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***Non linear dynamics in
semiconductor devices***

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Come nascono le parole della scienza

“non tentare le essenze, ma contentarsi delle affezioni quantitative”

(G Galilei, Lettera a M Welser , 1610)

Linguaggio
ordinario

MELA



Linguaggio fisico

Peso



Forma



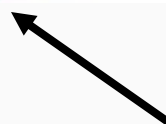
Colore



Sapore



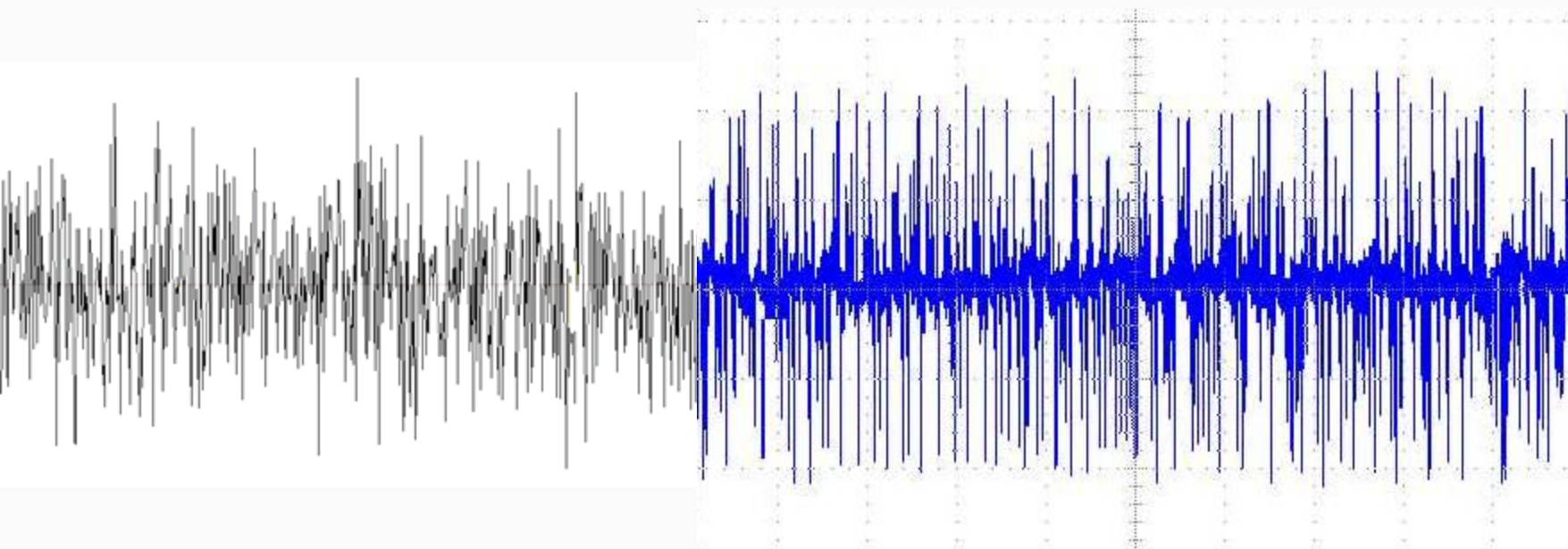
Newton



Posizione e velocità delle
molecole/atomi



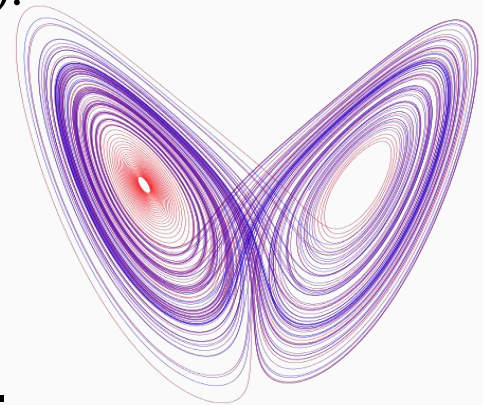
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Chaos general knowledge

- Chaos suggests complete disorder.
(A practical implication of chaos is that its presence makes it essentially impossible to make any long term predictions about the behavior of a dynamical system: while one can in practice only fix the initial conditions of a system to a finite accuracy, their errors increase exponentially fast).
- Butterfly effect 1960, E. Lorenz
- It must be sensitive to initial conditions.
- It must be topologically mixing, and
- Its periodic orbits must be dense.

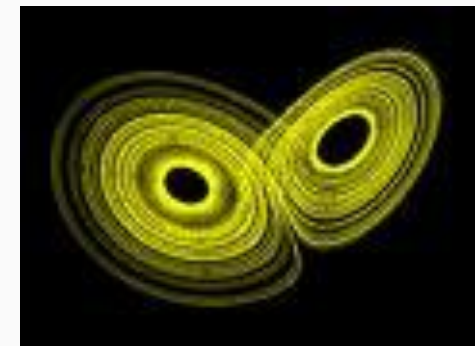
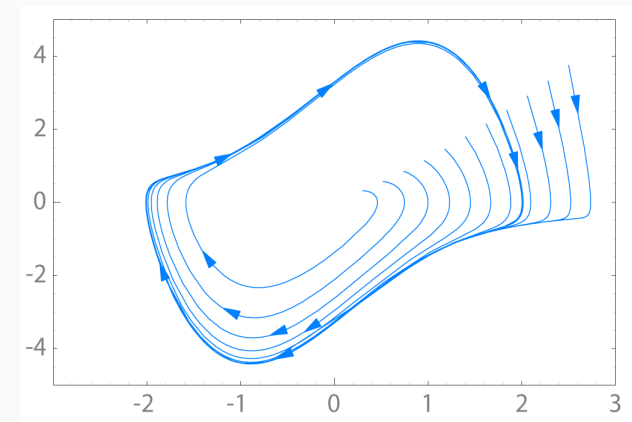




Attractor in phase space

Attractors were thought of as being geometrical subsets of the phase space

- ***Fixed point attractor***
- ***Limit cycle attractor (periodic)***
- ***Strange attractor***



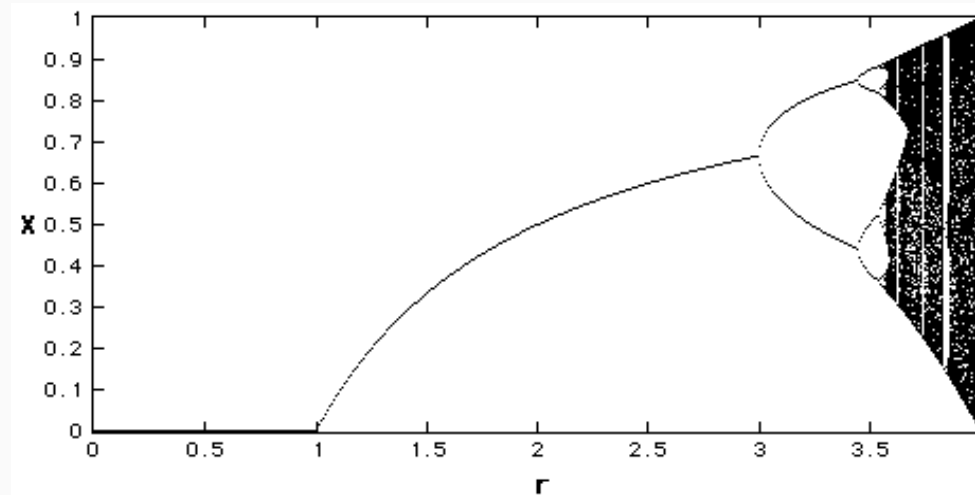
An attractor is informally described as **strange** if it has non-integer dimension or if the dynamics on it are chaotic.



Bifurcation diagram

The bifurcation is a period-doubling, a change from an N -point attractor to a $2N$ -point attractor, which occurs when the control parameter is changed.

A *Bifurcation Diagram* is a visual summary of the succession of period-doubling produced as a control parameter r increases



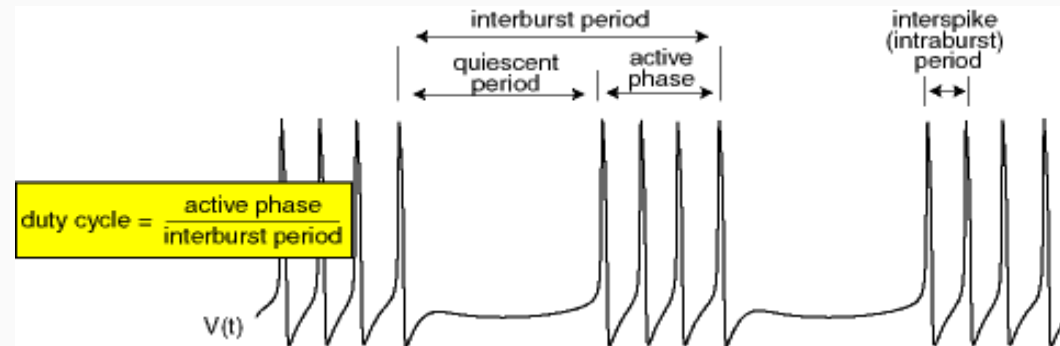
Long time series \longrightarrow Attractor \longrightarrow Bifurcation



The bursting

Bursting is a rapid signaling mode in neurons whereby clusters of two or more action potentials (spikes) are emitted as a single signaling event. A burst of two spikes is called a doublet, three spikes - triplet, four - quadruplet, etc

$$\begin{aligned} \dot{x} &= f(x, y) && \text{(fast spiking)} \\ \dot{y} &= \mu g(x, y) && \text{(slow modulation)} \end{aligned}$$





Bursting of homoclinic pulses using semiconductor laser

Periodic transition between a state of “quiescence” and state of repetitive “firing”/ “spiking”.

$$\dot{x} = f(x, y)$$

$$\dot{y} = g(x, y)\mu$$

- x fast variables → repetitive spiking
- y slow variables → spiking modulation
- μ small parameter ($\ll 1$)



Chaos general knowledge

Optically:

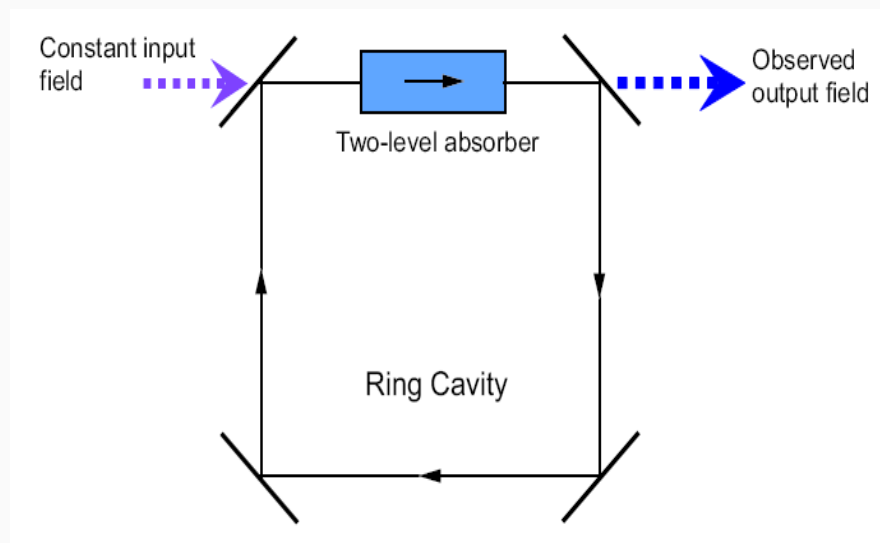
- **Chaos generation by optical feedback.**
- **Chaos generation by opto-electronic feedback.**



- The Dynamical System

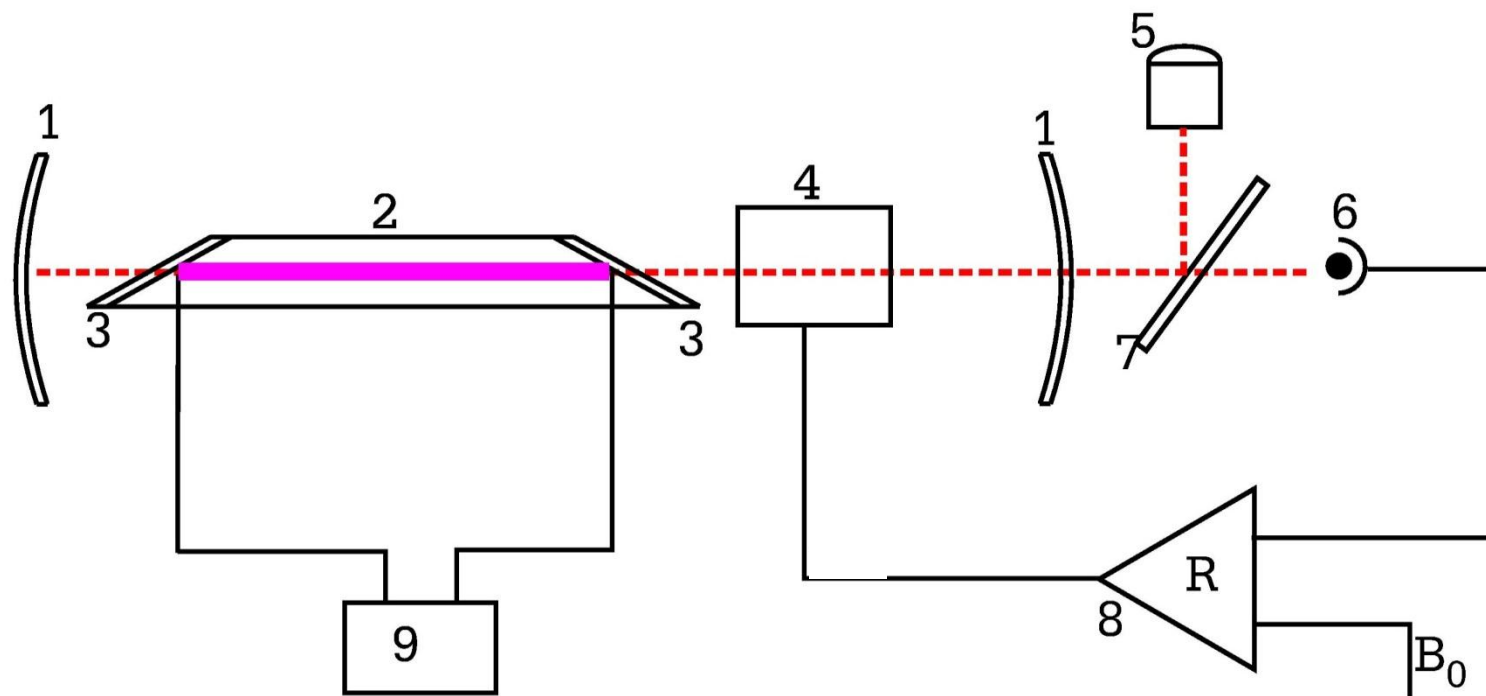
(The theory of TDS is the paradigm for modeling and studying phenomena that undergo spatial and temporal evolution)

- Ikeda scenario
(1973)





CO₂ laser with feedback



- | | |
|-------------------------------|------------------|
| 1- Laser mirror | 5- Power meter |
| 2- CO ₂ laser tube | 6- Detector |
| 3- Brewster window | 7- Beam Splitter |
| 4- Electro-optic modulator | 8- Amplifier |
| 9- Power supply | |

Control parameters: R



Introduction and basic definitions

- Chaos and MMOs generation by **opto-electronic** feedback
- Polarization dynamics by **optical feedback** in CO₂ laser)

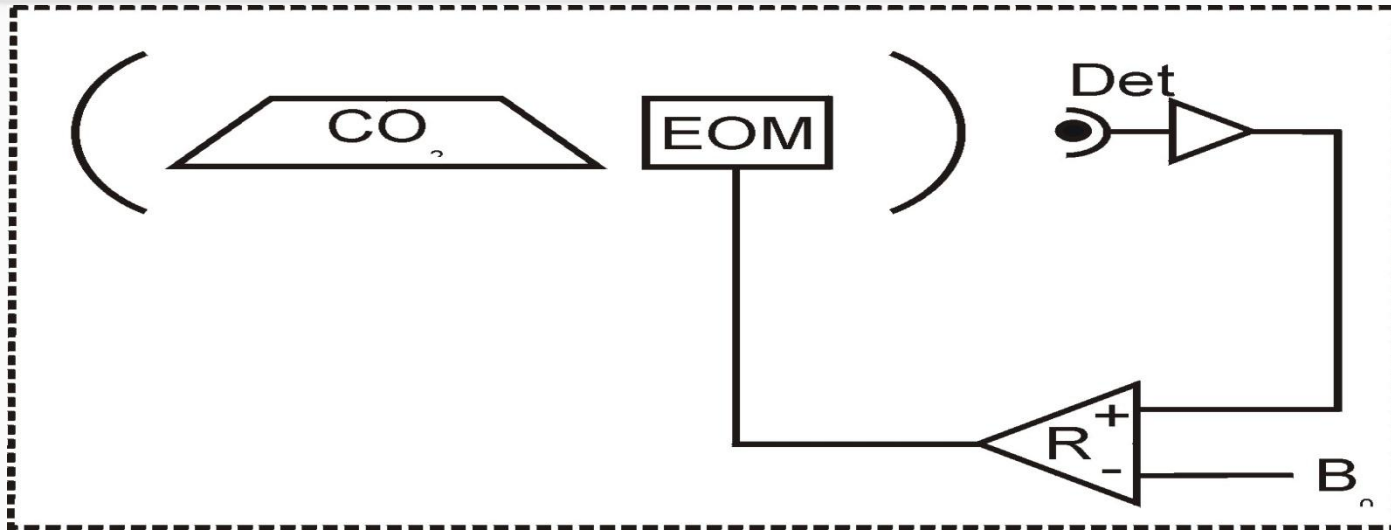


Outline - part 1: CO₂ laser

- Bursting control and synchronization in CO₂ lasers
- Conclusions



Hisroty-CO2 laser with modulation



3D model

$$\dot{x} = -k_0 x (1 - k_1 \sin^2 z) + Gxy$$

$$\dot{y} = -2Gxy - \gamma y + p_0$$

$$\dot{z} = \beta (-z + b_0 - R \cdot x)$$

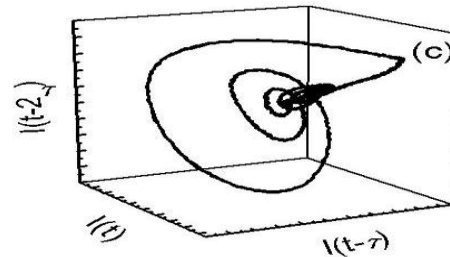
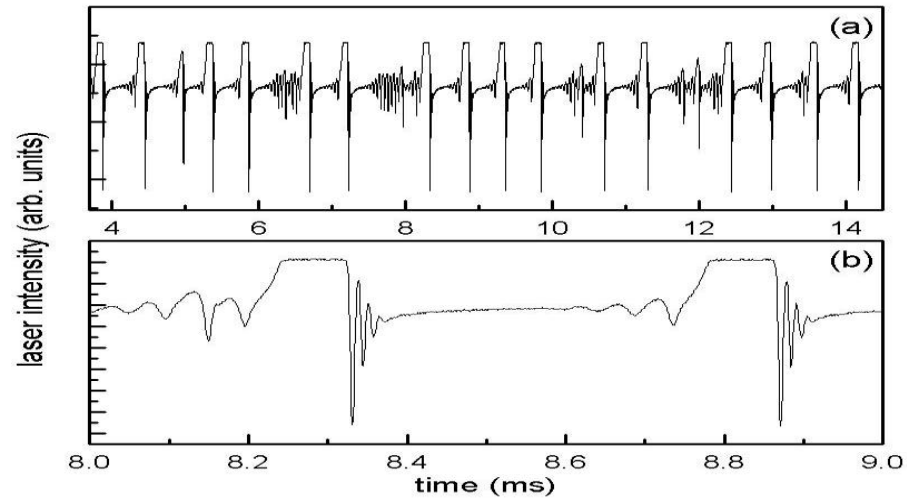
X laser intensity

Y population inversion

z feedback signal



Hisroty-CO₂ laser with modulation



Time series and reconstructed attractor



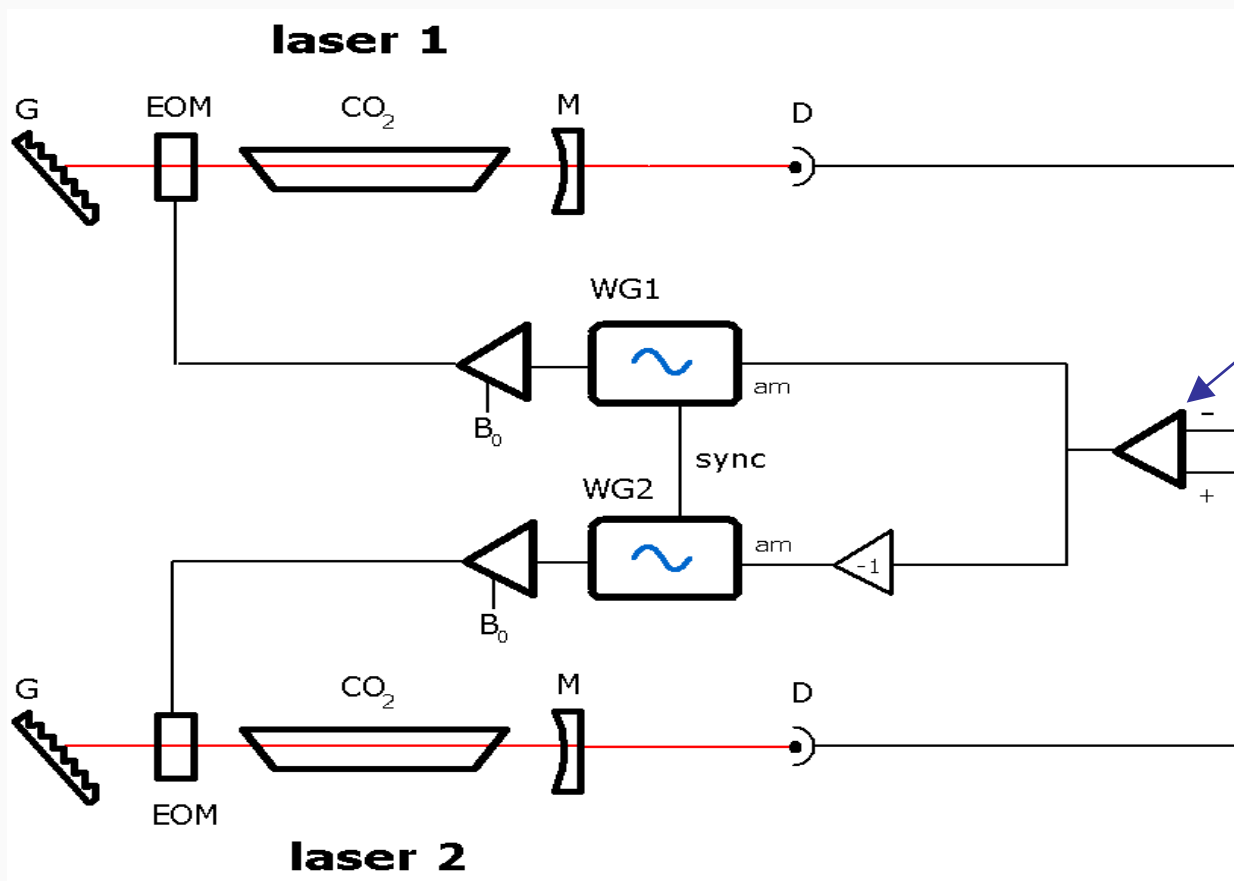
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Synchronization experiments in CO₂ lasers



Bidirectionally coupled modulated lasers

EXPERIMENTAL SETUP

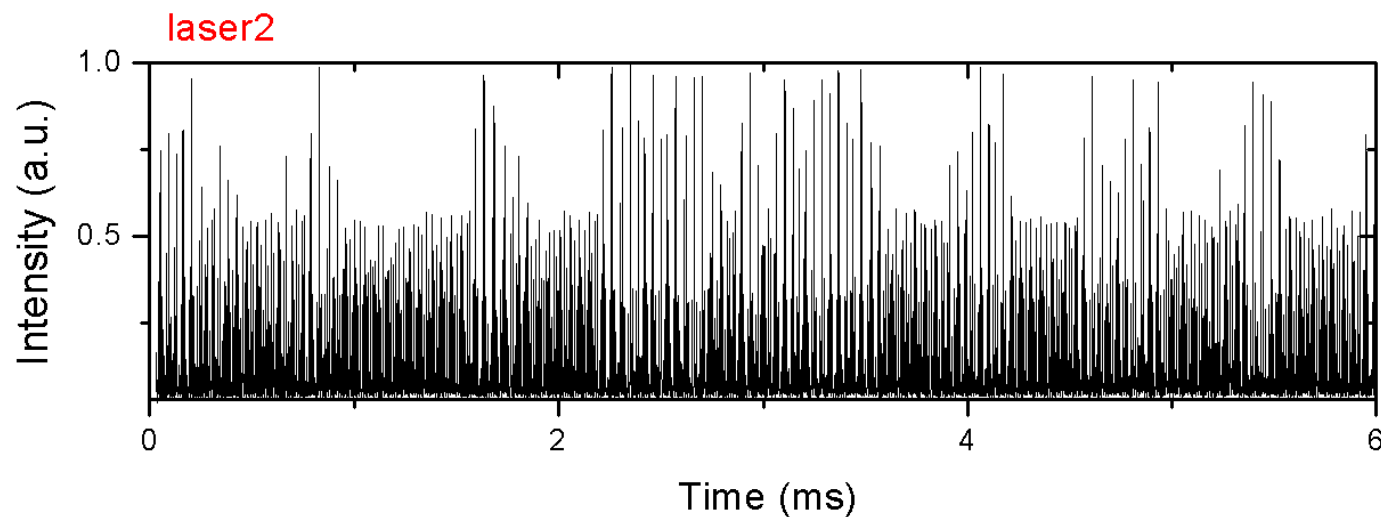
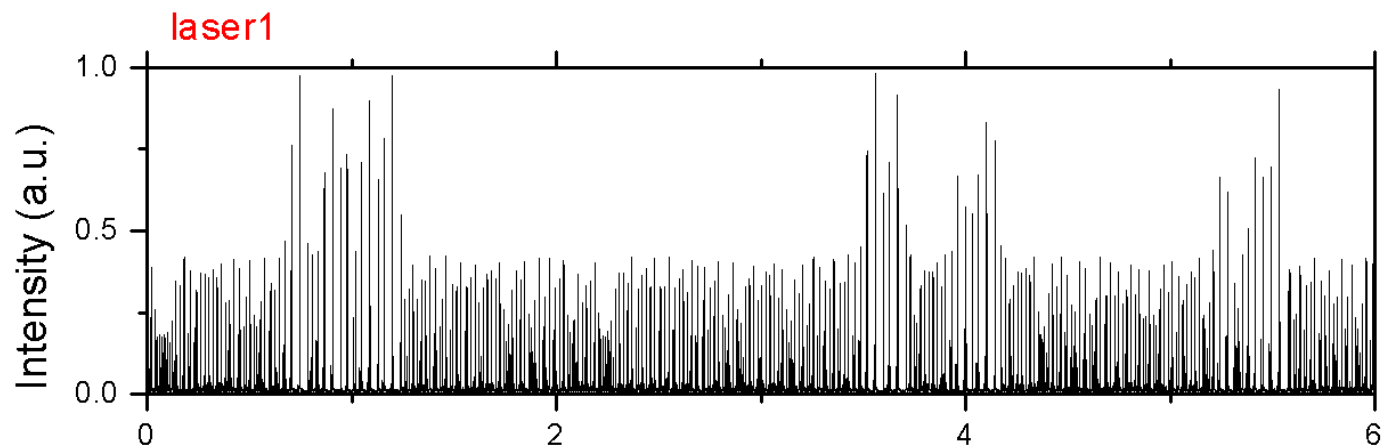


coupling
 ϵ



Time evolution of the lasers (1)

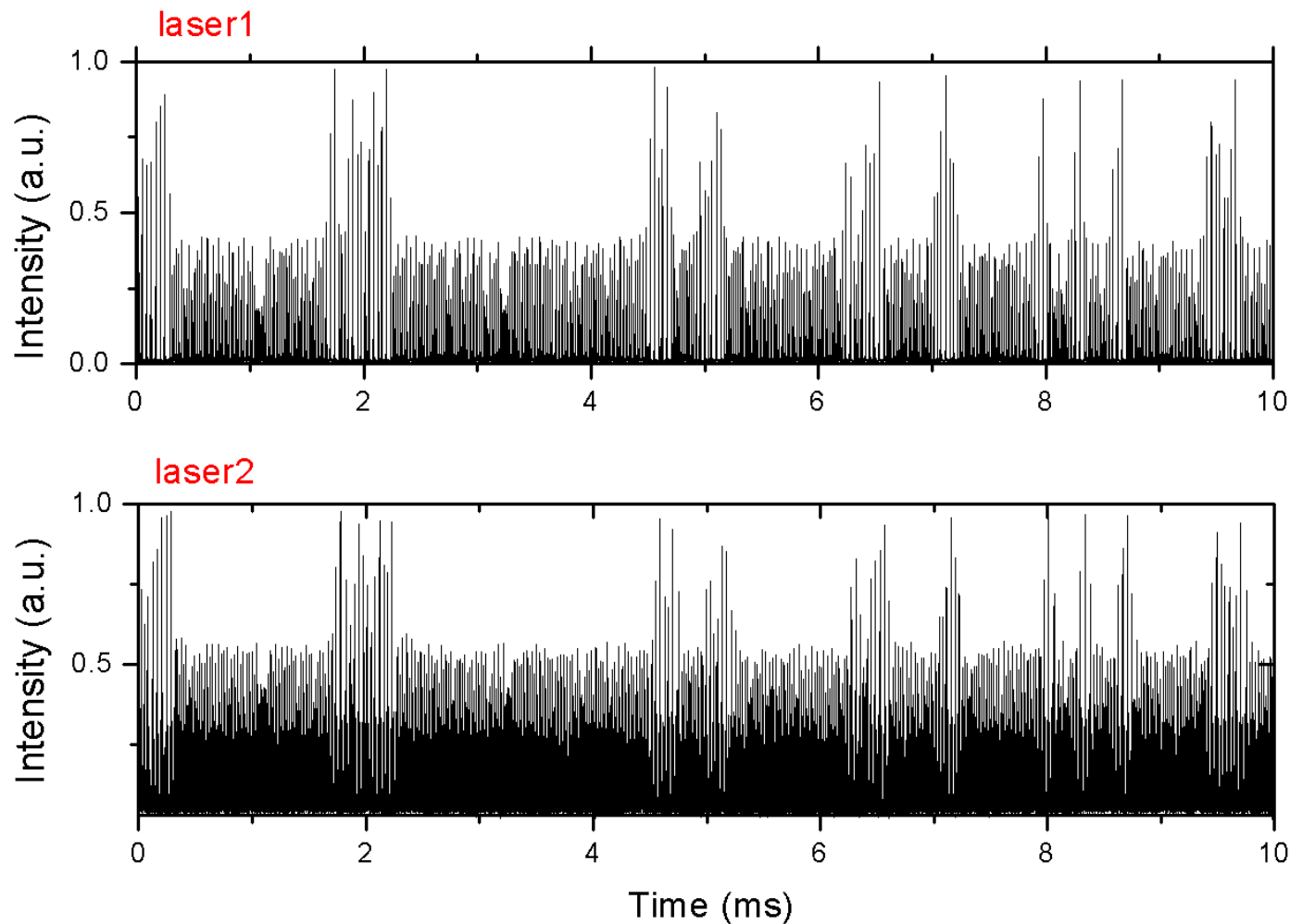
No couplig ($\epsilon=0$)





Time evolution of the lasers (2)

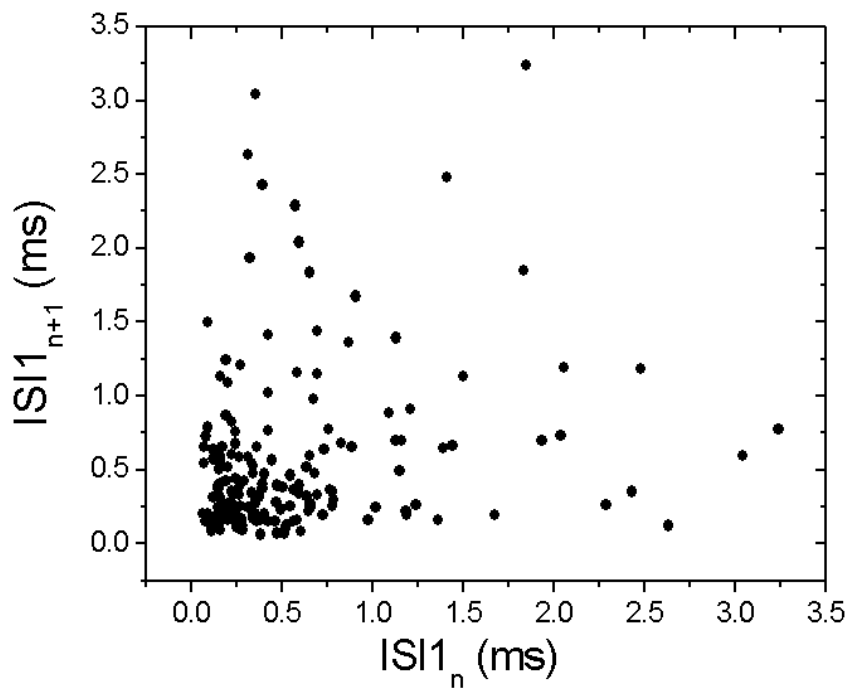
with couplig ($\epsilon=0.05$)



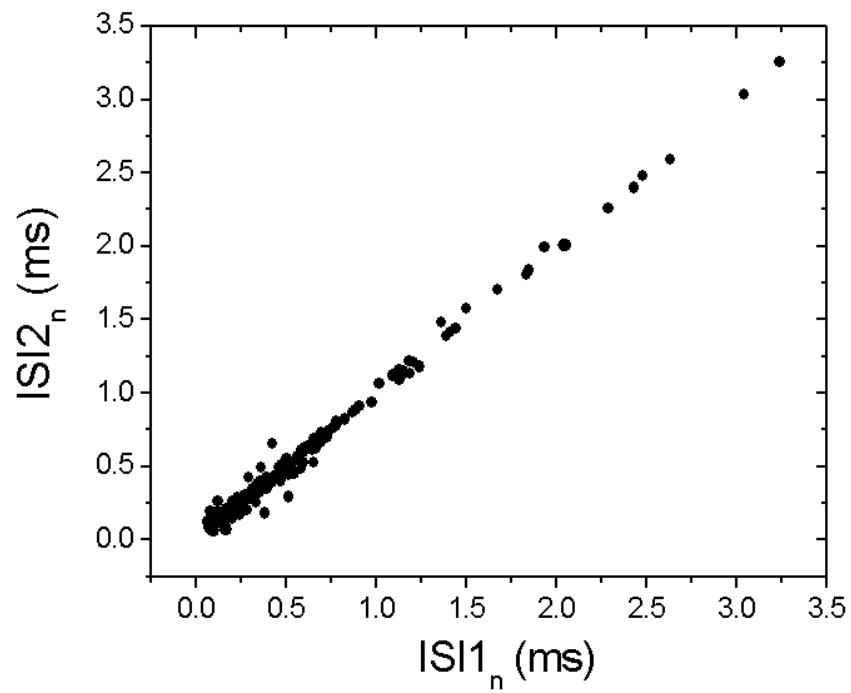


ISI distributions

Auto-correlation (single laser)

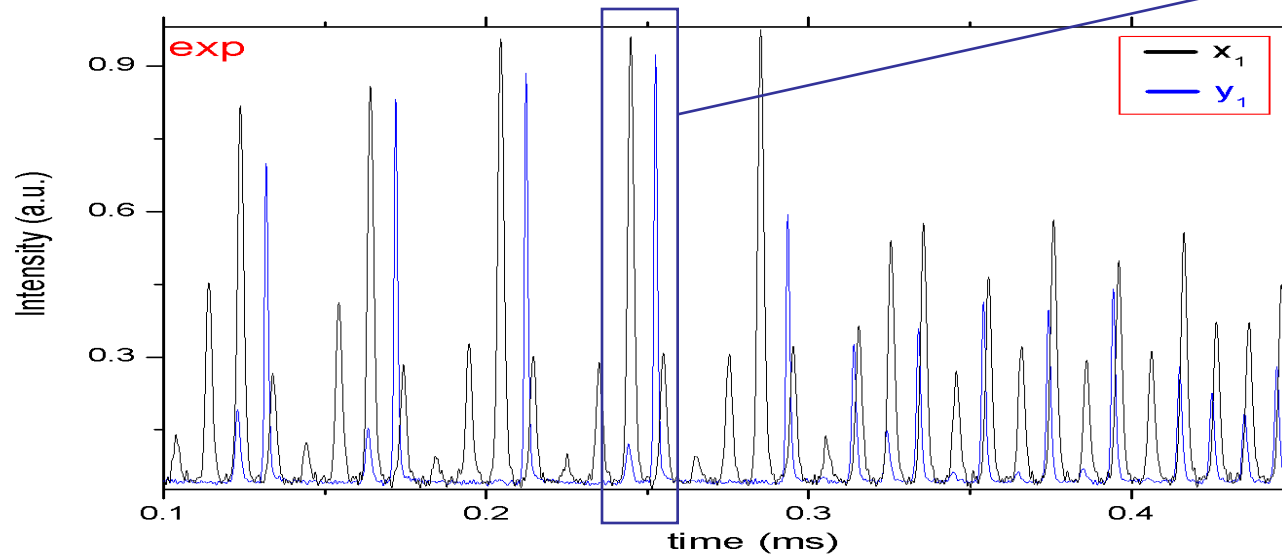
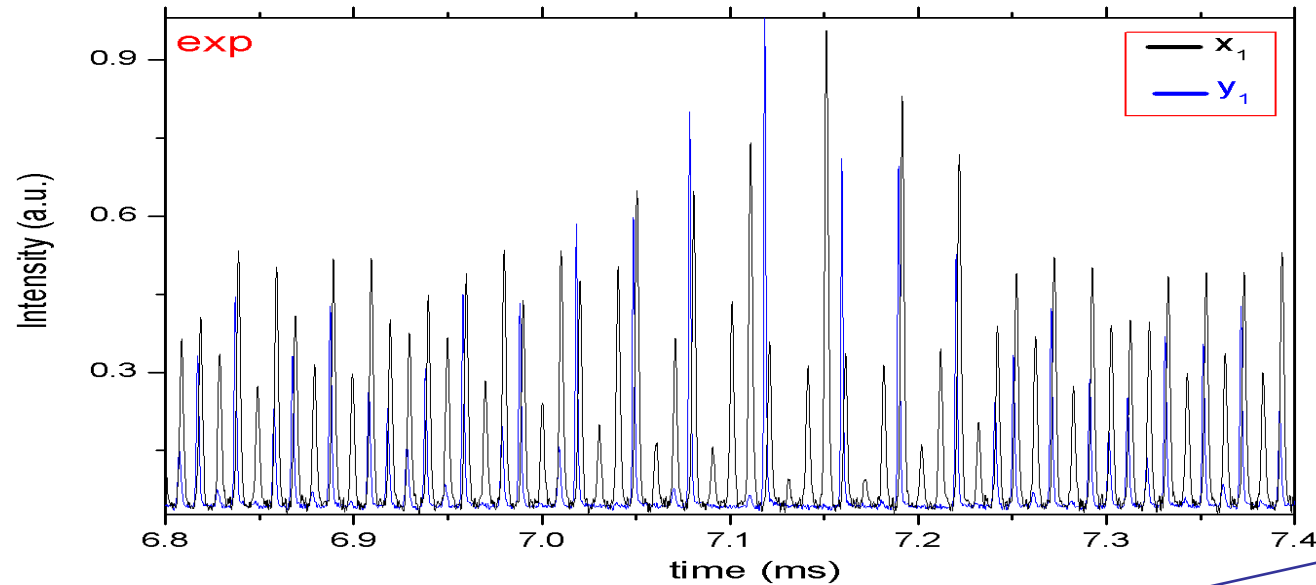


Cross-correlation (two lasers)





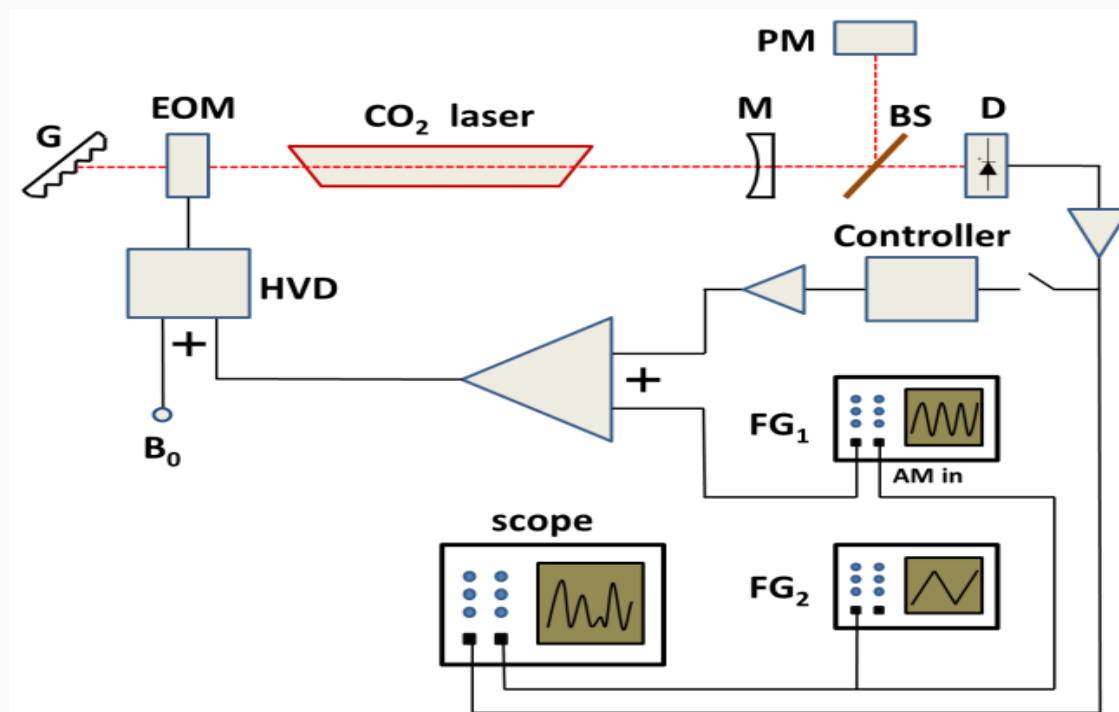
Anti-phase behaviour



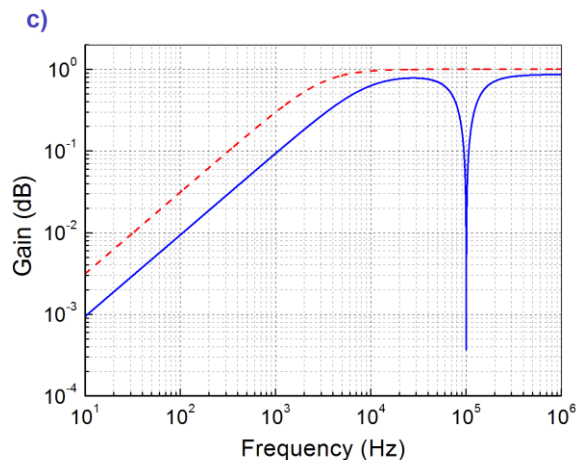
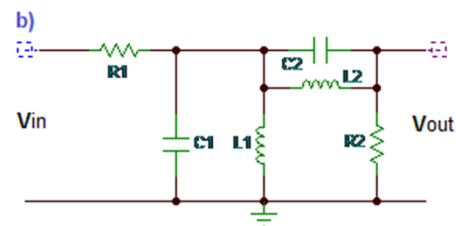
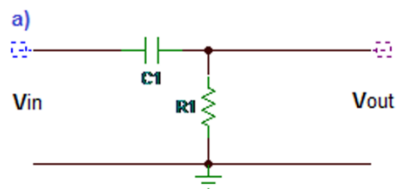
$$\Delta t = T_{\text{mod}}$$
$$\Delta \phi = 2\pi$$



Feedback Control of Bursting and Multistability



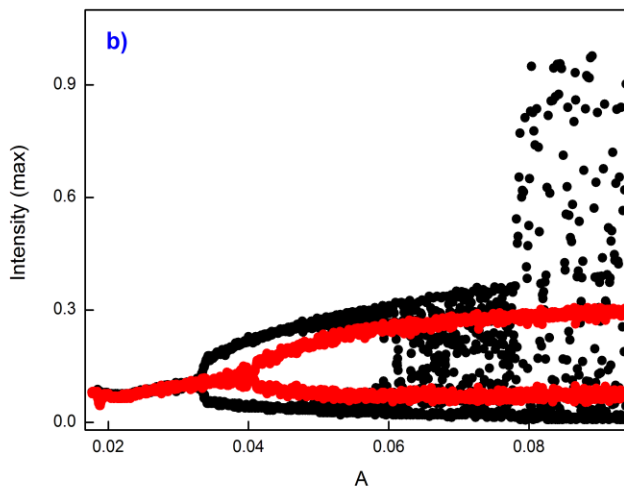
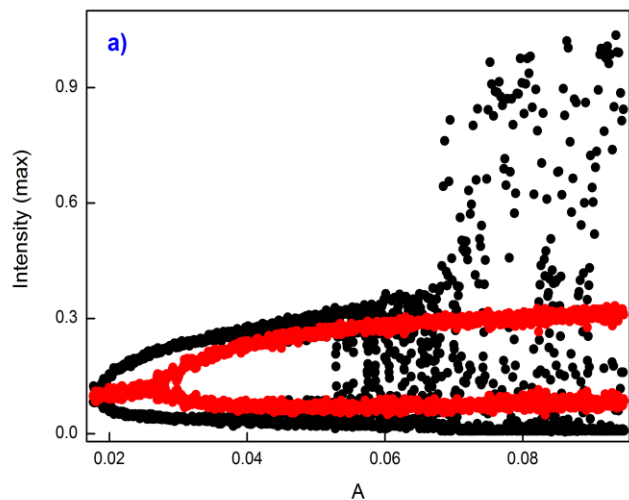
G, diffraction grating; EOM, electro-optical modulator; M, outcoupler mirror; BS, beam splitter; D, HgCdTe fast detector; controller, passive filters; FG₁ and FG₂, function generators; B₀, bias voltage; HVD, electro-optical modulator driver; scope, LeCroy digital oscilloscope WaveRunner LT342L (500MHz).



Electrical schemes of the linear filters used in the feedback loop. a) High pass (HP) filter with cut off frequency of 3kHz; $R_1=1\text{k}\Omega$, $C_1=50\text{nF}$. b) Notch filter with center frequency 100kHz; $R_1=1\text{k}\Omega$, $C_1=19\text{pF}$, $L_1=14.2\text{mH}$, $R_2=6.7\text{k}\Omega$, $C_2=200\text{pF}$, $L_2=12.4\text{mH}$. c) Amplitude response of the two filters, HP filter (dashed red line) and notch filter (solid blue line).



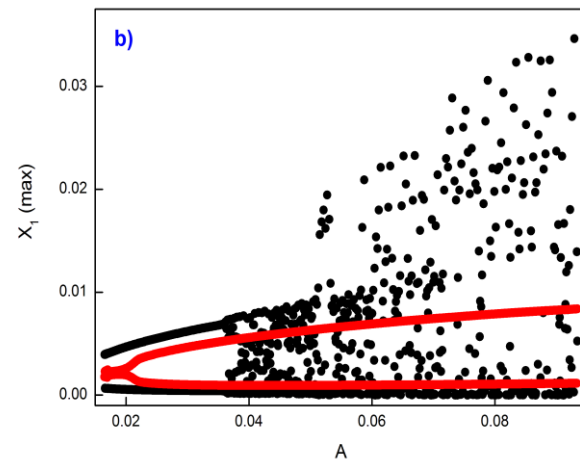
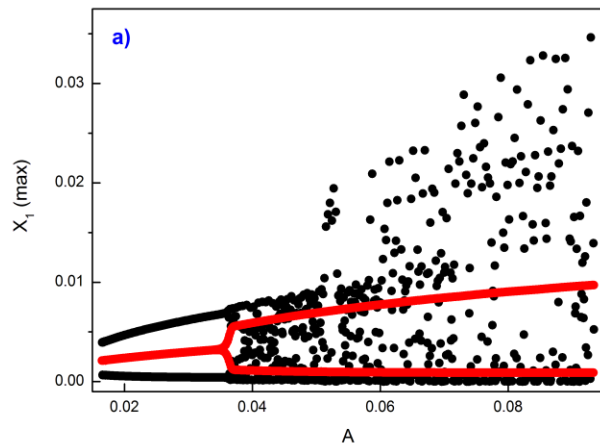
EXPERIMENTAL RESULTS



Panel a) shows the experiment based bifurcation diagram: laser intensity vs. driving amplitude A . Such a value has been normalized to $V_{\lambda/2}=2650V$ according to $A=\pi A_v/V_{\lambda/2}$, where A_v is the modulation voltage amplitude. The black trace denotes the uncontrolled scenario that exhibits interior crisis and the red trace denotes stabilized non-chaotic branch, due to the HP filter-based feedback control, cut-off frequency 3kHz. Panel b) shows another set of experiments where the black trace denotes the uncontrolled scenario and the red trace depicts the controlled non-chaotic branch due to the notch filter-based feedback control, notch frequency 100kHz.



Numerical Results



Panel a) shows numerical results corresponding to the experimental ones shown in Fig.3a (X_1 represents the laser intensity in the model): maximum perturbation amplitude 7.6% with respect to A . Panel b) shows numerical results corresponding to the experimental ones shown in Fig.3b: maximum perturbation amplitude 5.7%.

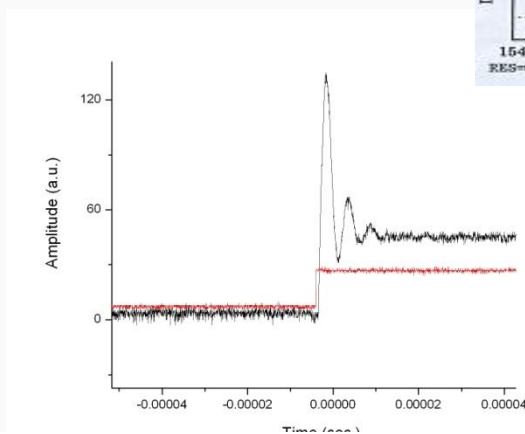
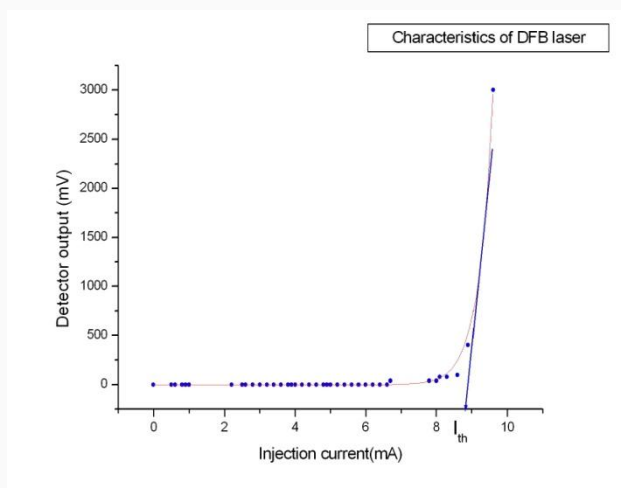
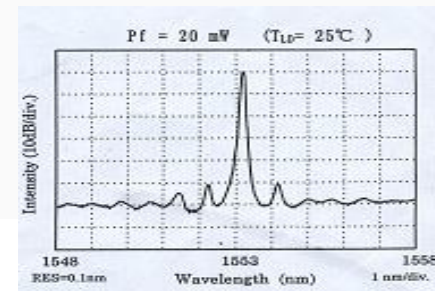


- Chaos Generation by optoelectronic feedback
in semiconductor devices

- Chaos generation in LD
- Chaos and MMOs in LED

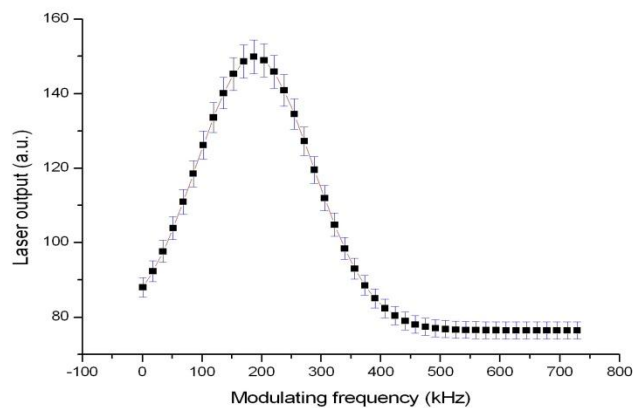


Distributed Feedback (DFB) semiconductor laser
The following measurements have been done:



Relaxation frequency (200kHz)

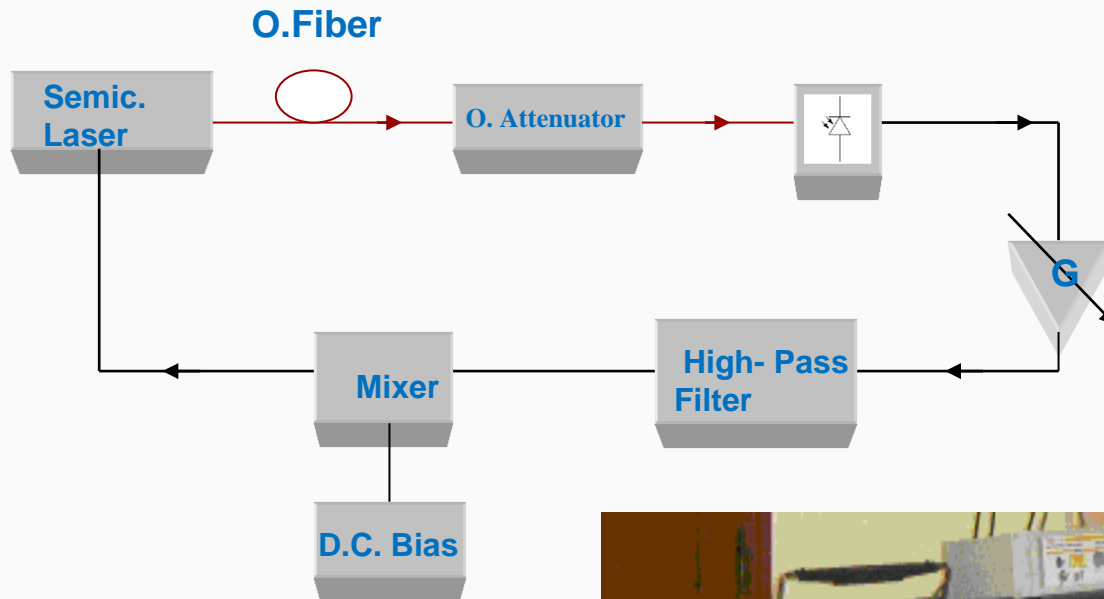
**Threshold current
8.9mA**



Frequency response (200kHz)



Chaotic spiking in semiconductor devices with optoelectronic feedback





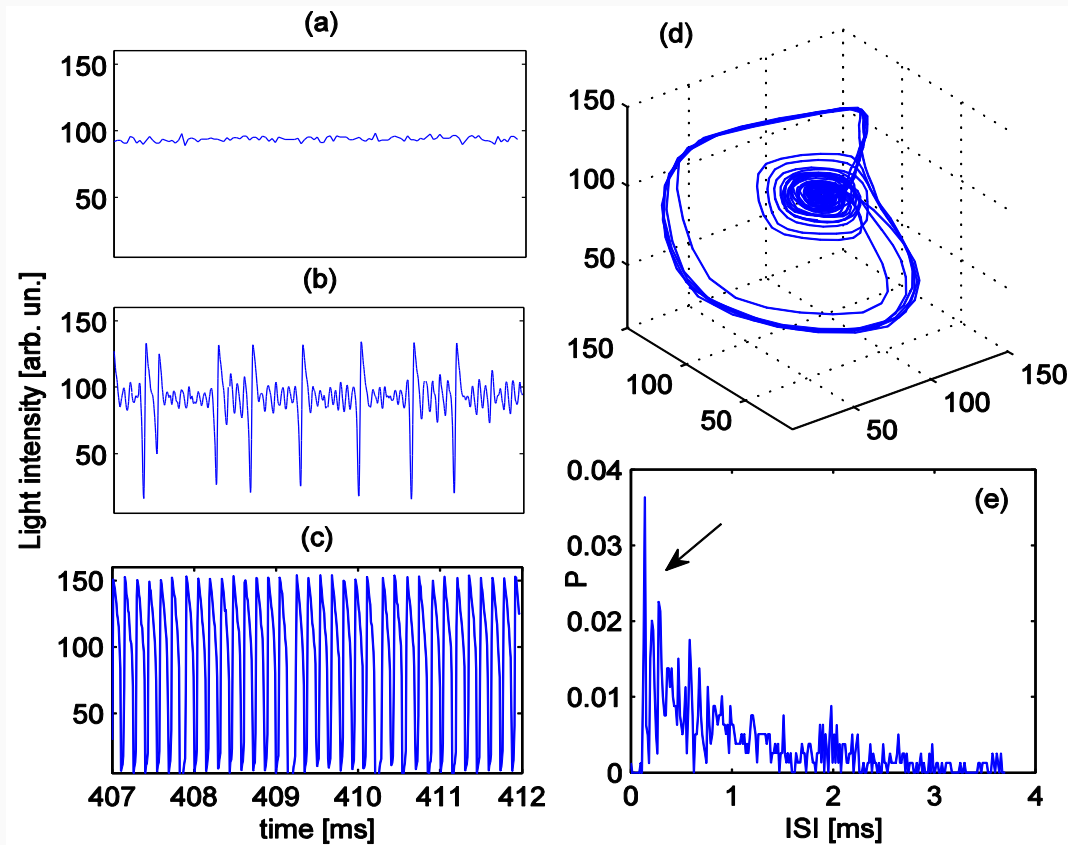
Experimental results

$I_{th}=8.300\text{mA}$

a) 8.700mA

b) 8.763mA

c) 9.050mA

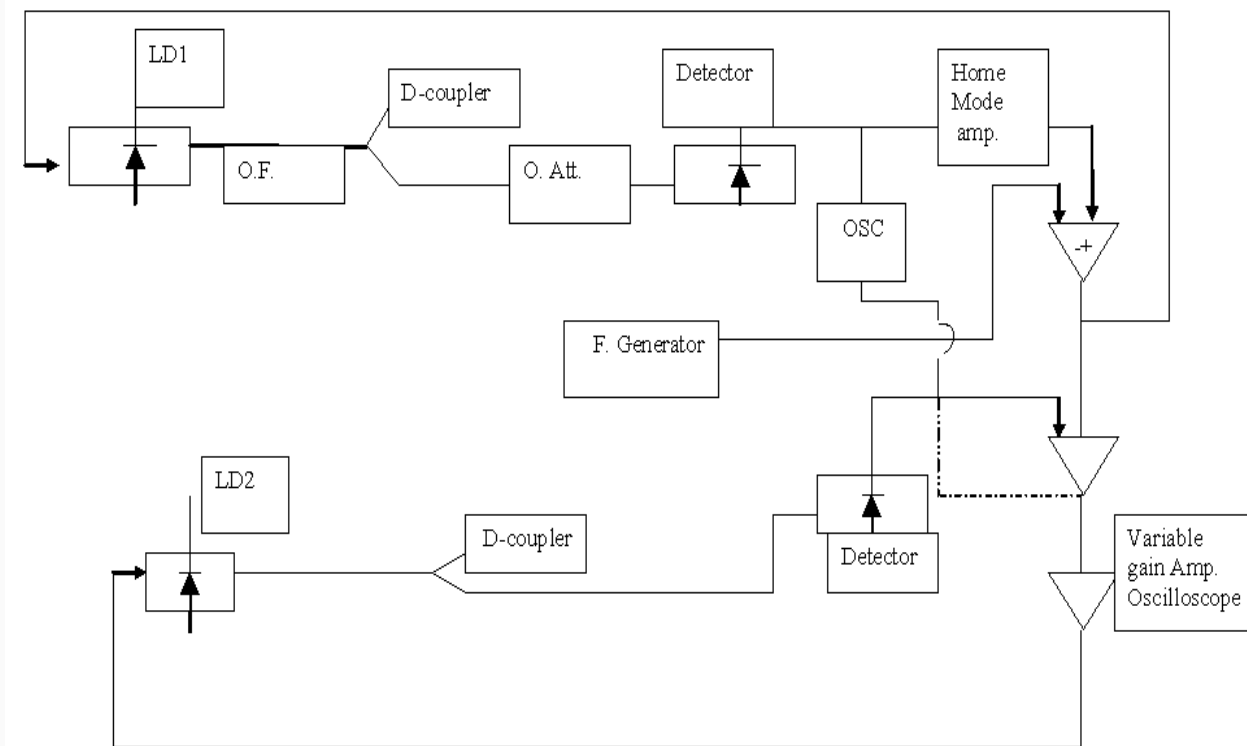


Transition from a stationary steady state to chaotic spiking and eventually periodic self-oscillations as the dc-pumping current is varied



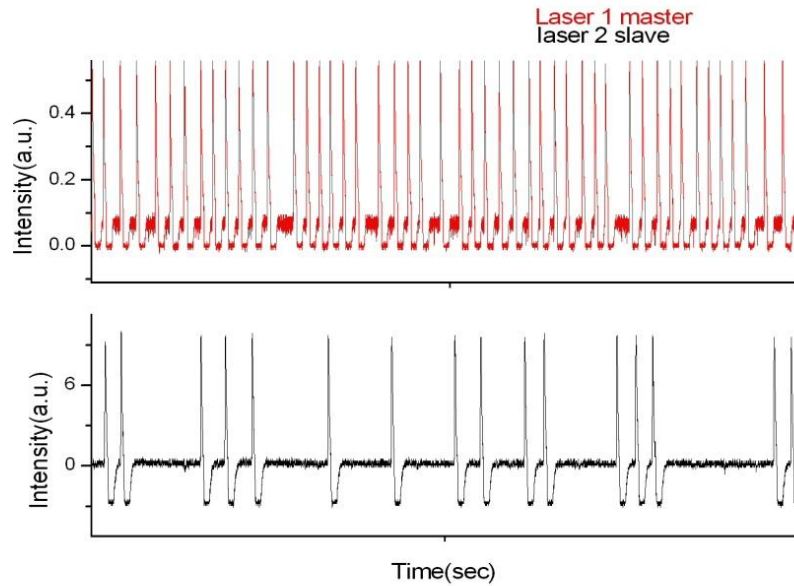
synchronization, LDs

Master – Slave configuration Unidirectional coupling

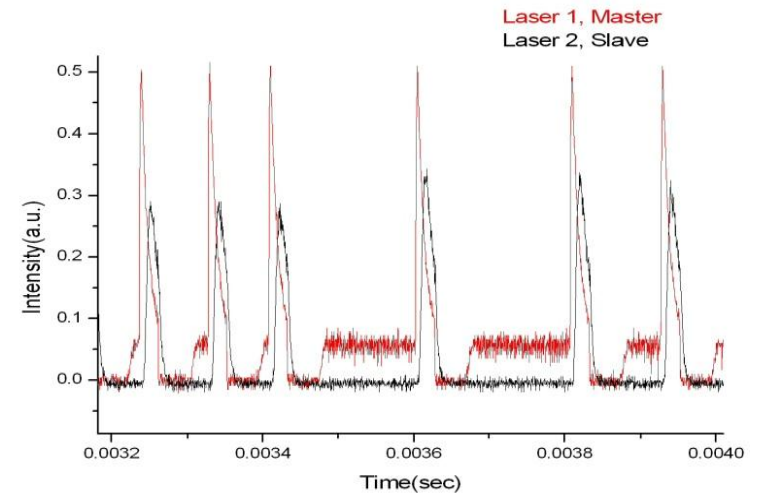




Synchronization



Master-slave configuration

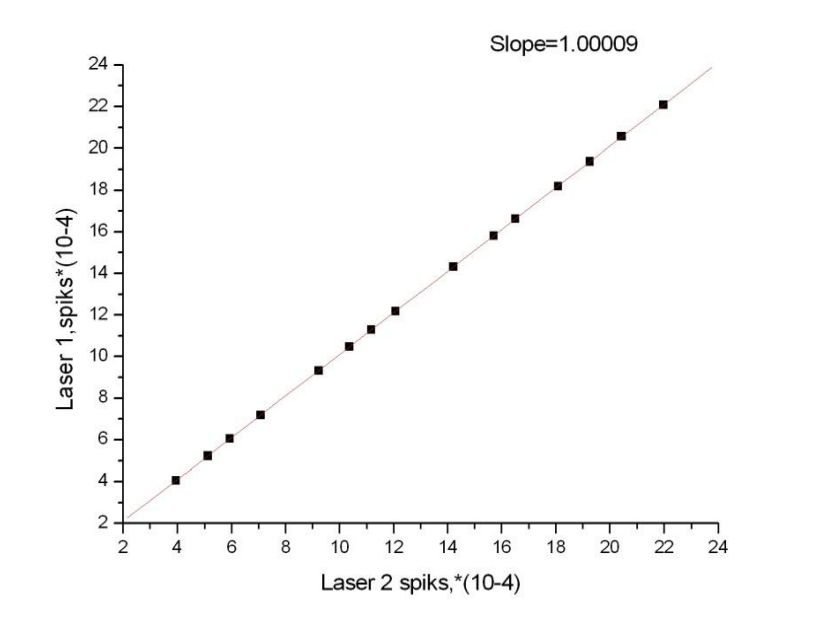
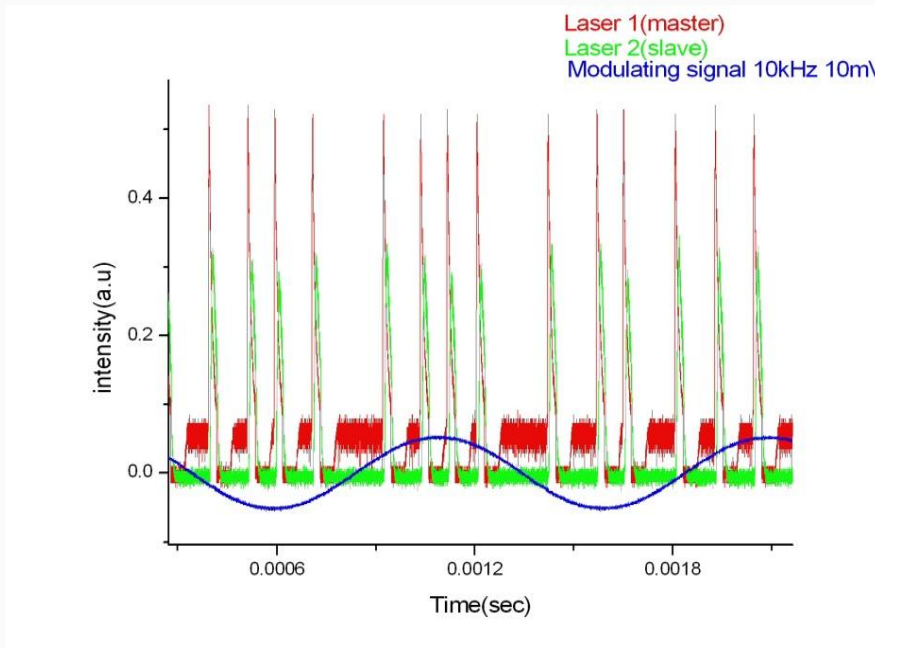


uncoupled and coupled oscillators (chaotic signals).



synchronization with modulation

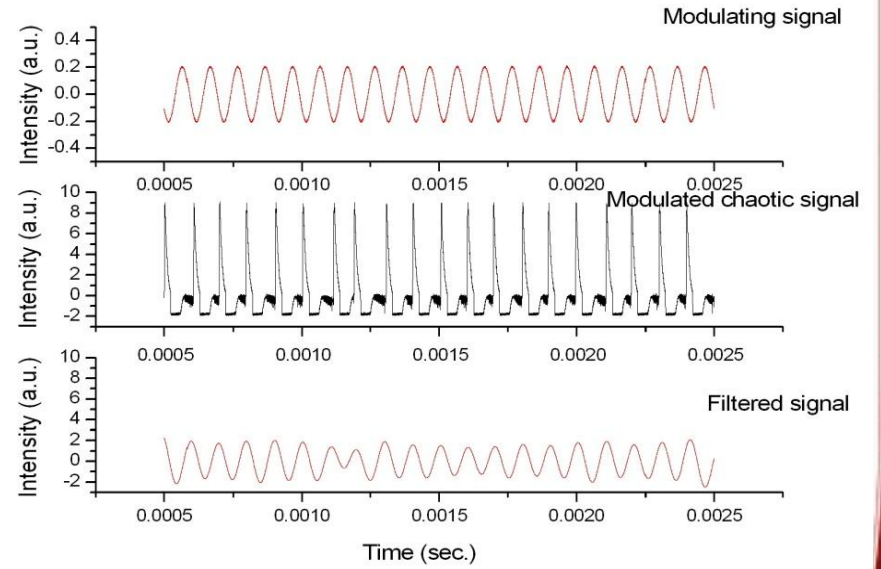
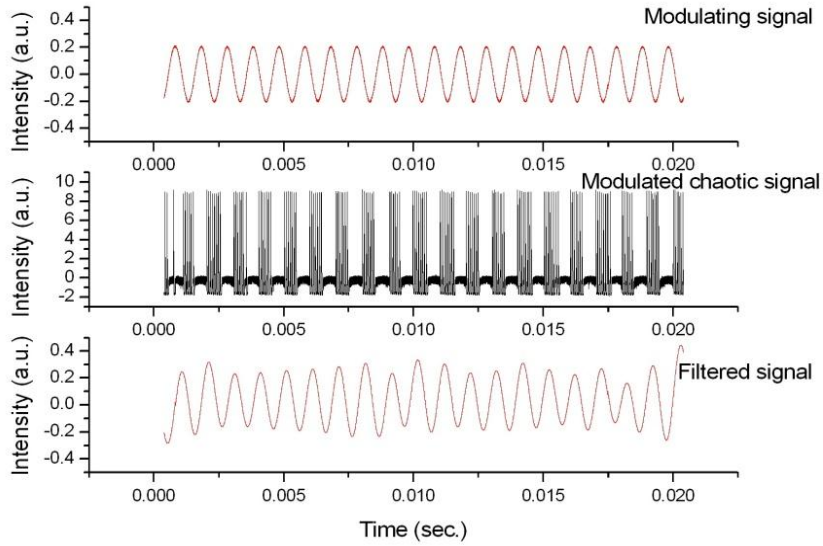
Adding forcing (modulation) frequency on the master oscillator



Forcing frequency is 10kHz and 10mV amplitude. The relation of spikes events time of the two oscillators



message decoding





Model of the laser diode + feedback

$$\dot{S} = [g(N - N_t) - \gamma_0]S$$

$$\dot{N} = \frac{I_0 + f_F(I)}{eV} - \gamma_c N - g(N - N_t)S$$

$$\dot{I} = -\gamma_f I + k\dot{S}$$

S = photon density

N = carrier density

I = high-pass feedback current



Non linear feedback function

$$x = (g / \gamma_c)S$$

$$y = (g / \gamma_0)(N - N_t)$$

$$w = (g / k\gamma_c)I - x$$

$$t' = \gamma_0 t$$

$$\gamma = \gamma_c / \gamma_0 = 10^{-3}$$

$$\varepsilon = \gamma_f / \gamma_0 = 2 * 10^{-5}$$

dimensionless variables :

$$\dot{x} = \gamma(x - y)$$

$$\dot{y} = \delta - \gamma(y + f(w) - x)$$

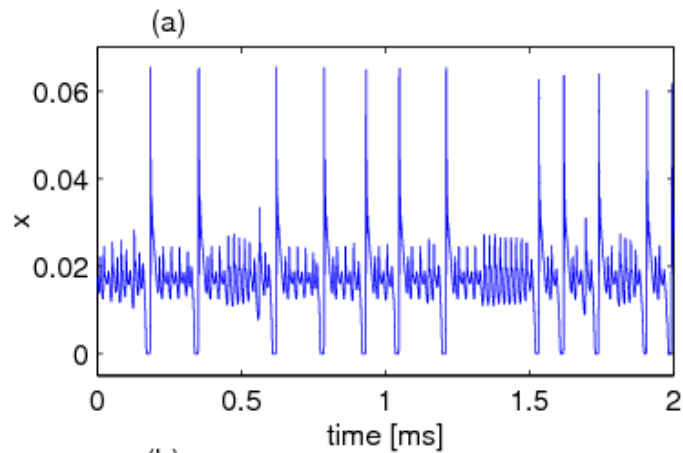
$$\dot{w} = \varepsilon(w)$$

$$\dot{x} = \gamma(x - y)$$

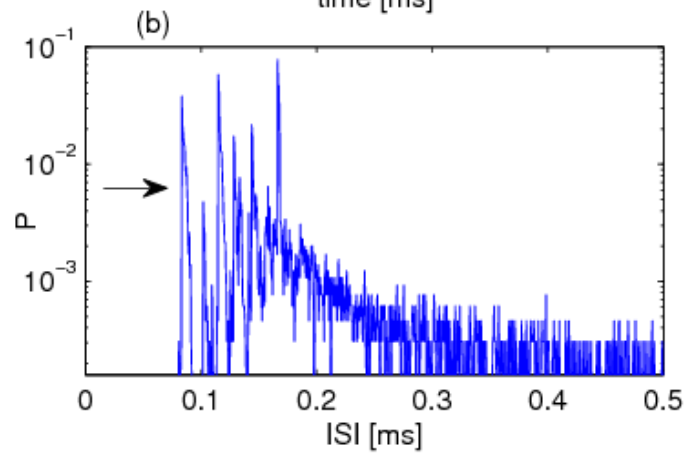
$$\dot{y} = \delta - \gamma(y + f(w) - x)$$



Numerical results



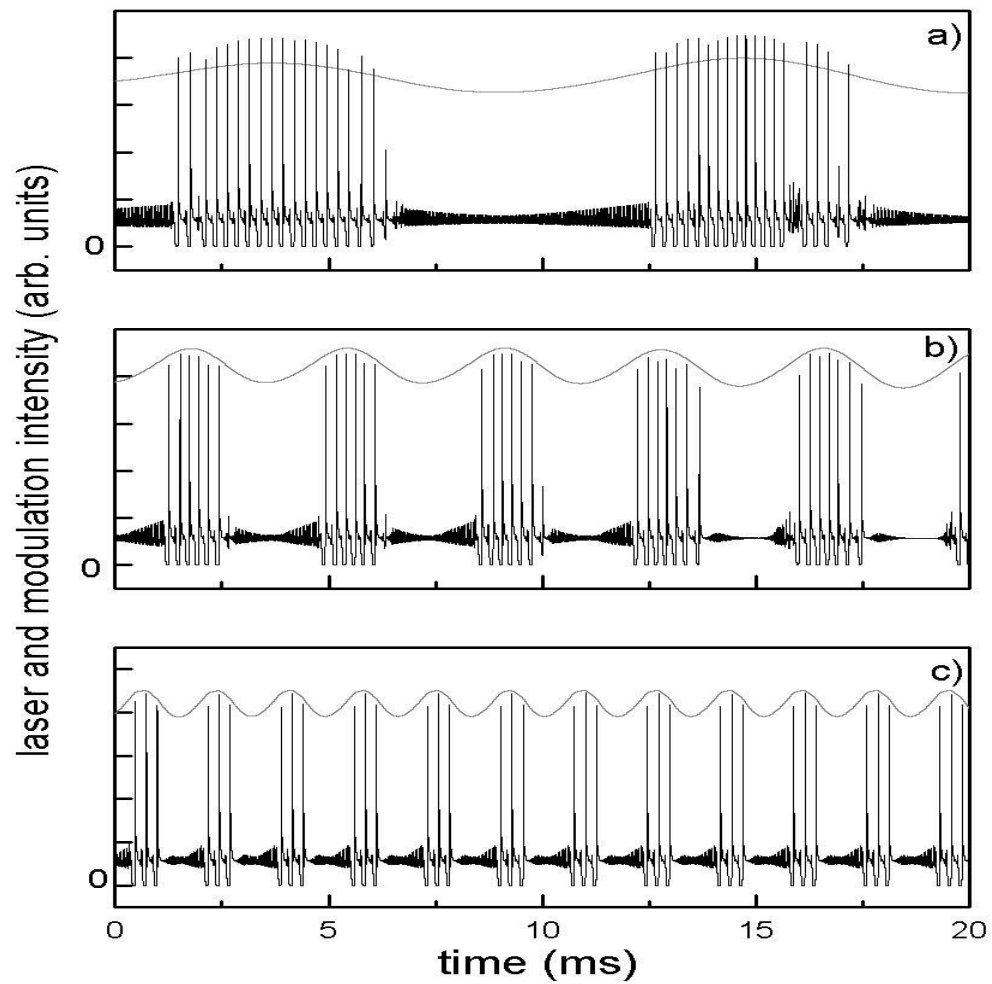
Chaotic spiking regime



ISI distribution



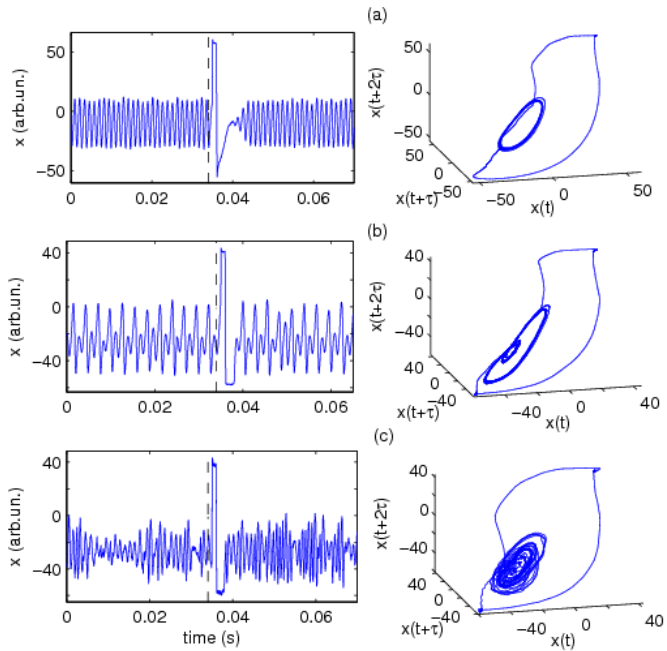
Numerical results



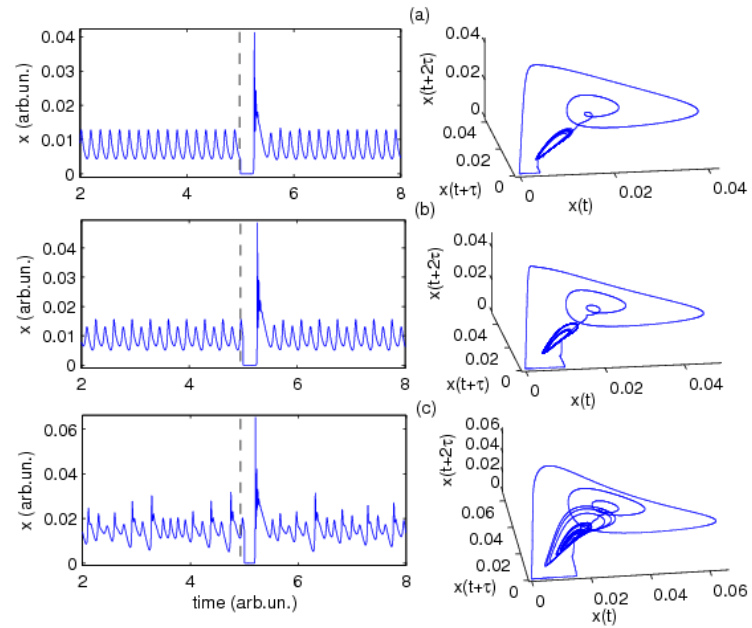


Excitability of attractors (general case)

Exp.

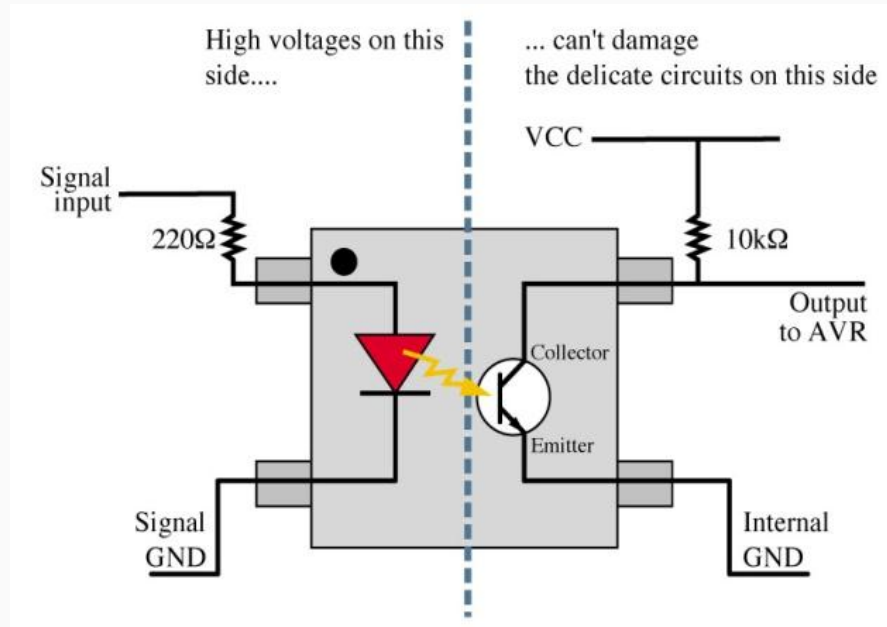


Num.





Mixed Mode Oscillations (MMOs) in optical isolators (LED+Detector)



An optical isolator is a small four-terminal device that includes a **LED** and a light detector.

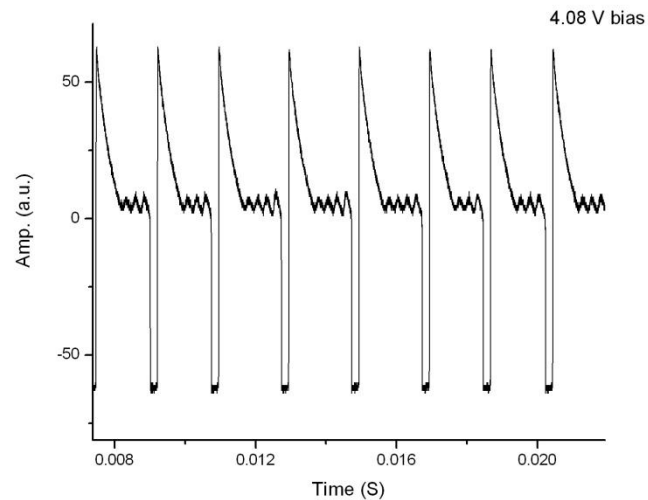
Feedback scheme as with the laser diode



MMOs

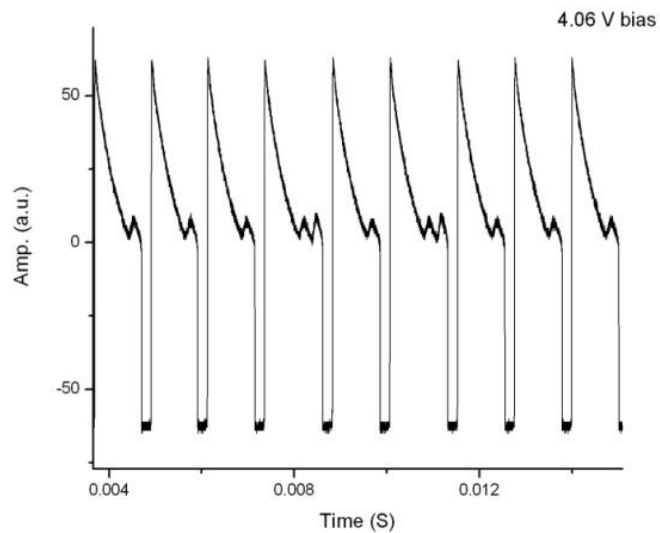
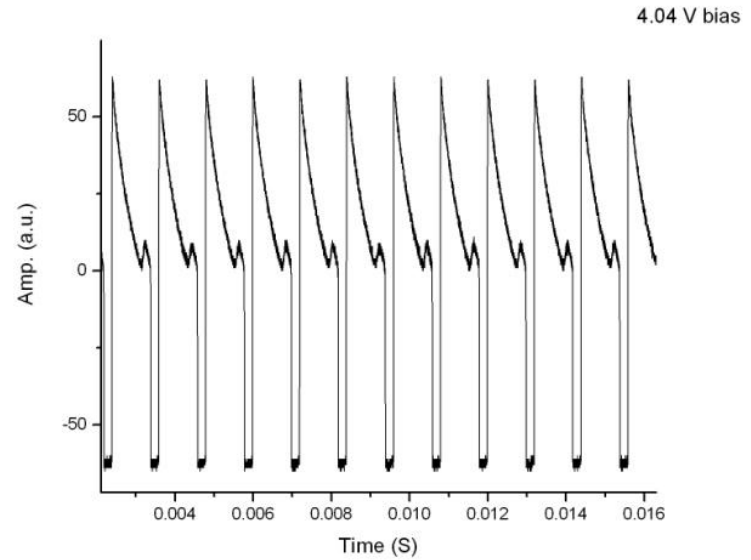
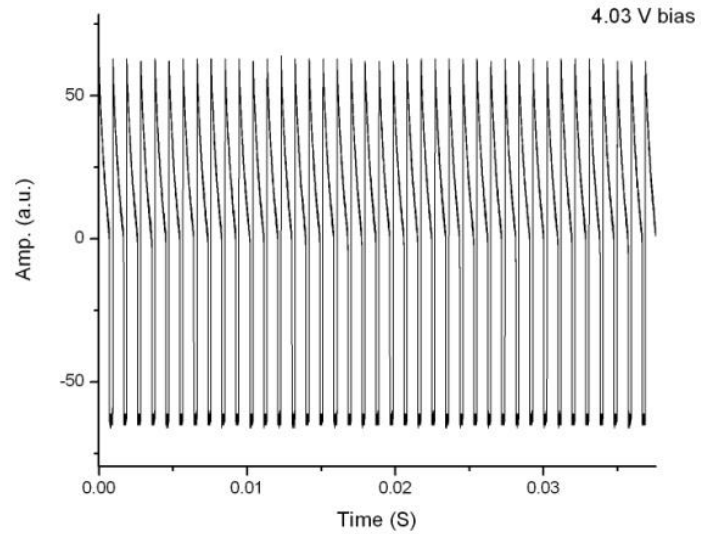
Mixed Mode Oscillations (MMOs) = mixture of two distinct kinds of oscillations (small and fairly harmonic oscillations and large amplitude relaxation spikes)

Exp. MMOs



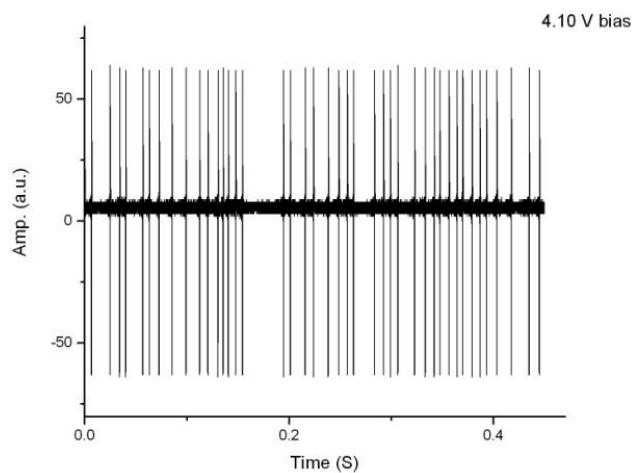
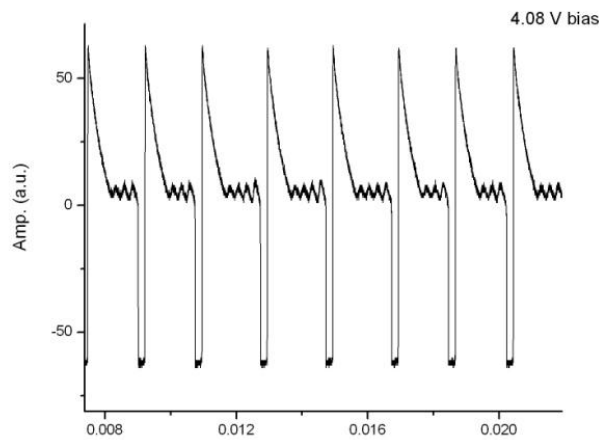
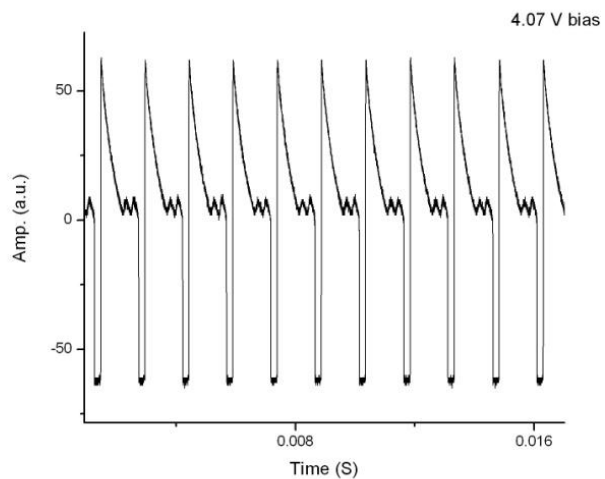


Exp. evidence of MMOs (I)

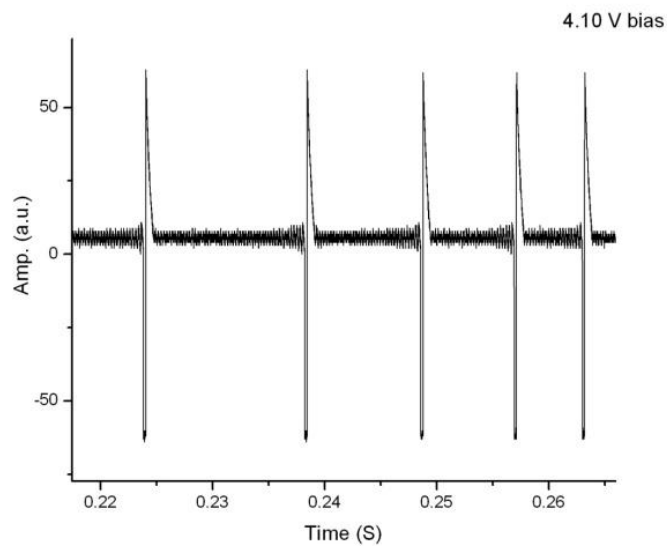




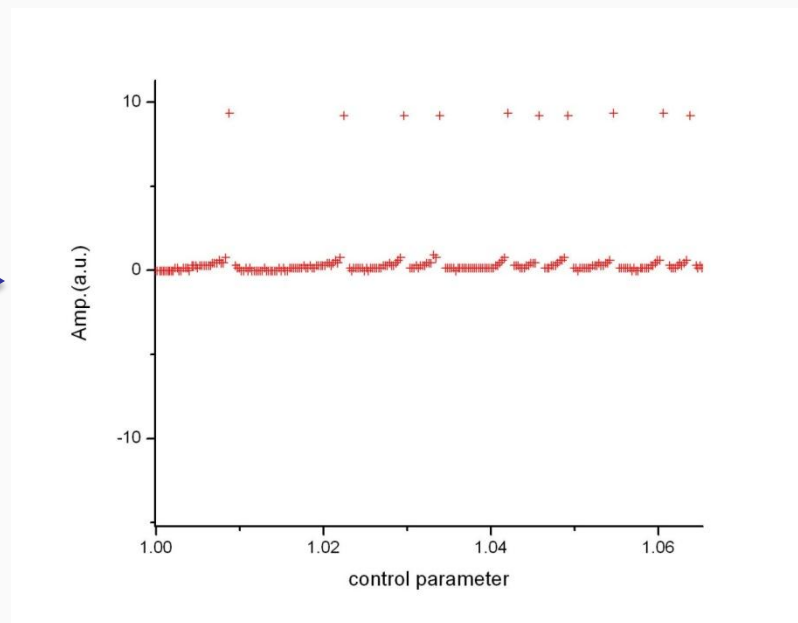
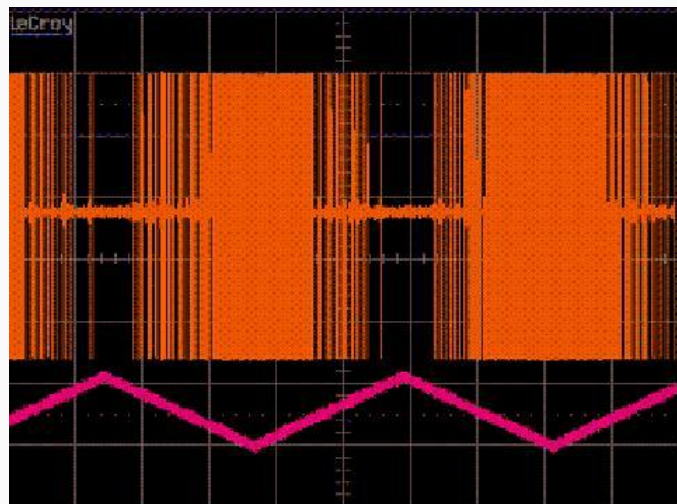
Exp. evidence of MMOs (II)

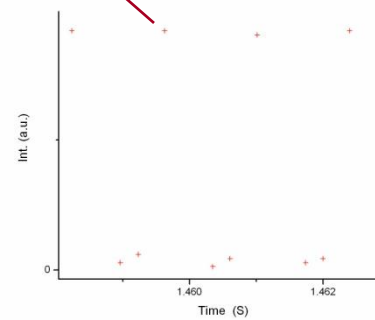
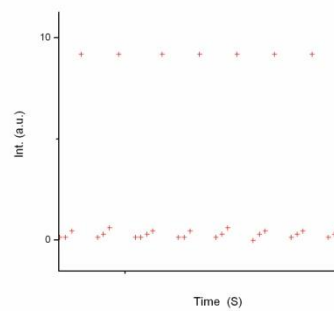
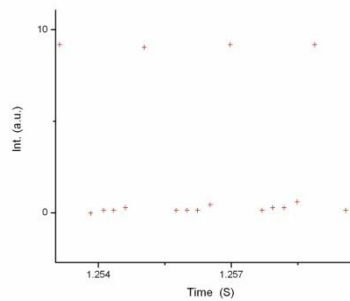
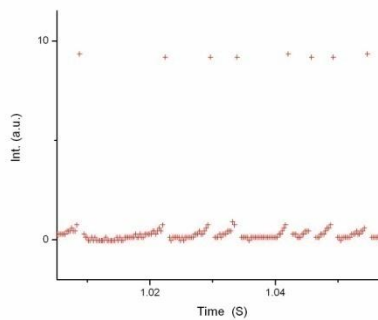
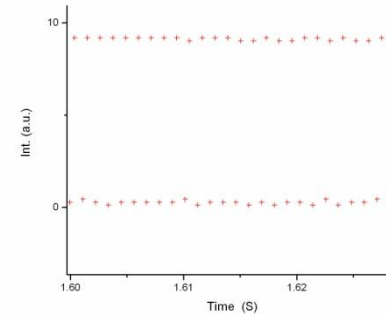
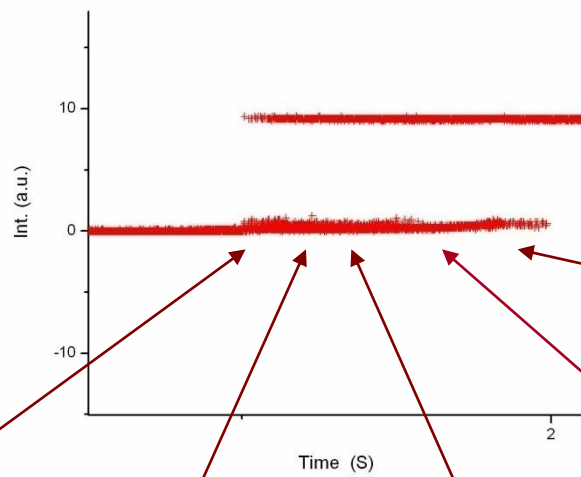


time
expansion



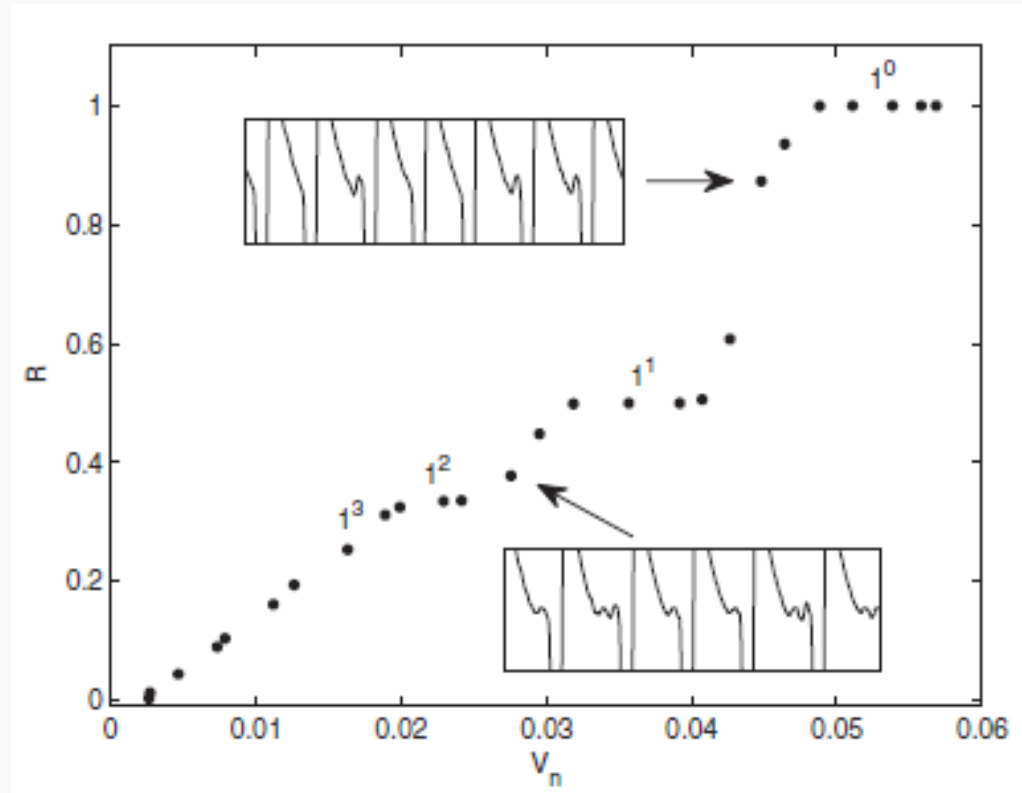
Chaotic behavior







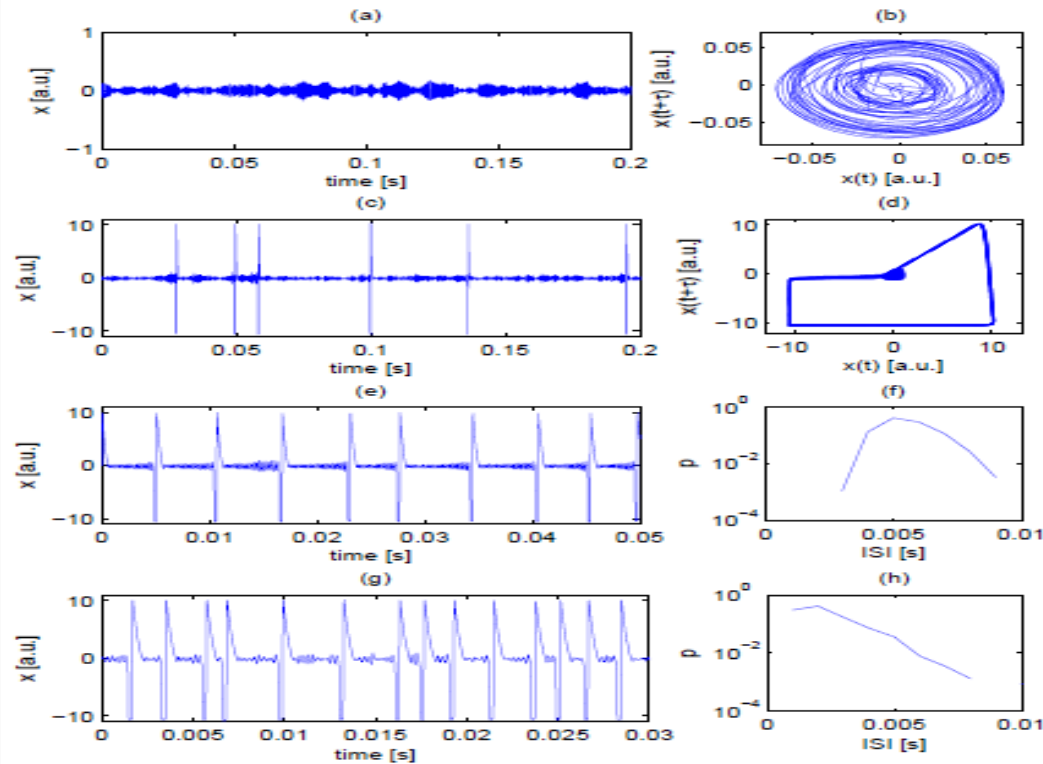
Winding number L^S



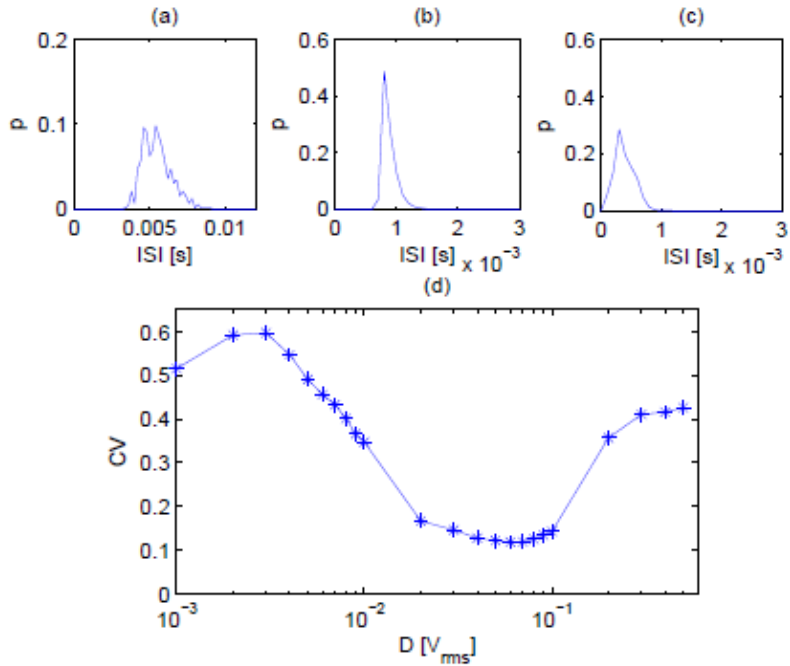
Plot of the winding number R as a function V_n . Insets: Experimental time series of the optical intensity corresponding to $V_n = 0.0448$ and 0.0275 .



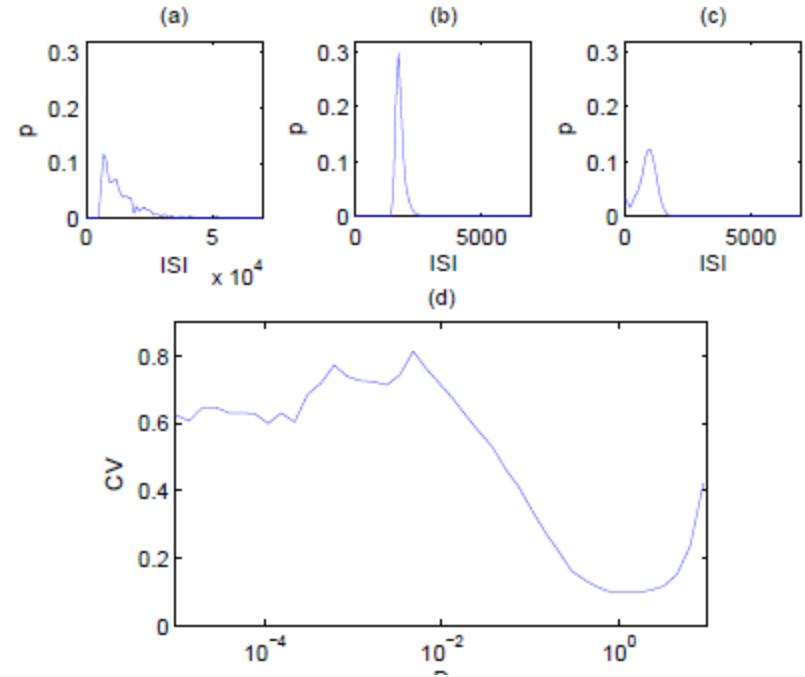
Noise effects on excitable chaotic attractors in coupled light emitting diodes



Experimental time series for a single LED with opto-electronic feedback in various dynamical regimes: the regime of small-amplitude excitable chaotic attractor in the absence (a) and in the presence of noise (c) and the corresponding reconstruction of the phase-spaces in (b) and (d); the regime of chaotic spiking in the absence (e) and in the presence of noise (g) and the corresponding probability distributions for the inter-spike intervals in (f) and (h).



Experimental results for light emitting diode with optoelectronic feedback: probability distributions of the inter-spike intervals for the noise intensity (a) $D = 0 \text{ V}_{\text{rms}}$, (b) $D = 0:04 \text{ V}_{\text{rms}}$ and (c) $D = 0:4 \text{ V}_{\text{rms}}$; (d) coefficient of variation versus noise intensity. The bin length are $h = 210 \times 10^{-4} \text{ [s]}$ in (a) and $h = 10 \times 10^{-4} \text{ [s]}$ in (b) and (c).

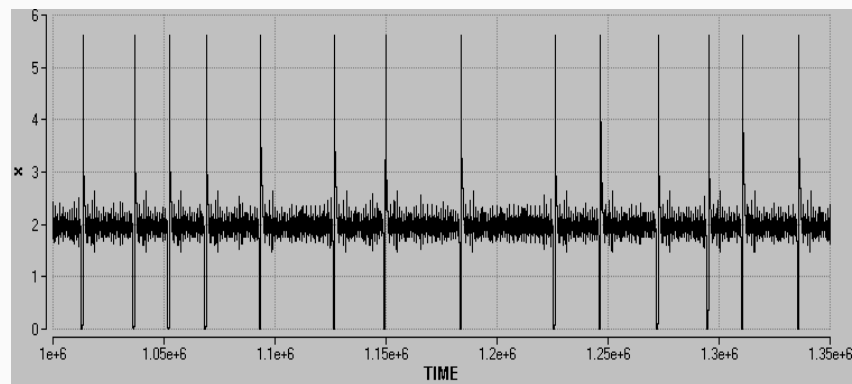
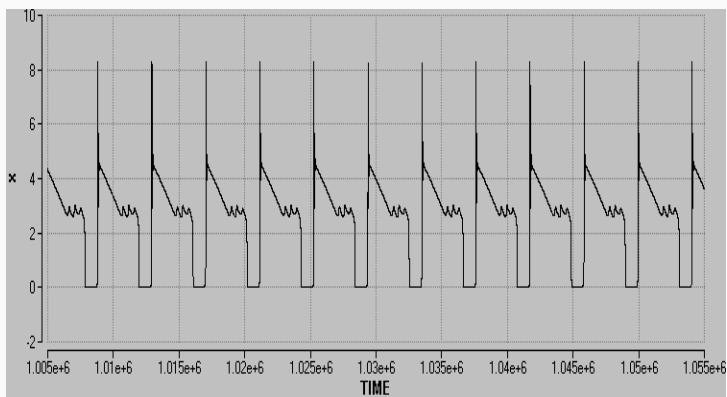
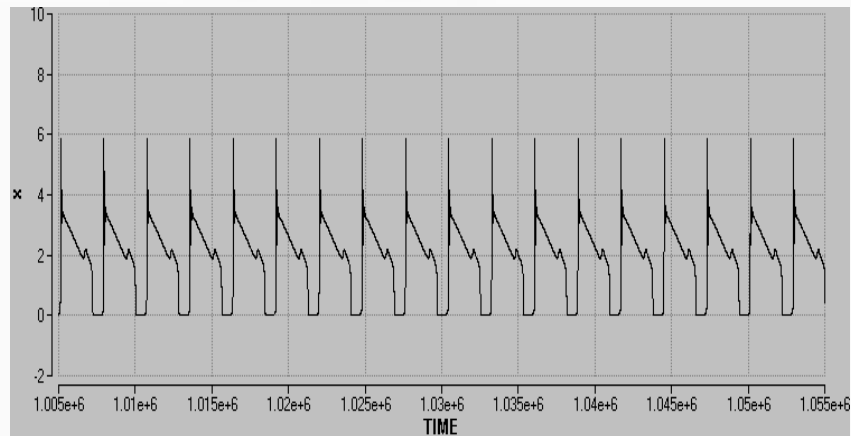


Numerical simulation of a model for light emitting diode with optoelectronic feedback: probability distributions of the inter-spike intervals for the noise intensity (a) $D = 0$, (b) $D = 1:4$ and (c) $D = 6:825$; (d) coefficient of variation versus noise intensity. The bin length are $h = 2103$ in (a) and $h = 50$ in (b) and (c). System parameters are: $\tau = 0:01$, $s = 0:2$, $\gamma = 5 \times 10^{-4}$, $\beta = 2:5$, $\alpha = 1:001481$ and $\delta = 0:01$.



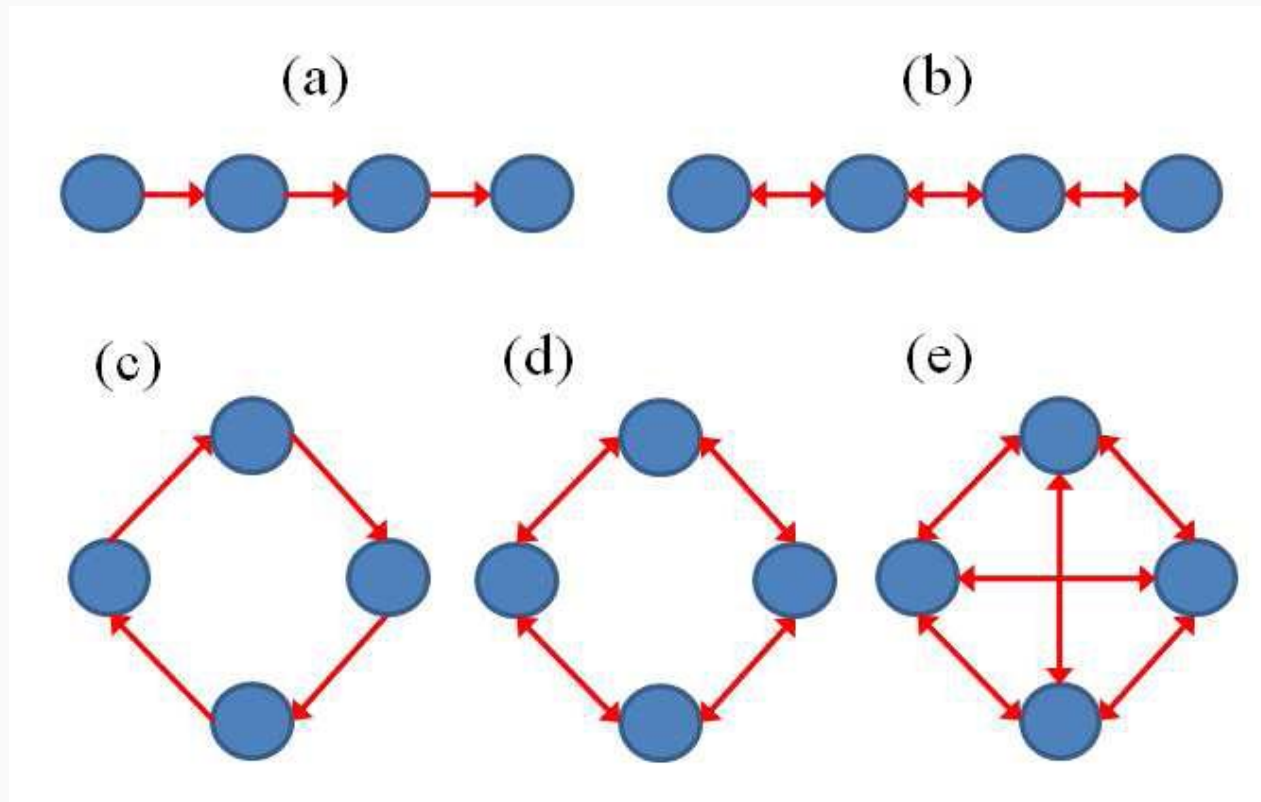
Model of the LED + feedback

$$\dot{N} = -\gamma_{sp}N + \frac{\mu N(V_d - V_{bi})}{\Delta^2},$$
$$C\dot{V}_d = \frac{V_0 - V_d + f_F(V_f)}{R} - \frac{e\mu NS(V_d - V_{bi})}{\Delta}$$
$$\dot{V}_f = -\gamma_f V_f + k\Phi,$$





Synchronization configurations





Synchronization patterns in arrays of homoclinic chaotic systems

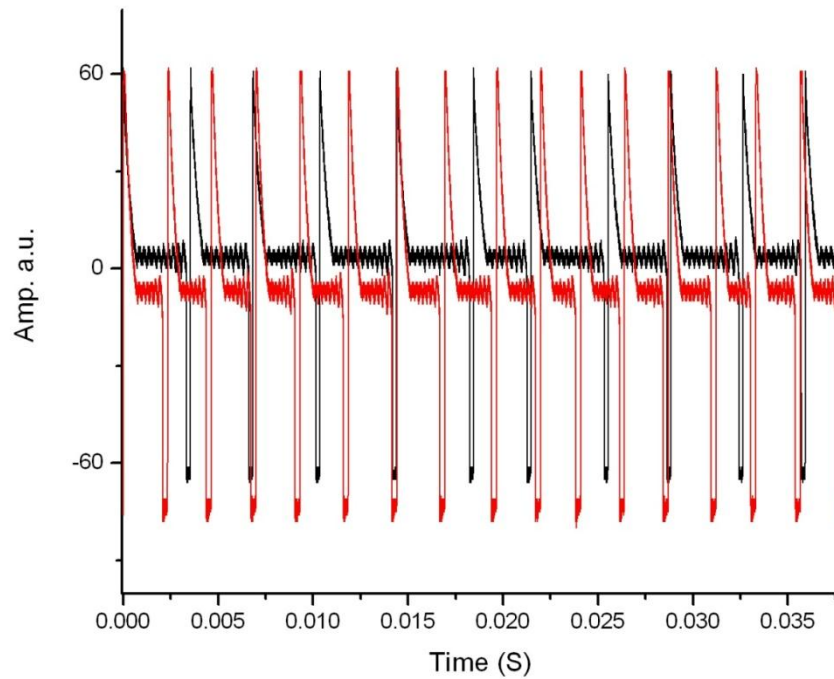
$$x^i$$

with

$$x^i + \varepsilon \left(x^{i+1} + x^{i-1} - 2 \langle x^i \rangle \right)$$



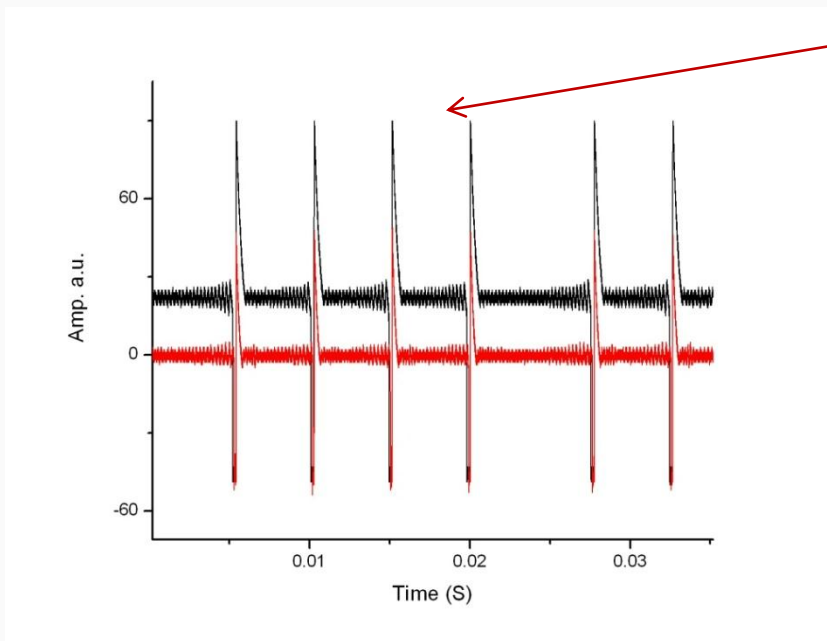
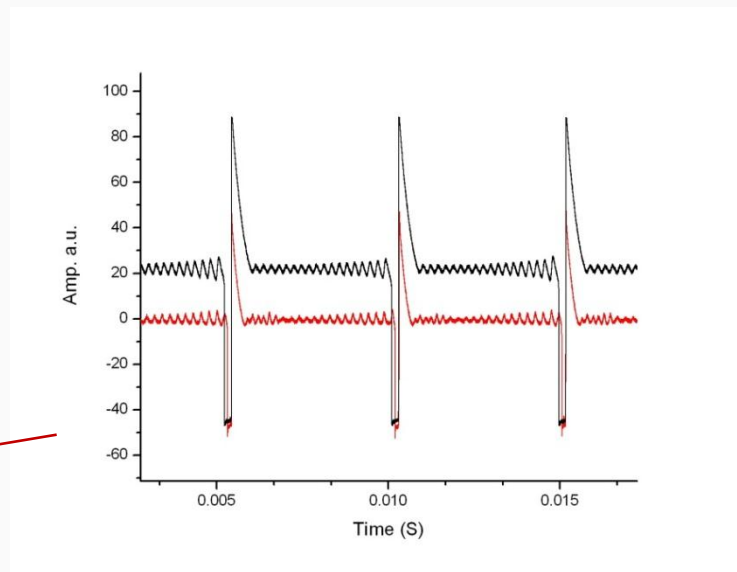
Free running behavior of two independent oscillators





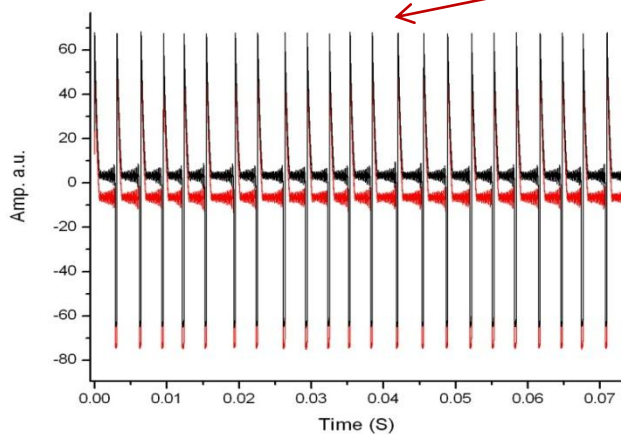
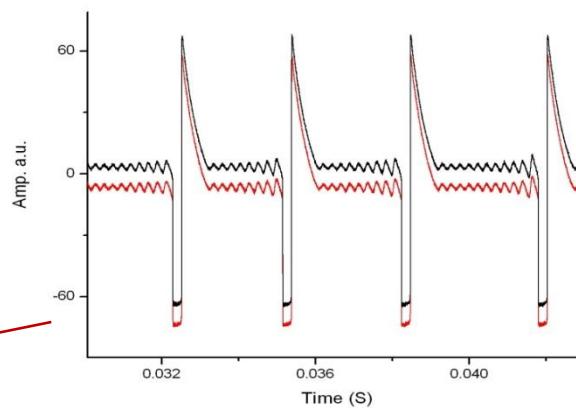
Exp. evidence of MMOs synchronization

Phase synchronization





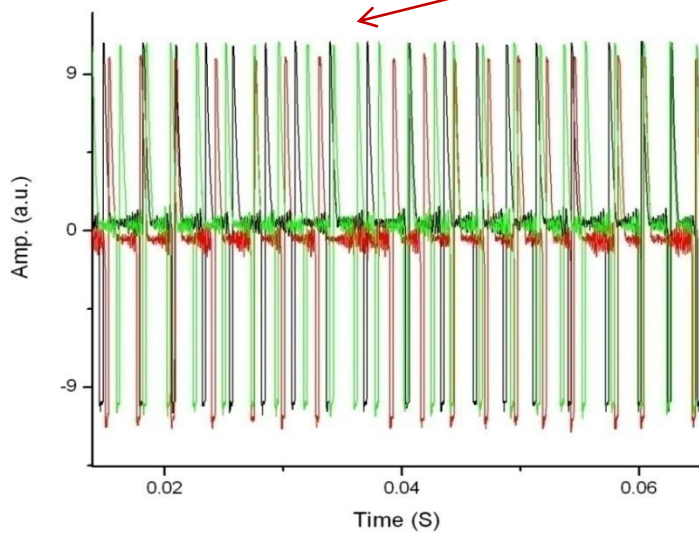
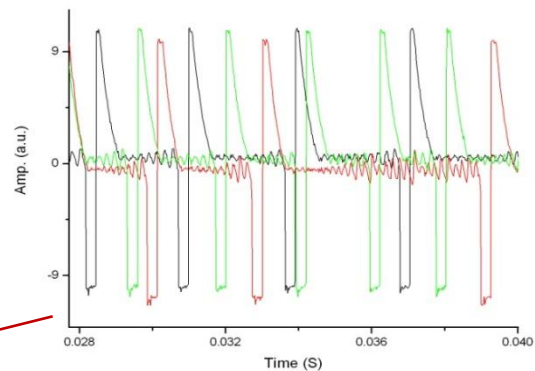
Complete synchronization





Exp. evidence of MMOs synchronization

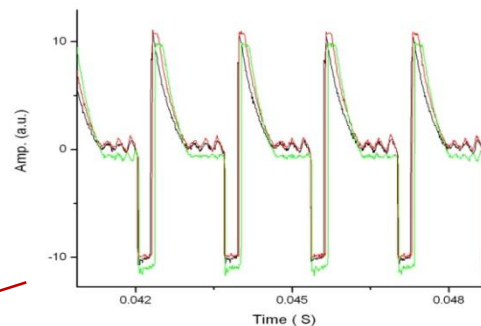
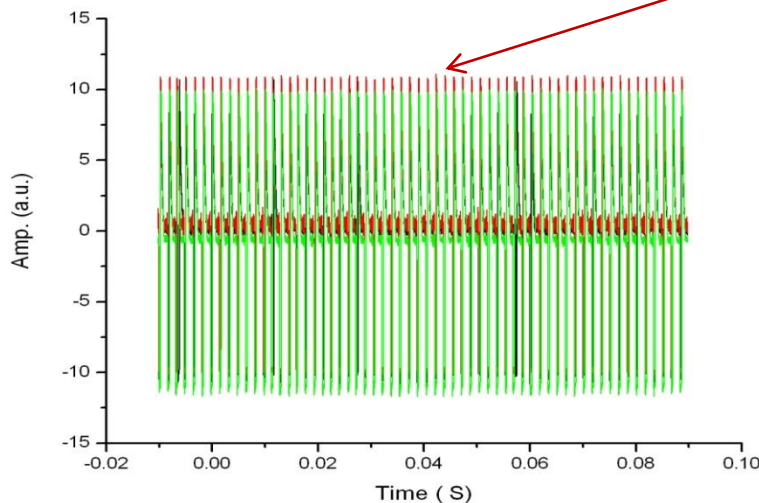
Free-running behavior





Exp. evidence of MMOs synchronization

Nearest Neighbor - coupling

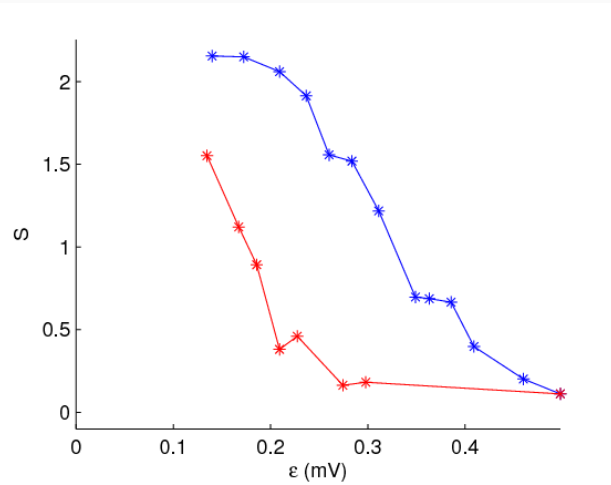


Synchronization among three oscillators

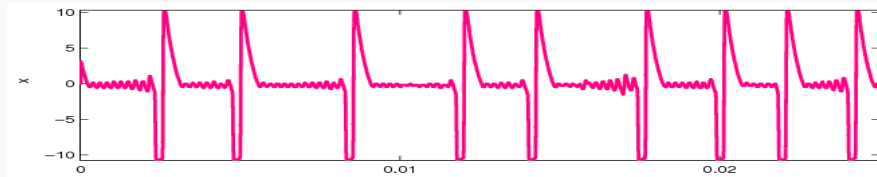


Synchronization

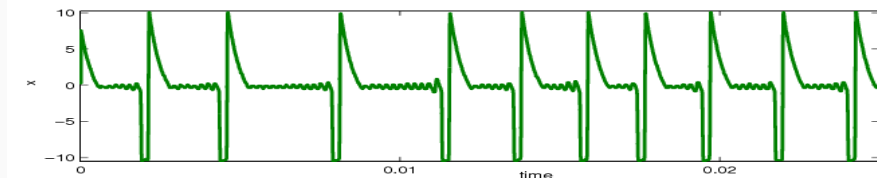
2 LEDs



Entropy S versus coupling strength shows the transition to synchronization

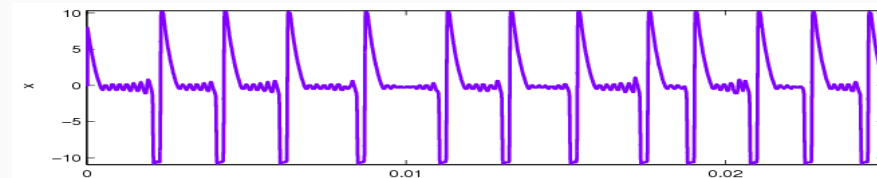


1

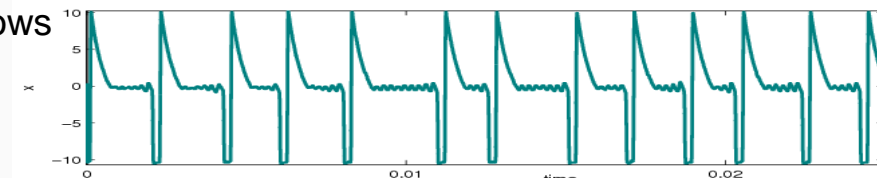


2

Free



1

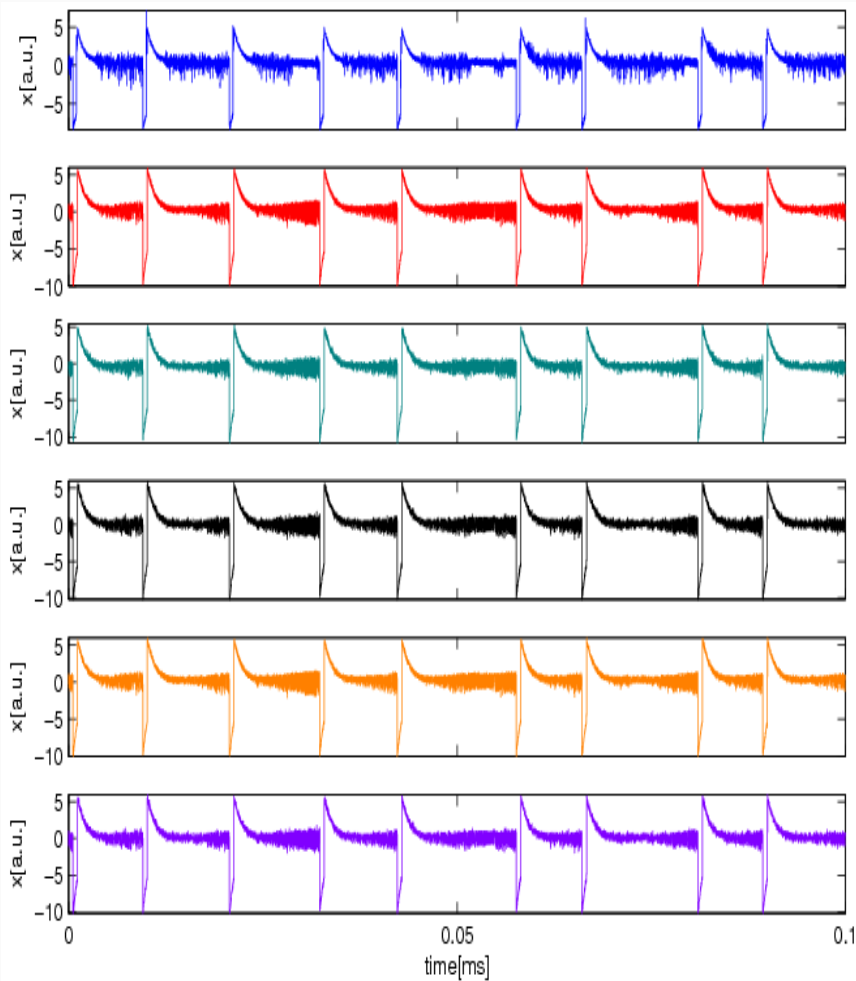


2

Coupled



Synchronization

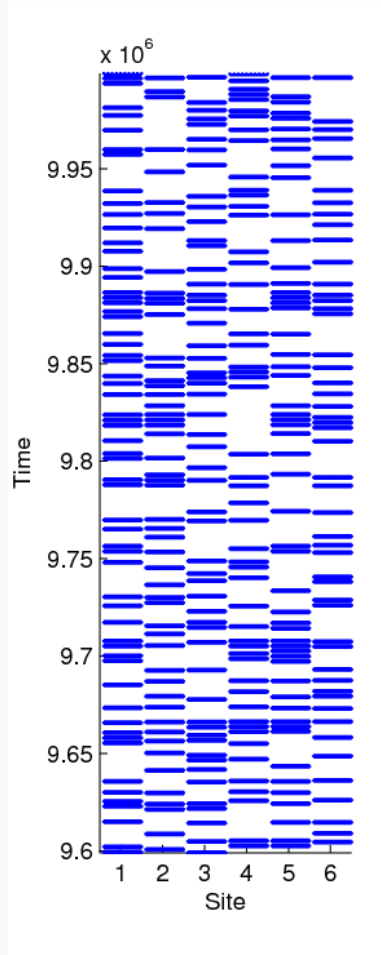


- 1
- 2
- 3
- 4
- 5
- 6

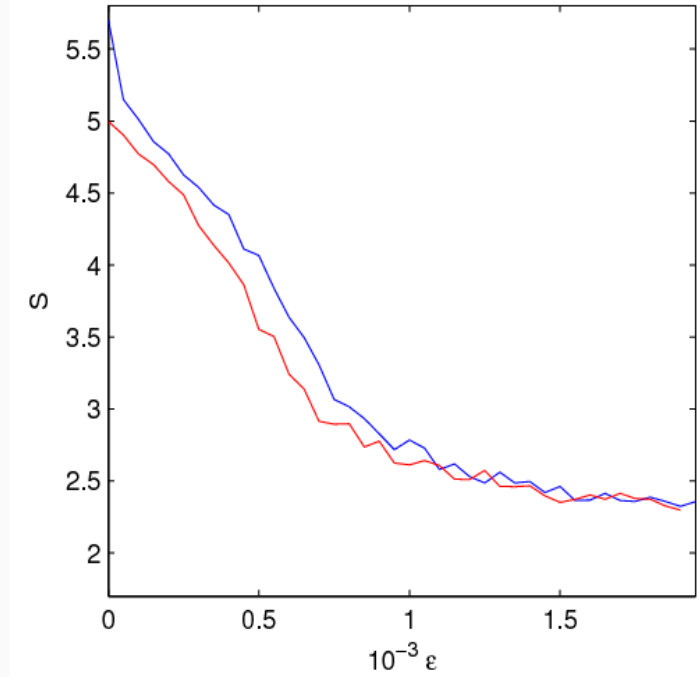
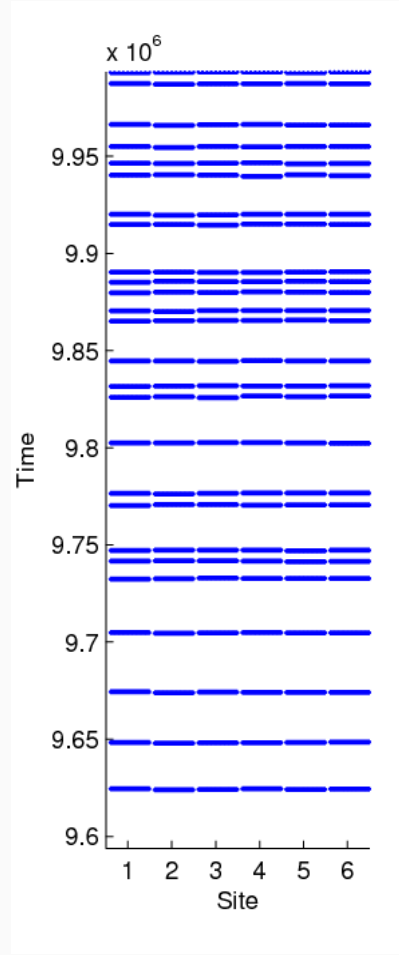


Synchronization

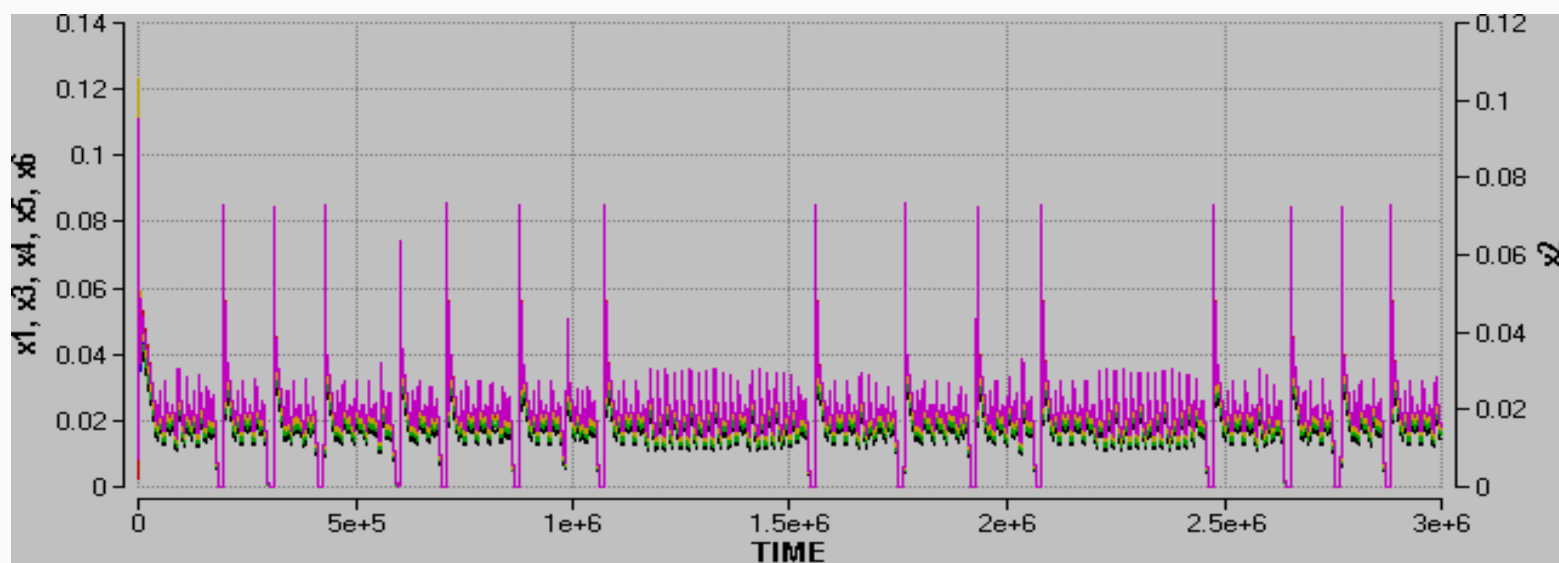
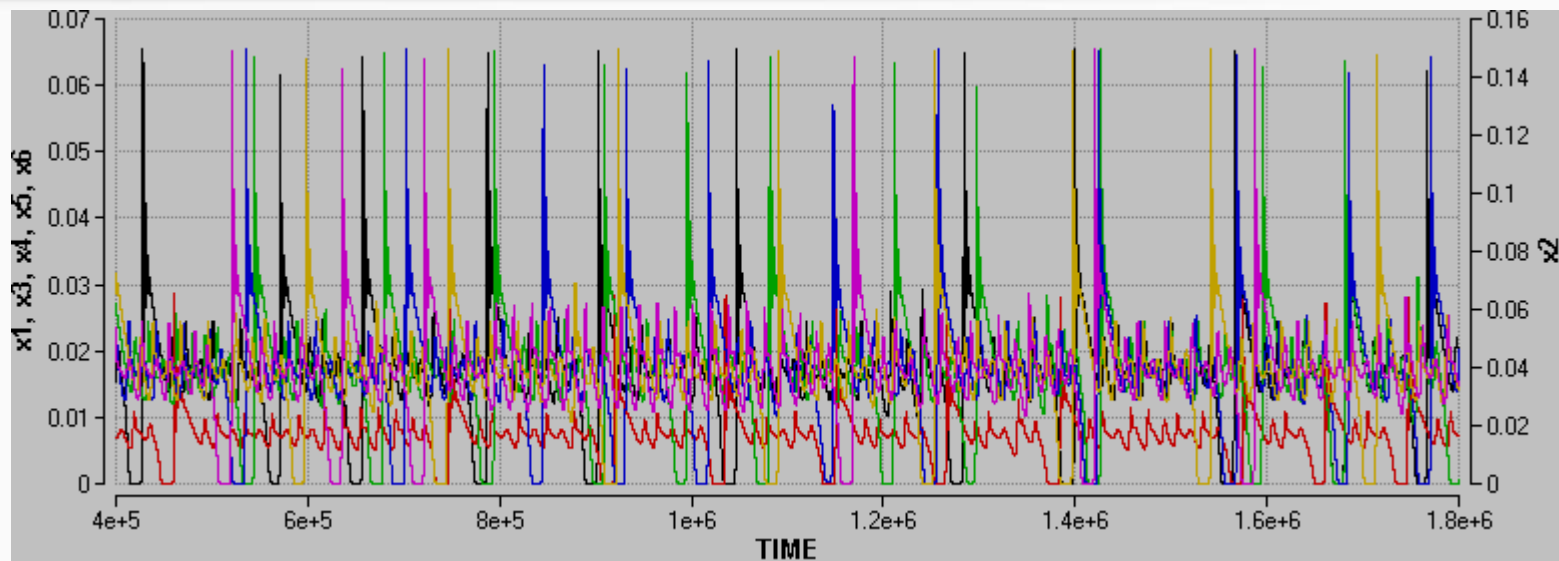
$$\tilde{K}=\varepsilon=0$$



$$\tilde{K}=\varepsilon= 2 \cdot 10^{-3}$$

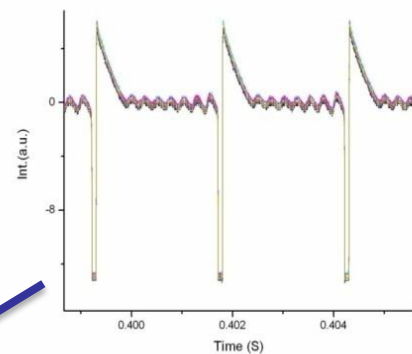
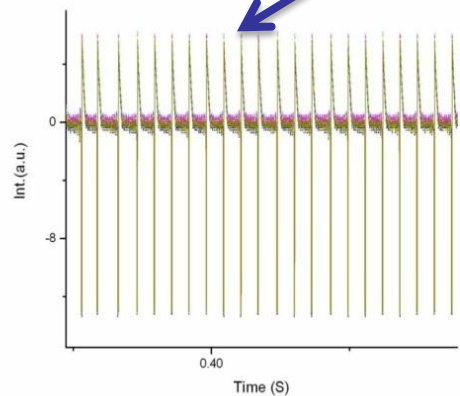
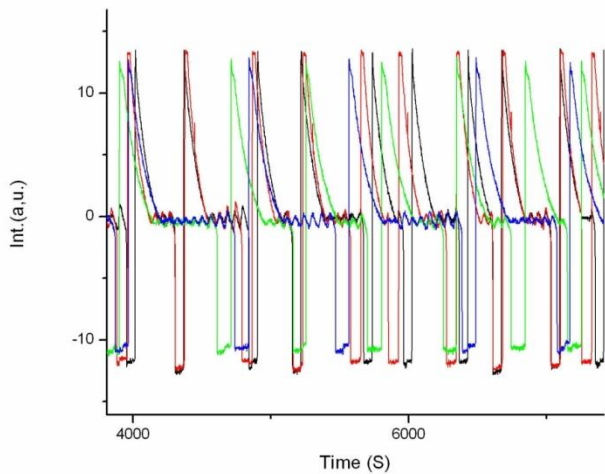
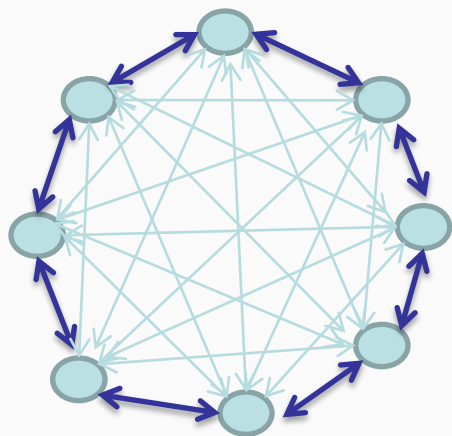


Entropy S versus coupling strength is used to describe the transition to synchronization





8 oscillators synchronization





Conclusions

- 1) From CO₂ to Semiconductor laser and LED with feedback
- 2) MMOs and chaotic spiking sequences in semiconductor light sources (lasers and LEDs) with AC-coupled feedback, **phase independent**)
- 3) Time scale determined by the high-pass filter; the erratic spike occurrence evidenced by means of ISI distribution.
- 4) By tailoring the filter, ISI's last a few ms, thus matching the neuron behavior; whence we can easily build large coupled arrays to model brain areas.



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