The Einstein Coefficients

Let N_1 and N_2 be the number of atoms per unit volume present in the energy levels E_1 and E_2, respectively. Let $u(\omega)d\omega$ be the energy per unit volume of the incident radiation between frequency ω and $\omega + d\omega$.

$\omega = (E_1 - E_2)/\hbar$

(i) An atom in the lower energy level E_1 can absorb the incident radiation and go to the higher energy level E_2. The number of atoms undergoing absorptions per unit time per unit volume is given by

$$R_{abs} = B_{12} N_1 u(\omega) \qquad (1)$$

The Einstein Coefficients

(ii) In a reverse process, an atom in the higher energy level E_2 can emit radiation at frequency ω and go to the level E_1. This is stimulated emission. The number of atoms undergoing stimulated emissions per unit time per unit volum e is given by

$$R_{st} = B_{21} N_2 u(\omega) \qquad (2)$$

(iii) An atom in the upper state E_2 can also make a spontaneou s emission at frequency ω and go to the level E.The number of atoms undergoing stimulated emissions per unit time per unit volum e is given by

$$R_{sp} = A N_2 \qquad (3)$$

At thermal equilibriu m, the number of upward transitio ns should be equal to downward transitio ns. If they are not equal, radiation energy wil l grow or sink which is not consistent with the assumption of thermal equilibriu m.

The Einstein Coefficients

$$AN_2 + B_{21}N_2u(\omega) = B_{12}N_1u(\omega)$$

$$u(\omega) = \frac{A}{(N_1/N_2)B_{12} - B_{21}} \qquad (4)$$

Using Boltzmann's law, under equilibrium, the ratio of N_2 and N_1 is

$$\frac{N_2}{N_1} = e^{-(E_2-E_1)/k_BT} = e^{-\hbar\omega/k_BT} \qquad (5)$$

k_B = Boltzmann's constant $= 1.38 \times 10^{-23}$ J/K.

According to Planck's theory, the radiation energy density per unit frequency interval is

$$u(\omega) = \frac{\hbar\omega^3 n_0^3}{\pi^2 c^3} \frac{1}{e^{\hbar\omega/k_BT} - 1} \qquad (6)$$

The Einstein Coefficients

Comparing Eq. (4) and (6), we see that

$$B_{21} = B_{12} \equiv B$$

$$A_{21} = \frac{B\hbar\omega^3 n_0^3}{\pi^2 c^3}$$

The coefficien ts A and B are called Einstein coefficien ts.

Observatio ns :

(i) The stimulated emission rate per atom is same as the absorption rate per atom.

(ii) At thermal equilibriu m, the ratio of spontaneou s to stimulated emission is

$$R = \frac{AN_2}{BN_2 u(\omega)} = e^{\hbar\omega / k_B T} - 1$$

If $\hbar\omega >> k_B T$, $R >> 1$ and spontaneou s emission dominates the stimulated emission.

Example 1

Consider an optical source at T = 1000k. Calculate the ratio of the number of spontaneous to stimulated emission for a wavelength of 1.55 microns. Assume $k_B = 1.38 \times 10^{-23}$ J/K, $\hbar = 1.054 \times 10^{-34}$ Js.

$\lambda = 1.55$ microns imples $f = c/\lambda = 3 \times 10^8 / (1.55 \times 10^{-6}) = 1.9355 \times 10^{14} Hz$

The ratio of the number of spontaneous to stimulated emission is given by

$$R = \exp(\hbar 2\pi f / k_B T) - 1$$

$$= \exp(1.054 \times 10^{-34} \times 2 \times \pi \times 1.9355 \times 10^{14} / (1.38 \times 10^{-23} \times 10^3)) - 1$$

$$= 1.08 \times 10^4$$

Therefore at optical frequencies and at this temperature, spontaneous emission dominates and hence the light from usual light sources is incoherent

Population inversion

Light amplification can take place only if $R_{st} > R_{abs}$ or

$$N_2 > N_1.$$

Under thermal equilibrium, it is $N_1 > N_2$. Therefore, N_2 should be increased by external means. The condition $N_2 > N_1$ is referred as *population inversion*.