long axis of the rod and a distance ***a*** from one end (Fig. 9).

Solution: Let us assume that the rod is lying along the ***x*** axis, that ***dx*** is the length of one small segment, and that ***dq*** is the charge on that segment. Because the rod has a charge per unit length **λ**, the charge *dq* on the small segment is **$dq = \lambda dx$**
The field ***d*E** due to this segment at ***P*** is in the negative ***x*** direction (because the source of the field carries a positive charge ***Q***), and its magnitude is:

$$dE = k\, dq / x^2 = k \lambda\, dx / x^2$$

The total field at ***P*** due to all segments of the rod, which are at different distances from ***P***, is given by :

$$E = \int k \lambda\, dx / x^2$$

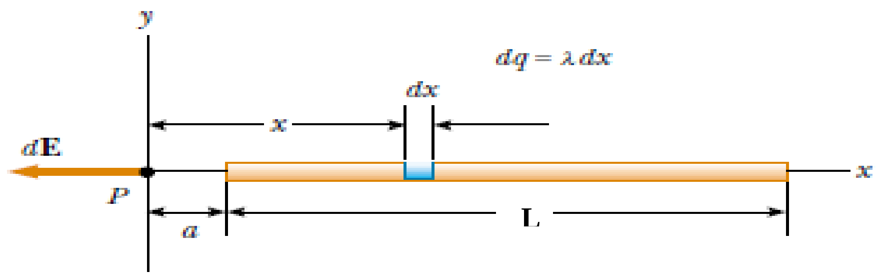

Figure 9 The electric field at *P* due to a uniformly charged rod lying along the *x* axis. The magnitude of the field at *P* due to the segment of charge *dq* is kdq/x^2. The total field at *P* is the vector sum over all segments of the rod.

where the limits on the integral extend from one end of the rod (**x=a**) to the other(**x=L+a**). The constants $\mathbf{k_e}$ and **λ** can be removed from the integral to yield

$$E = k\lambda \int dx / x^2 = k\lambda\, [-1/x]$$

$$E = k\, \lambda\, \{(1/a)-(1/(L+a)\}$$

$$E = k\, Q/a\, (L+a)$$

Where we have used the fact that the total charge ***Q* =λL .** If ***P*** is far from the rod (**a>>L),** then the L in the denominator can be neglected, and

$E = k\,Q / a^2$ .This is just the form you would expect for a point charge.

EXAMPLE 6:

A ring of radius ***a*** carries a uniformly distributed positive total charge ***Q***. Calculate the electric field due to the ring at a point ***P*** lying a distance ***x*** from its center along the central axis perpendicular to the plane of the ring (Fig. 10).

Solution The magnitude of the electric field at ***P*** due to the segment of charge ***dq*** is:

$$dE = k\, dq / r^2$$

This field has an ***x*** component $dE_x = dE\cos\theta$ along the axis and a component $dE_\perp$ perpendicular to the axis. As we see in Figure 10b, however, the resultant field at ***P*** must lie along the ***x*** axis because the perpendicular components of all the various charge segments sum to zero. That is, the perpendicular component of the field created by any charge element is canceled by the perpendicular component created by an element on the opposite side of the ring. Because and $r=(x^2+a^2)^{1/2}$ and $\cos\theta = x/r$, we find that:

$$dE_x = dE\cos\theta = (k\, dq/ r^2)\, x/ r = k\, x\, dq / (x^2 + a^2)^{3/2}$$

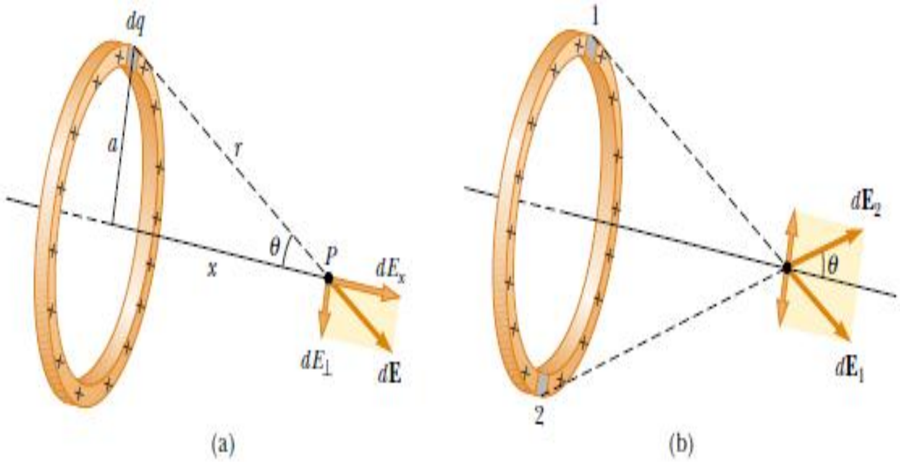

Figure 10 A uniformly charged ring of radius *a*. (a) The field at *P* on the *x* axis due to an element of charge *dq*. (b) The total electric field at *P* is along the *x* axis. The perpendicular component of the field at *P* due to segment 1 is canceled by the perpendicular component due to segment 2.

All segments of the ring make the same contribution to the field at ***P*** because they are all equidistant from this point. Thus, we can integrate to obtain the total field at ***P*** :

$$E_x = \int k\,x\,dq / (x^2 + a^2)^{3/2} = \{ k\,x / (x^2 + a^2)^{3/2} \} \int dq$$

$$E_x = k\,x\,Q / (x^2 + a^2)^{3/2}$$

This result shows that the field is zero at $x = 0$