1--Nonlinear dynamics in Optics

2-Chaos in optics and

application to Cognitive processes

FT Arecchi

University of Firenze and CNR-INO,

E-mail: tito.arecchi@ino.it

Homepage: www.ino.it/home/arecchi

Firenze, October2012

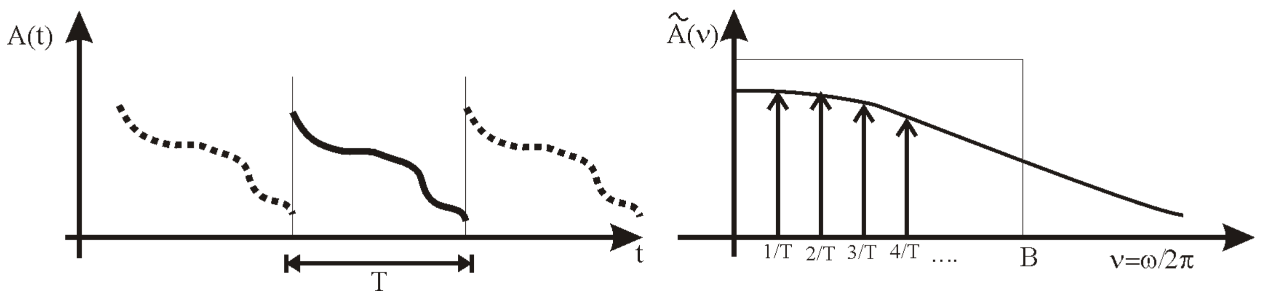
Nonlinear dynamics in Optics

0 - HOW TO ORGANIZE OBSERVATIONS

Degrees of freedom of a signal (Shannon)

Two real n° for each Fourier component, namely, modulus and phase

$$A(\omega) = |A(\omega)| e^{i\varphi(\omega)} \quad .$$



A measuring device has a band *B* and is active for a time *T*

Beyond *T* , assume $A(t)$ repeats left and right, thus use a ***series expansion*** : keep only harmonics $A(\nu)$,

spaced by *1/T*, and only those within *B*, for a total *B·T.* Each one yields two reals, thus the n° *N* of degrees of freedom is (***Shannon***):

$$N = 2BT$$

Apply to TV screen. Let us visualize a chessboard of white-black squares.

Visual permanence time ***t_p=1/15 s ;*** to have motion feeling, need 2 squares within t_p, hence *T=1/30 s.* TV channel band ***B=5MHz***. Thus

$$N = 2 \cdot 5 \cdot 10^6 \cdot \frac{1}{30} \cong 0{,}3 \cdot 10^6$$

N° horizontal lines ***$N^{1/2}$=600***

Space case :optical resolution (pixel)

Image formationthrough a lens (Abbe ,1880) .

Shine an obiect with a plane wave; information elements will scatter the single ***k*** direction to many waves of equal color but different directions *k* .The amplitude $\tilde{A}(\vec{k})$ results from summing the different features *A(r)* of the 2D domain $\vec{r}(x, y)$ with different phases **(Fourier transform)**

$$A(k) = \int A(r) e^{ik \cdot r} dr \; .$$

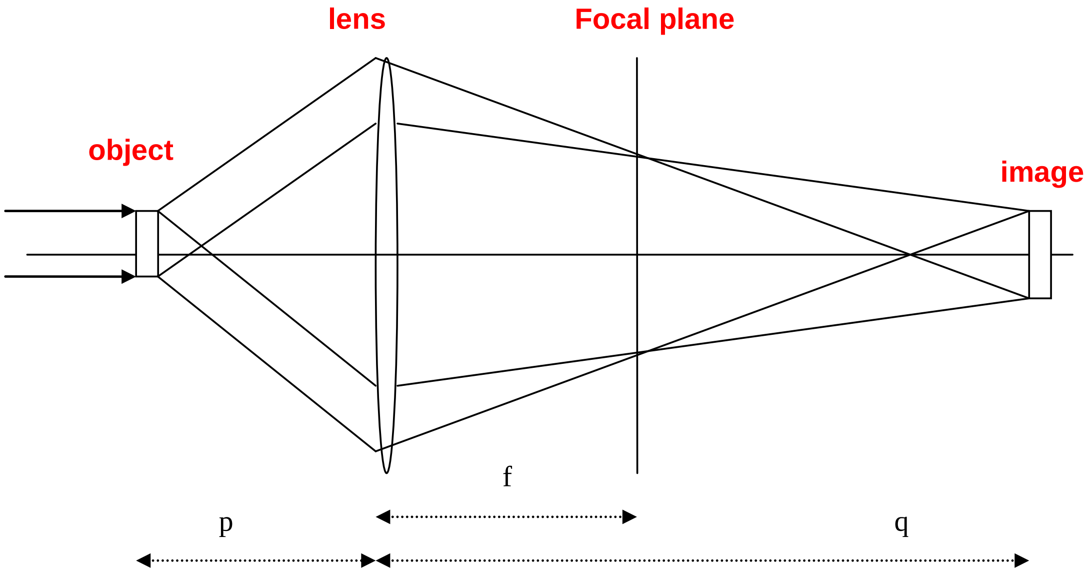
Let's go through a lens.Each plane wave converges to a point of the focal plane (= Fourier plane).

However a detector senses NOT the ***amplitude but the energy*** of each wave (phase lost).
We do not get the signal on the focal plane , but let each point act as a source of a spherical wave .The spherical waves interfere with their mutual phases ; in particular they recombine on the image plane yielding an image similar (besides a ***magnification***)to the object.Object and image planes are related by ***Gauss formula***

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

p, q and *f* being the distances of object, image and focus from the lens.Thus the object is an inverse Fourier transform

$$A(r) = \Im^{-1}\left[A(k)\right] = \int A(k) e^{-ik \cdot r} dk$$



- Image formation (***Abbe***) : each scattered wave is focused on a point on the focus; points on ***f*** are sources of spherical waves which sum with the right phases only on the image

- ***Image formation***

- $$\underbrace{A(r)}_{object} \Rightarrow\Rightarrow \underbrace{A(k)}_{focal.plane} \Rightarrow\Rightarrow \underbrace{A(r)}_{image}$$

- this occurs only in the visible where transparent media ***refract***, i.e. bend the rays (lens).

- At X rays no lenses. Going in the far field collect all the square Fourier transform

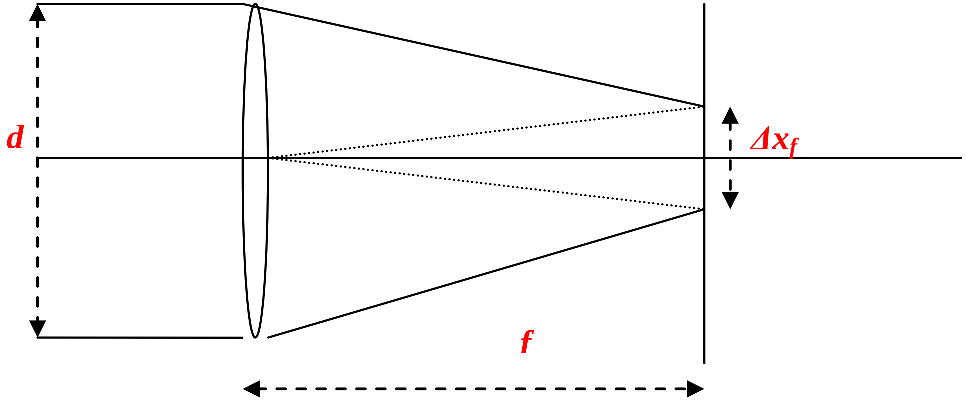
$$\left|A(k)\right|^2$$

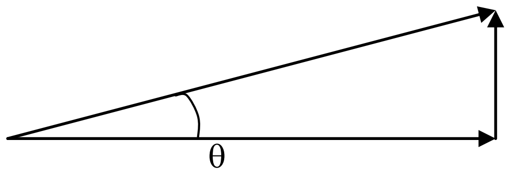
- ***Problem of phase*: *how to reconstruct the signal in crystallography***

PSF(point spread function) and space Shannon (pixels) .

Due to limited *d*, large angles are lost .
A plane wave in Fourier is not integrated between $\pm\infty$ but between $\pm d/2$; as a rect function angles are spread in a cone

$$\theta = \Delta k / k = \frac{2\pi / d}{2\pi / \lambda} = \frac{\lambda}{d} .$$





$\Delta k = 2\pi/d$

$k_0 = 2\pi/\lambda$

- Have a spot

- $$\Delta x_f = f \cdot \theta = {\lambda f}/{d} \cdot$$

- Thus the focal image of a plane wave is Δx_f called ***PSF*** (point spread function):

- The Fourier band is not $\Delta k = \infty$ yielding $\Delta x_f = 0$, ma $\Delta k = 1/\Delta x_f = d/\lambda f$

The number of distinct spots (***pixel*** = picture elements) with an object of length L è $L/\Delta x_f$.
Multiply by 2 (amplitude+phase) and get the n° N of degrees of freedom od a 1-dimensional image .(***Shannon*** ***)***

$$N = 2L \cdot \Delta k = 2L \frac{d}{f\lambda}$$

For 2D have N^2

For a slide of side *d=3cm* and focus *f=10 cm*, in green light **(λ=0,5 micrometer)**
$N=10^4$ thus **$N^2=10^8$ pixel=100 megapixel.**

In microscopes ***d=f*** **(** in jargon: numerical ***numerical aperture =1*****)**

Nonlinear dynamics in Optics

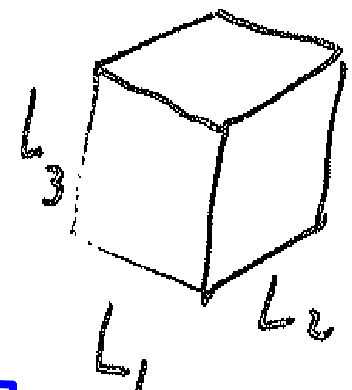
1-LASER BASICS

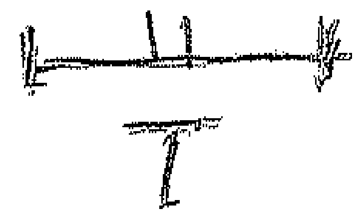
The e.m. field and its quantization

$$E(x,y,z,t) = \sum_{\vec{k}} E(\vec{k}, t)\, e^{i(k_1 x + k_2 y + k_3 z)}\, e^{-i\omega_k t}$$

$$\boxed{\omega_k = c k}$$

$$k_i = n_i \frac{2\pi}{L_i} \quad (i = 1, 2, 3)$$





$$\boxed{\delta^3 k = \frac{(2\pi)^3}{V}}$$

$$\omega_i = n_i \frac{2\pi}{T}$$

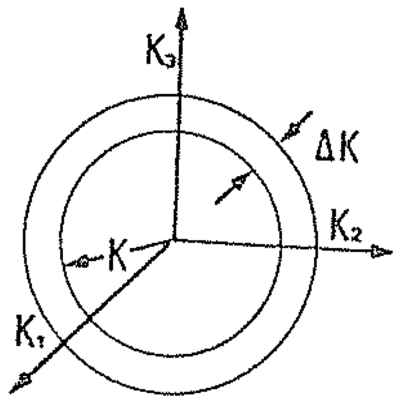


Fig. 1

Rayleigh

$$M = 2 \cdot \frac{4\pi k^2 \Delta k}{\delta^3 k} = 8\pi \frac{\nu^2}{c^3} \Delta\nu \, V$$

$$= 8\pi \frac{V}{\lambda^3} \cdot \left(\frac{\Delta\nu}{\nu}\right)$$

Planck 1900

$$\frac{dW}{d\nu} = \frac{dM}{d\nu} \cdot h\nu \cdot \bar{n}_1(\nu) = \frac{8\pi\nu^2}{c^3} \cdot V \cdot h\nu \frac{1}{e^{h\nu/kT} - 1}$$

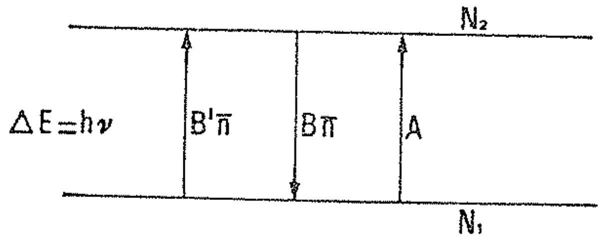


Fig. 2

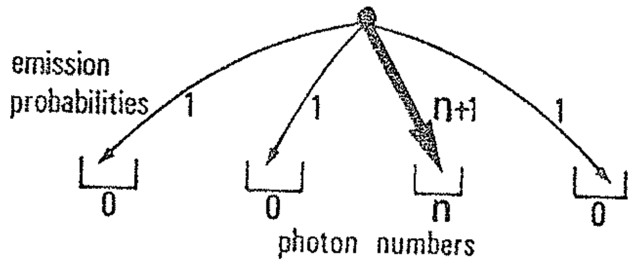


Fig. 3

Einstein 1915

$$\frac{N_2}{N_1} = e^{-\Delta E / kT} \longrightarrow k_{\text{Boltzmann}}$$

$$N_1 B \bar{n} = N_2 B \bar{n} + N_2 \cdot A$$

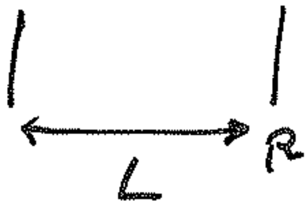
$$\bar{n} = \frac{A/B}{\frac{B'}{B}\frac{N_1}{N_2} - 1} \equiv \bar{n}_1(\nu) \cdot \Delta\nu$$

$$B' = B, \quad \boxed{\frac{A}{B} = \frac{8\pi\nu^2}{c^3} V \Delta\nu = M}$$

LASER

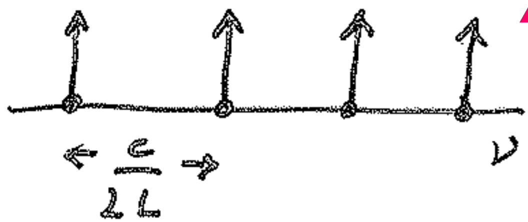
$n > M$

Fabry-Perot cavity



$$t_c = \frac{L}{c}\,\frac{1}{1-R}$$

$R < 1$

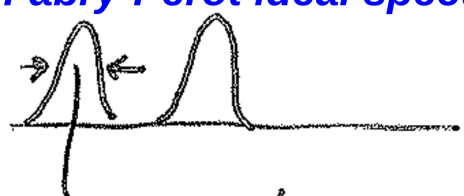


$$m\,\frac{\lambda}{2} = L \;\rightarrow\; \Delta\nu_{i,i+1} = \frac{c}{2L}$$

FSR

[Free spectral range]

Fabry-Perot ideal spectrum



$$\text{Finesse} = \frac{\Delta\nu_{i,i+1}}{\Delta\nu_c} = \frac{\pi}{1-R}$$

$$\Delta\nu_c = \frac{1}{2\pi}\,\frac{1}{t_c}$$

Fabry-Perot real spectrum

Rate

$$\dot{n} = BN \cdot n - n/t_c + BN_2$$

$$\dot{N} = -BN \cdot n + P - AN_2$$

Threshold

$$N \geq \frac{1}{B \cdot t_c}$$

Pump

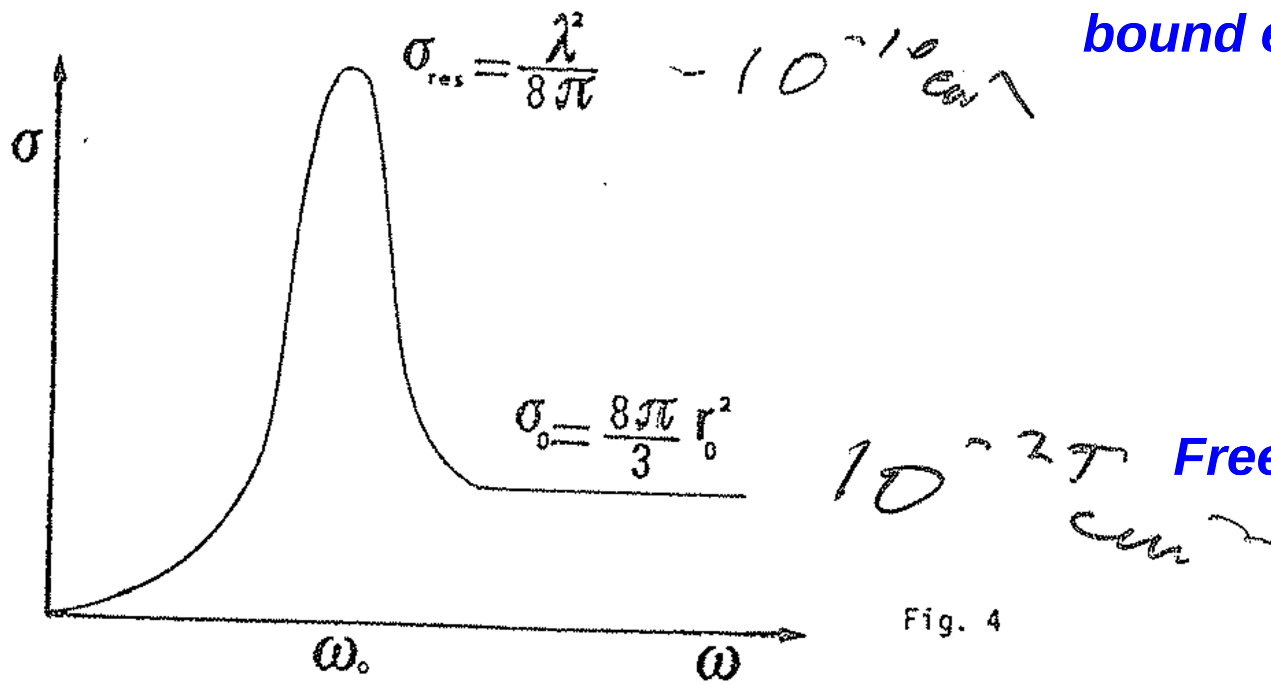
$$P \geq BNn > \frac{n}{t_c} > \frac{M}{t_c}$$

Concept of cross section

$$Bn = \sigma \phi = \sigma \cdot c \frac{n}{V} \longrightarrow \boxed{\sigma = \frac{BV}{c}}$$

1/s cm² 1/cm²·s

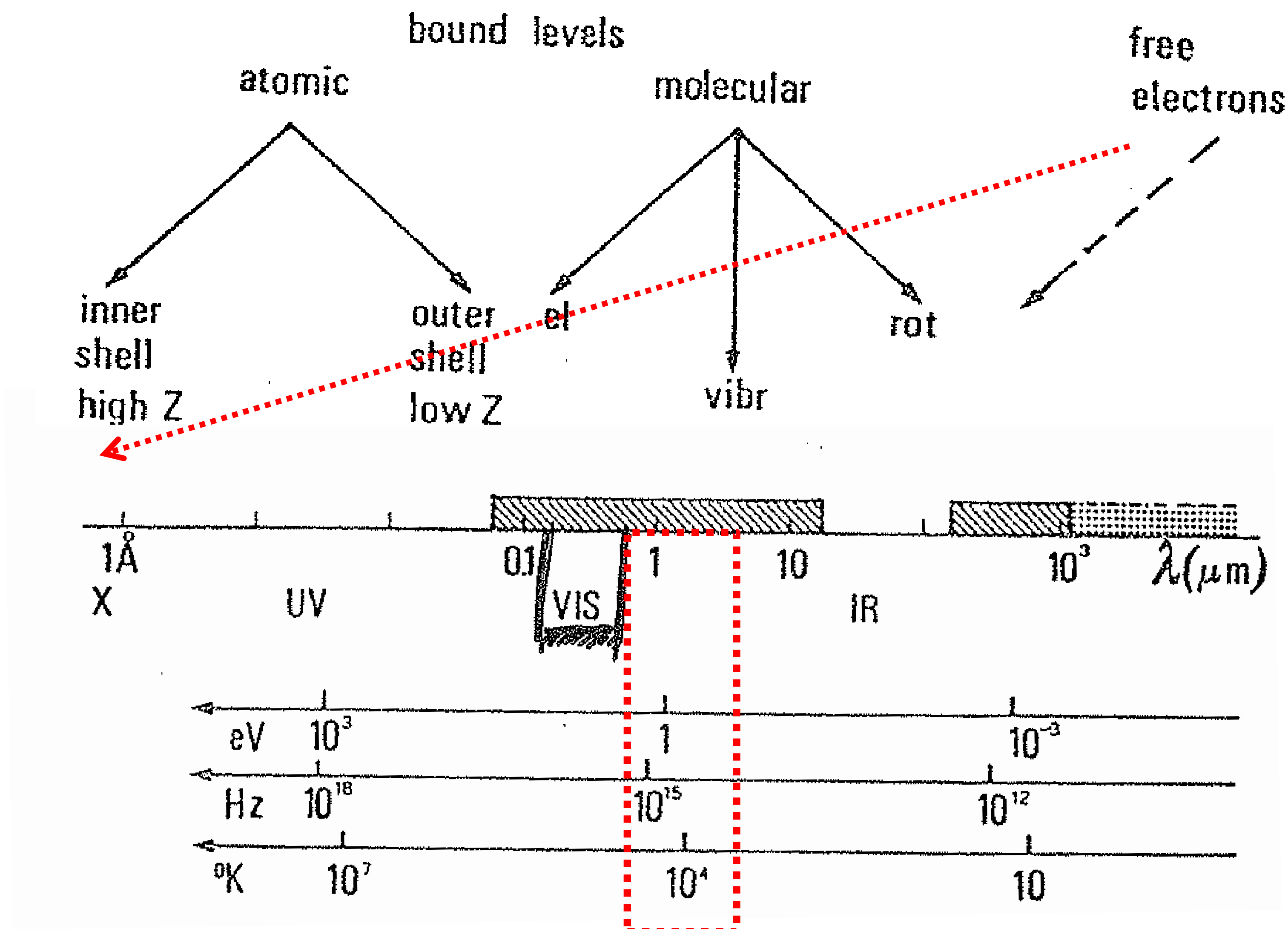
bound electrons



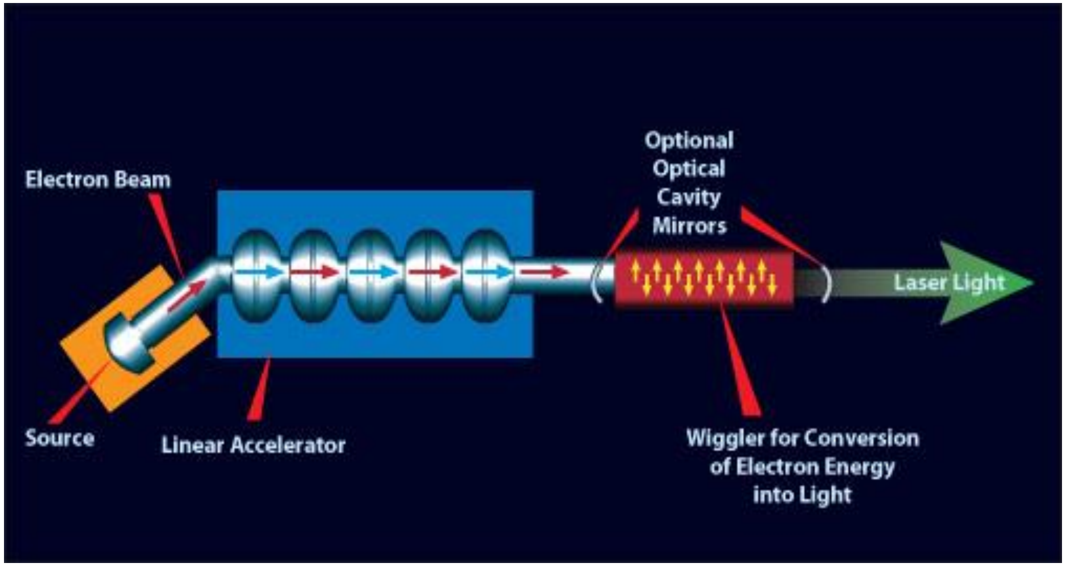
Free electrons

Fig. 4

LASERS

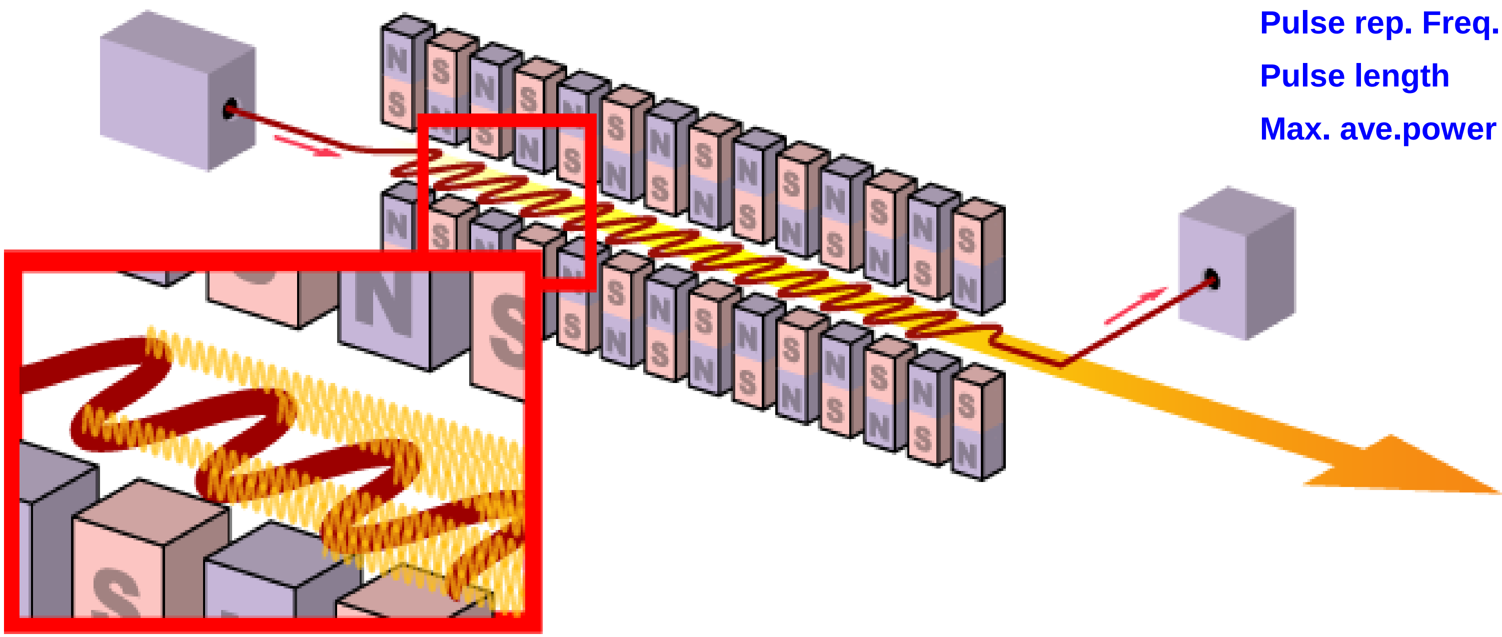


bound levels
atomic
molecular
free
electrons
inner
shell
high Z
outer
shell
low Z
el
vibr
rot
1Å
0.1
1
10
10^3
$\lambda(\mu m)$
X
UV
VIS
IR
eV
10^3
1
10^{-3}
Hz
10^{18}
10^{15}
10^{12}
°K
10^7
10^4
10



Wavelength range (IR)	1-14µm
Energy/pulse	120 µJ
Pulse rep. Freq.	Up to 75 MHz
Pulse length	500-1700 fs
Max.ave. power	>10 kW

Wavelength range (UV/VIS)	250-1000 nm
Energy/pulse	20 µJ
Pulse rep. Freq.	Up to 75 MHz
Pulse length	300-1700 fs
Max. ave.power	>1 kW



--electron energy E,
-- period of the undulator magnet λ_u
-- its magnetic field B,

$$\lambda = \lambda_u (1 + K^2)/2\gamma^2$$ **[double Doppler]**

λ_u = *undulator period (e.g. = 1cm)*

$K = eB\lambda_u/(2\pi mc)$ = *pitch parameter (e.g.=2)*

$\gamma = E/(mc^2)$ = *relativistic factor (e.g. =10^3 for E=0.5 GeV)*

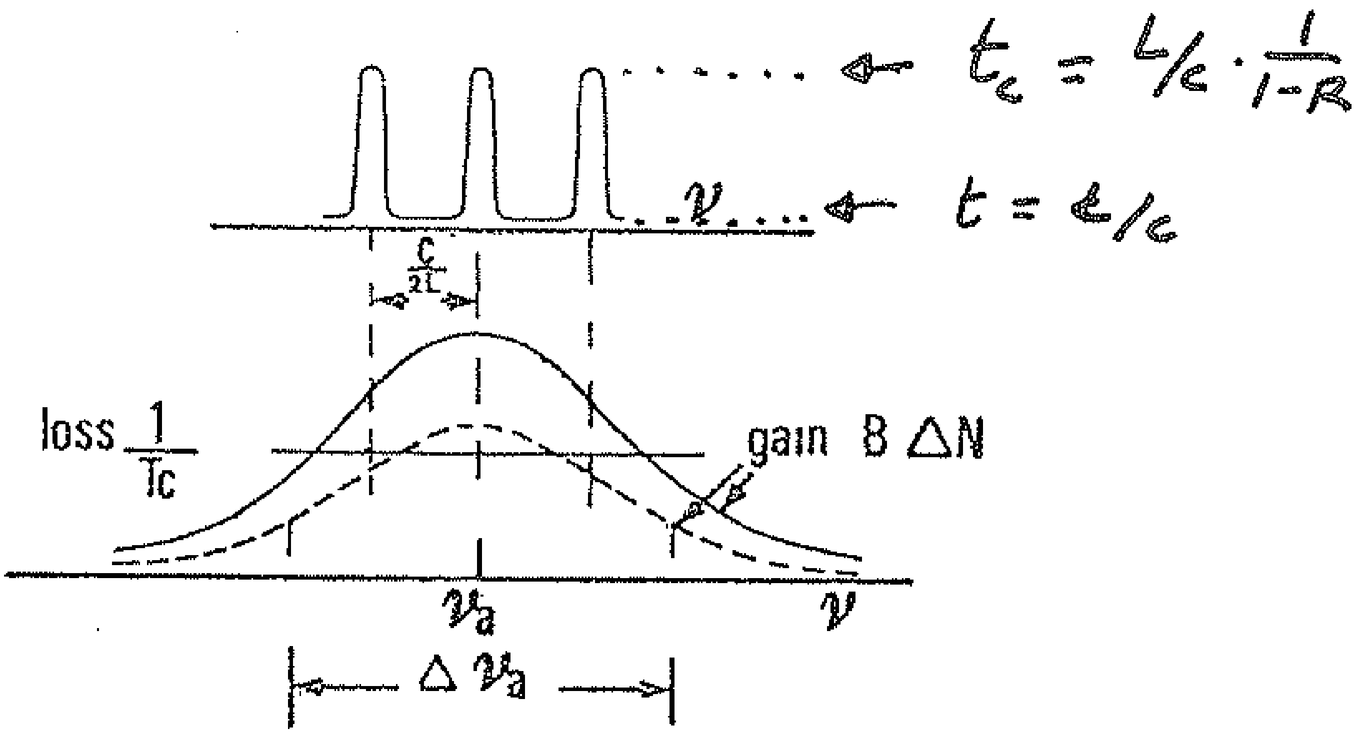
N = number of undulator periods

drawback of FELs : setups large and expensive, used only at few large facilities. Most ambitious project currently pursued in Hamburg : goal hard X-ray output , wavelengths down to 0.085 nm and pulse duration below 100 fs. So far, wavelengths down to 6.5 nm achieved

Linac Coherent Light Source (LCLS) at SLAC- Stanford.

Electron Beam	
Electron energy, GeV	*14.3 (γ=2.8x10^4)*
Peak current, kA	*3.4*
Pulse duration, fs	*230*
Undulator	
Period, cm	*3*
Field, T	*1.32*
K	*3.7*
Gap, mm	*6*
Total length, m	*100*
Radiation	
Wavelength, nm	*0.15*
Bunches/sec	*120*
Average Brightness	*4x10^{22}*
Peak power, GW	*10^{10}*

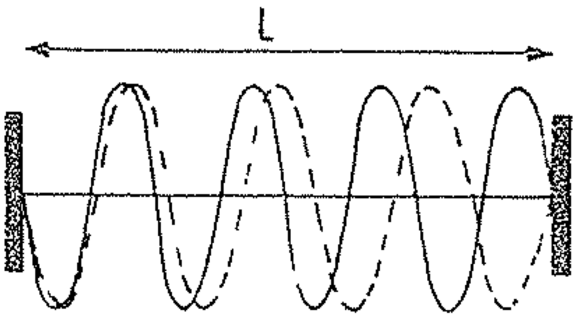
Threshold: Gain = BN > $1/t_c$



As gain increases , from 1 to 3 lasing modes

Two adjacent modes

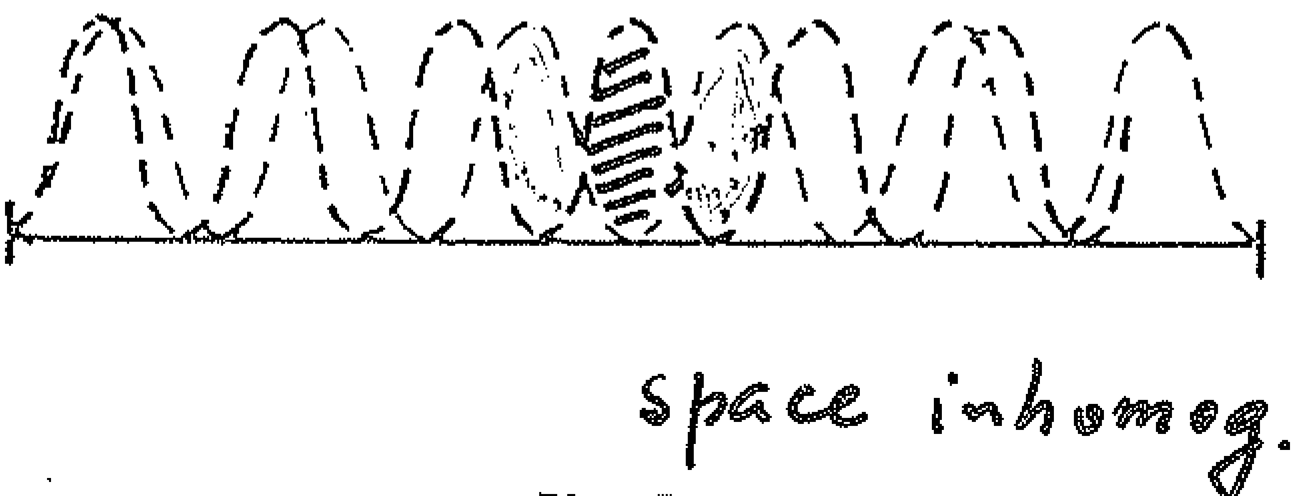
have different wavelength



Since pop. depletion (saturation) depends on local intensity

different modes exploit different regions of the laser material

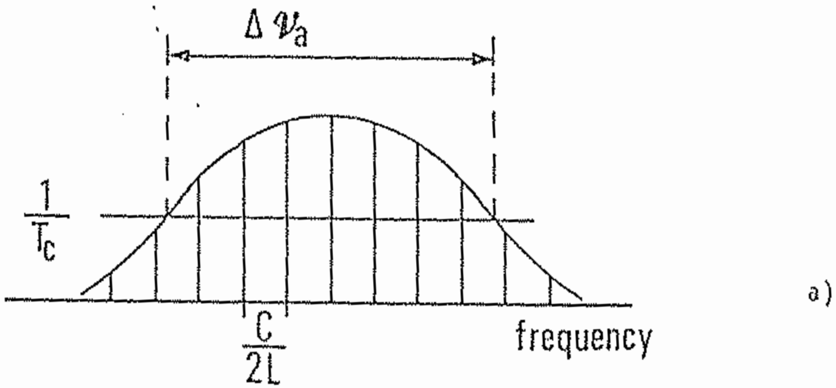
(space inhomogeneity)



MODE LOCKING → SHORT PULSES

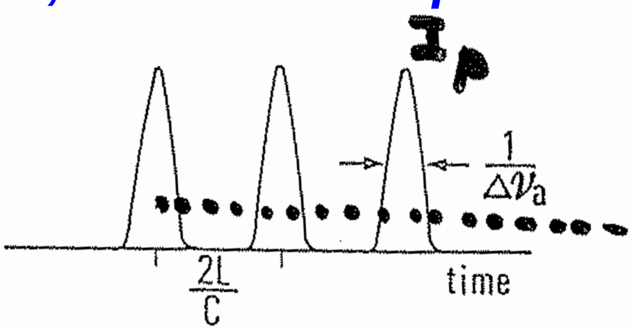
N # of modes above threshold = material width/mode separation

$$N = \frac{\Delta\nu_a}{c/2L}$$



If modes coherent in phase, then Fourier spectrum of repetitive pulses

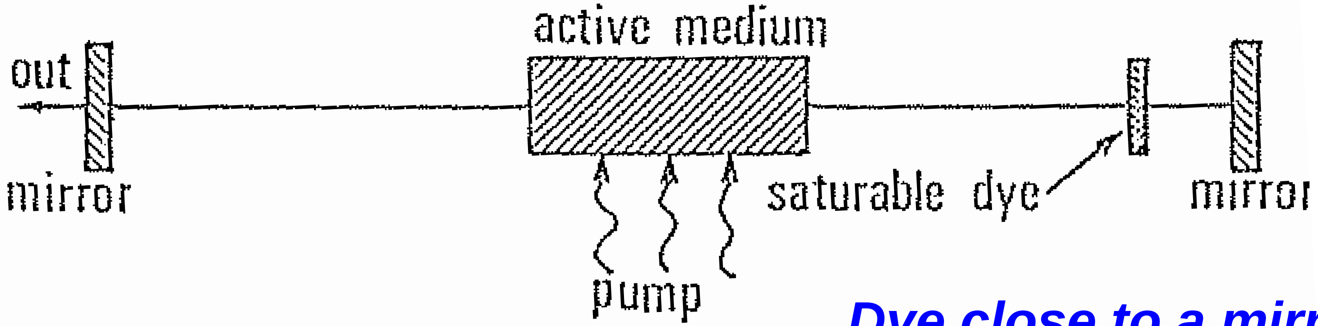
$$\Delta t = \frac{1}{N}\,\text{Sep.}$$



$$I_p = N^2 I_1$$

$$\langle I \rangle = N I_1$$

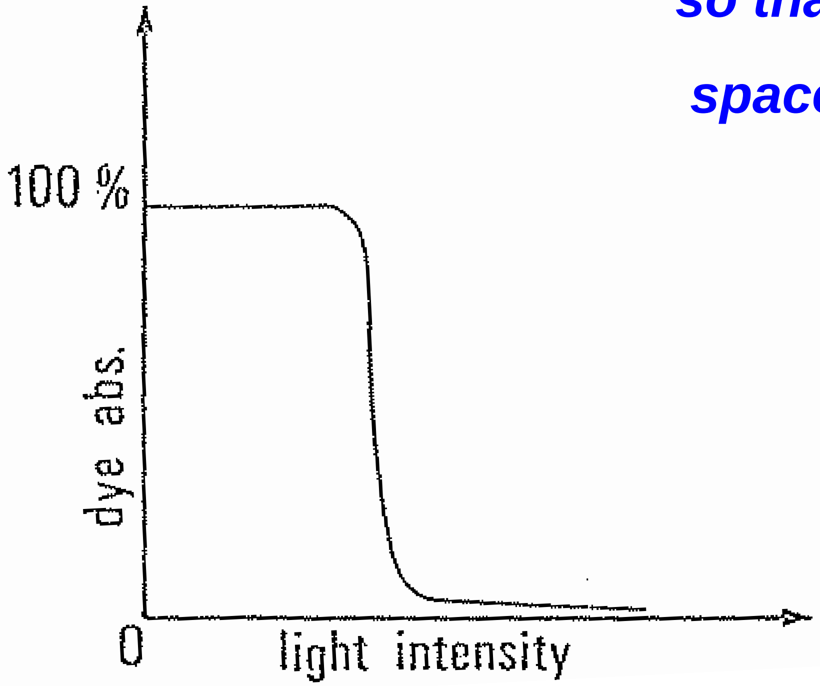
Pulse duration= (pulse separation)/N



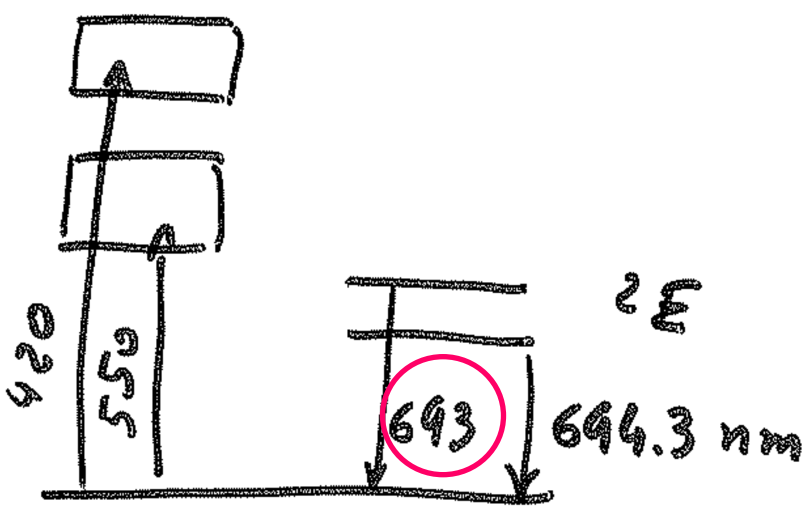
Dye close to a mirror,

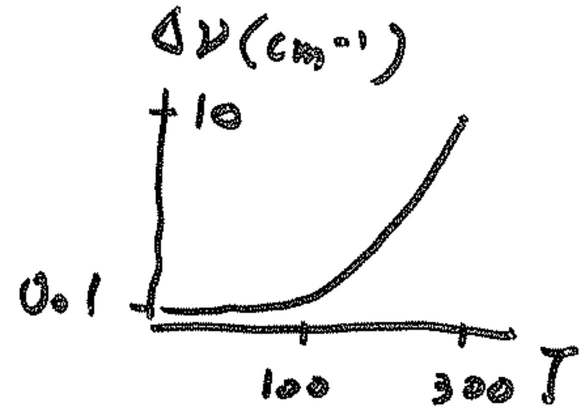
so that all modes have same

space phase

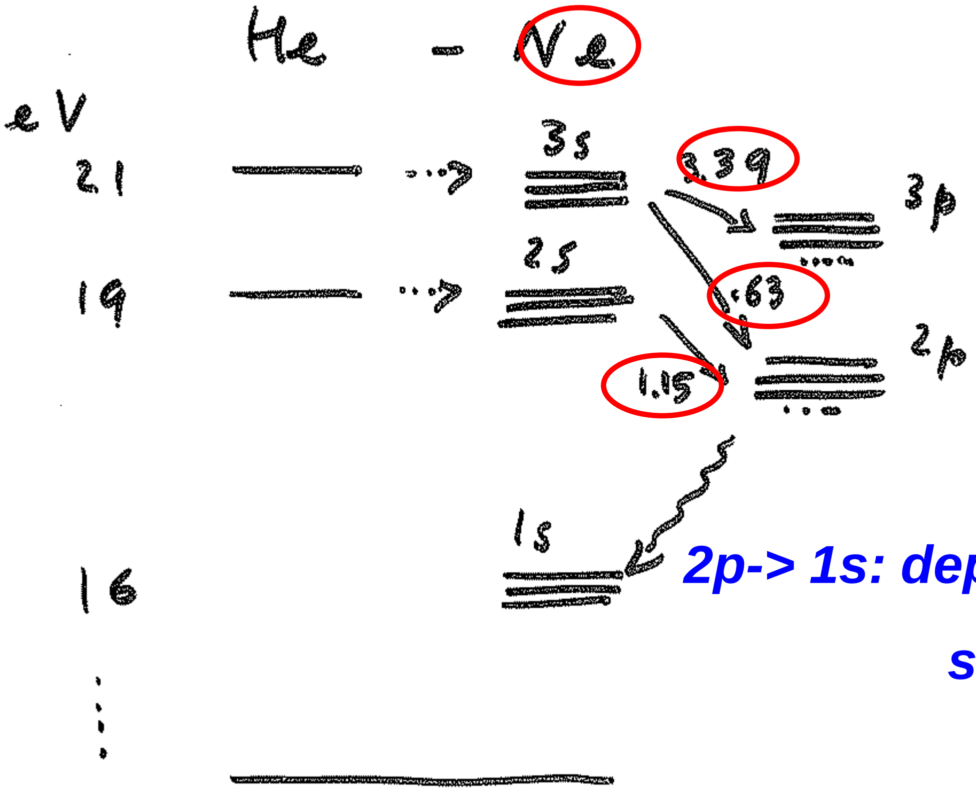


Ruby $Cr^{3+} : Al_2O_3$

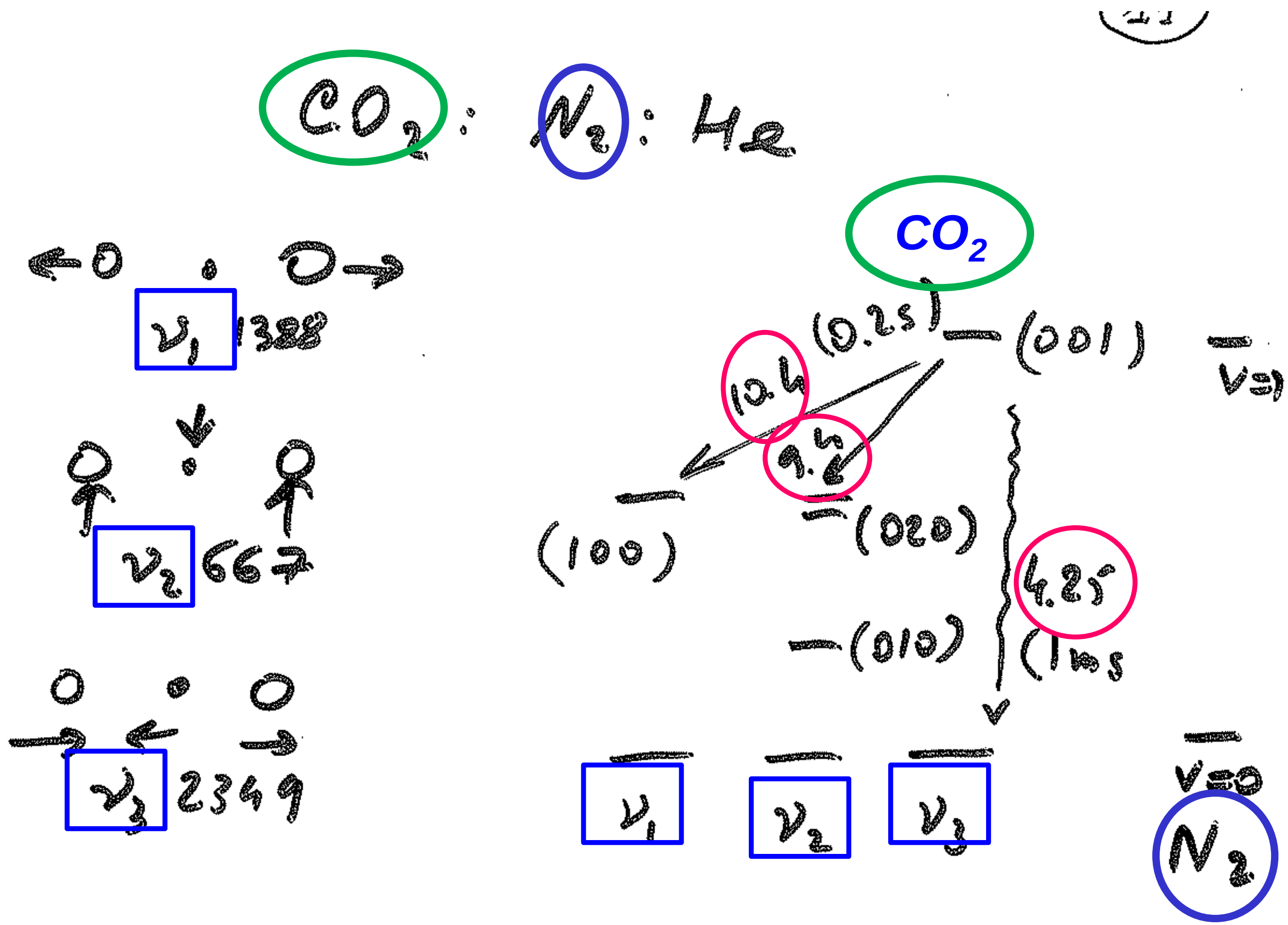




Radiative transitions



2p-> 1s: depletion of final state



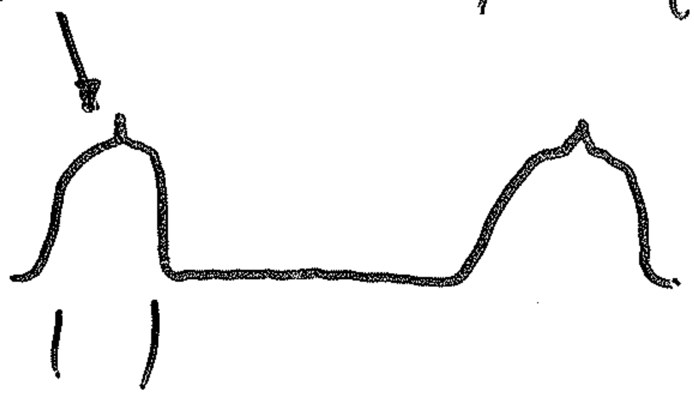
$\nu_{1\text{-}3}$ are three vibrational modes :transition frequencies(cm^{-1})

and ◯ corresponding wavelengths (micron)

$\tau^{-1}_{coll\ vib} \sim 10^{?}\ s^{-1}/torr$

$\tau^{-1}_{coll\ rot} \sim 10^{7}\ s^{-1}/torr$

$T_2^{-1} = 7\ MHz/torr$ (Lamb dip)



$\Delta\nu_D \sim 50\ MHz$

$\Delta\nu_{rot} = 50\ GHz$

$E_{rot} = \frac{\hbar^2}{2I} J(J+1)$

$\nu = \Delta E \sim J+1$

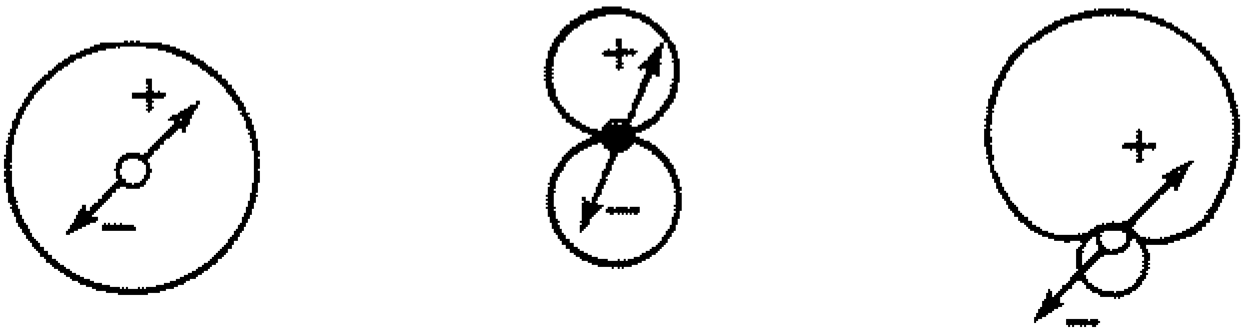
$\Delta\nu \sim const$

Dipole moments and linewidths of laser materials

Let us evaluate the dipole source due to a radiating atom.

It will be the expectation value of the dipole operator over the atomic state $|\psi\rangle$, i.e.,

$$\langle d\rangle = e\langle r\rangle = e\,\langle\psi|r|\psi\rangle$$



$\psi = |s\rangle$ $\quad$ $\psi = |p\rangle$ $\quad$ $\psi = a|s\rangle + b|p\rangle$

$\langle r\rangle = 0$ $\quad$ $\langle r\rangle = 0$ $\quad$ $\langle r\rangle \neq 0$

$$\langle r\rangle \equiv \int \vec{r}\,|\psi(\;)|^2\, d^3r$$

Fig. 1.8

Atomic wavefunctions for pure s and p states and for a linear combination with amplitudes a and b

When interacting with a constant E field, an atom is driven back and forth in a reversible way as shown in fig. 1.9 at a rate

$$\Omega = \mu E / h \qquad (CmVm^{-1}/Js = s^{-1}) \tag{1.21}$$

where $\mu = e\langle s|r|p\rangle$ is the transition matrix element (taken for simplicity as a real number) and Ω is called the Rabi frequency. The wavefunctions at different times are sketched in fig. 1.9 and any time the instantaneous wavefunction $\psi(t)$ is given by

$$|\psi(t)\rangle = a(t)|s\rangle + b(t)|p\rangle, \tag{1.22}$$

where a(t) and b(t) have sinusoidal variations as shown in the figure. Correspondingly the induced dipole

$$\langle d\rangle = \mu(a^*b + ab^*) \tag{1.23}$$

[because of the quantum evolution of a and b due to the field E]

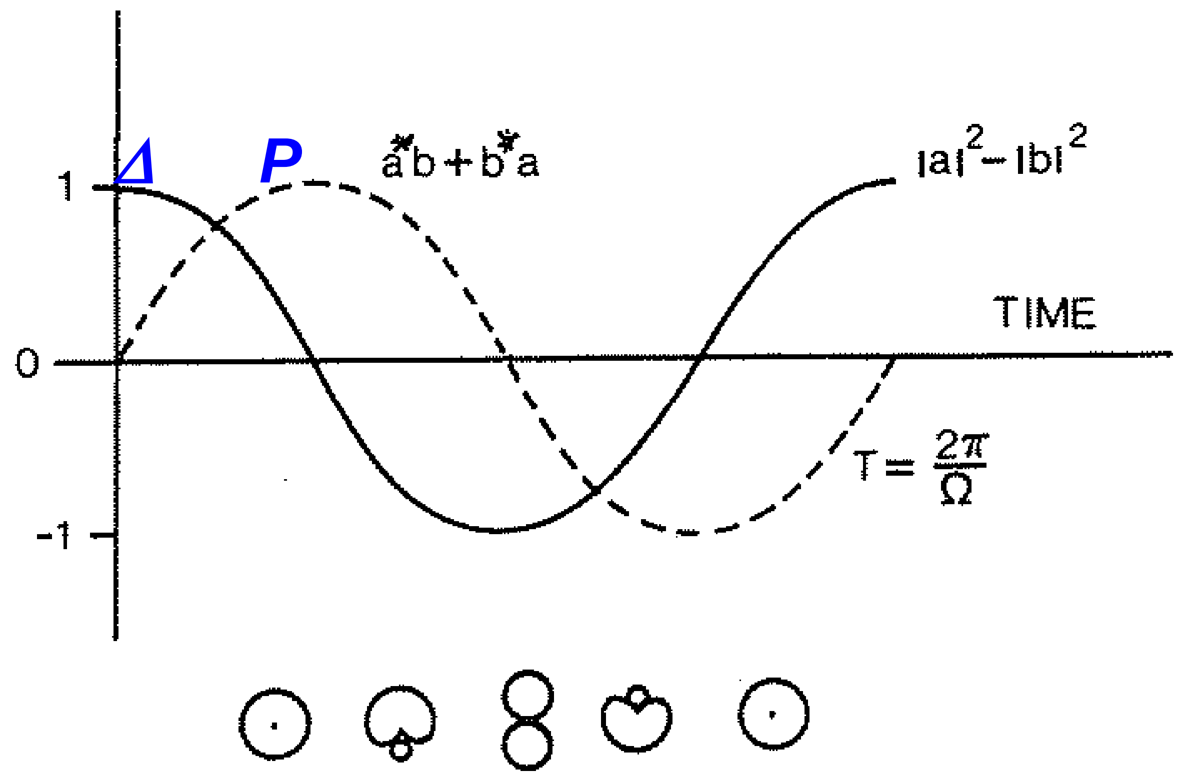


Fig. 1.9

Δ P

Population difference (solid line) induced dipole (dashed line) and atomic orbitals (ff.below) for different time during a Rabi period goes as the dashed line in fig. 1.9. As shown in eq. (1.23) it depends on the phase of the wavefunction, whereas the population inversion $|b|^2 - |a|^2$ is phase independent. Both $|a|$ and $|b|$ return to equilibrium values $|a| = 1$, $b = 0$ by spontaneous emission processes

. On a

faster scale, phase destroying processes, as collisions, interrupt the coherent Rabi precession as shown in fig. 1.10. The average interruption time is called T_2, and its reciprocal is the homogeneous linewidth

$$1/T_2 = \Delta\nu_a \geqslant \Delta\nu_{sp}$$

Since $\langle d\rangle/\mu = \Omega T_2$, then the average dipole is $\langle d\rangle = \frac{\mu^2 E T_2}{h} = \frac{\mu^2 E}{h\Delta\nu}$

and the polarization (ρ being the atomic density)

$$P = \rho\langle d\rangle = \frac{\rho\mu^2}{h\Delta\nu} E \qquad (1.24)$$

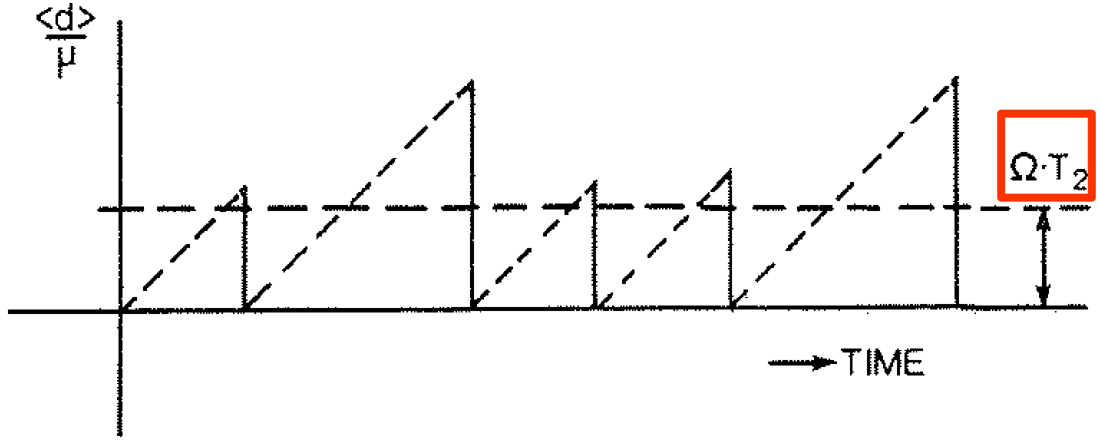


Fig. 1.10

Interruptions due to decay processes, giving a non zero polarization

The 3 so-called Maxwell-Bloch equations ruling field-atom interaction

$\Delta = \cos \Omega t$

$P = \sin \Omega t$

$\Omega = \mu E/h$

$[\Delta^2 + P^2 = 1 (\text{circle})]$

$dP/dt = \Omega\Delta$ (2)

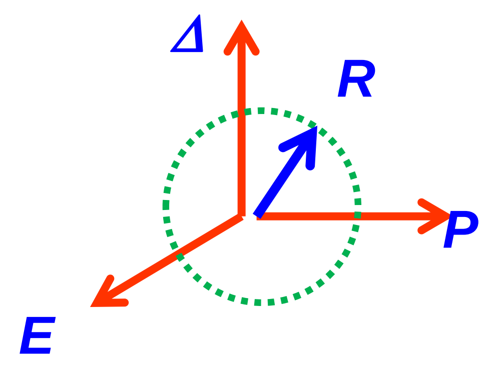
$d\Delta/dt = \Omega P$ (3)

On the other hand, from Maxwell, E evolution linear in P:

$dE/dt = P$ (1)

Thus, interaction atom-field as the motion of a vector R on a 3-D space (precession of R around E

at angular rate $\Omega = \mu E/h$)



Lasers of Class A, B and C: how time scales rule the dynamics

Couple e.m. field in cavity with N atoms.

Keeping only the first mode E which goes unstable, E has as linear source a polarization P. P depends on both E and population inversion Δ in a nonlinear way (terms in) as shown by Maxwell-Bloch eqs.

Furthermore,we add dissipative terms to account for:

i) Losses of E, P and Δ, at rates k, $\gamma_\perp$, $\gamma_\|$ respectively

ii) incoherent supply of energy (pump)

$$\dot{E} = -kE + gP \,,$$

$$\dot{P} = -\gamma_\perp P + gE\Delta \,,$$

$$\dot{\Delta} = -\gamma_\| (\Delta - \Delta_0) - 4gPE \,.$$

Here, k, $\gamma_\perp$, $\gamma_\parallel$ are the loss rates for field, polarization and population; g is a coupling constant and Δ_0 is the population inversion that would be established by the pump, in absence of coupling with E.

While the terms in ( ) are due to field-atom coupling, the terms

In k, $\gamma_\perp$, $\gamma_\parallel$ represent dissipation, that is, coupling with the environment (precisely, the field escapes from the cavity; and polarization P and population Δ decay at different rates

The following classification has been introduced *(Arecchi, #111)*

Class A (e.g., He–Ne, Ar, Kr, dye): $\gamma_\perp \simeq \gamma_\parallel \gg k$
The two last equations can be solved at equilibrium (adiabatic elimination procedure) and one single nonlinear field equation describes the laser. $N = 1$

Class B (e.g., ruby, Nd, CO_2): $\gamma_\perp \gg k \lesssim \gamma_\parallel$
Only polarization is adiabatically eliminated [middle eq.] and the dynamics is ruled by two rate equations for field and population. $N = 2$ allows also for oscillations.

Class C (e.g., FIR lasers): $\gamma_\parallel \approx \gamma_\perp \simeq k$
The complete set of eqs. has to be used,

Class A: single eq.

dE/dt= f(E)

where

$$f(E) = -\alpha E \quad \text{below}$$

$$f(E) = +\alpha E - \beta |E|^2 E \quad \text{above}$$

} *threshold*

Class B: two coupled eqs:

$$\begin{cases} dE/dt = f(E,\Delta) \\ d\Delta/dt = g(E,\Delta) \end{cases}$$

or,calling $I=E^2$ the intensity

$$\begin{cases} dI/dt = F(I,\Delta) \\ d\Delta/dt = G(I,\Delta) \end{cases}$$

that in fact are the rate eqs. for n= I (photon n° n prop. to intensity I) and N=Δ , given beforehand

Table 2 summarizes the relevant parameters for the most common lasers

[remember $T_{cavity}=10^{-6}$ sec]

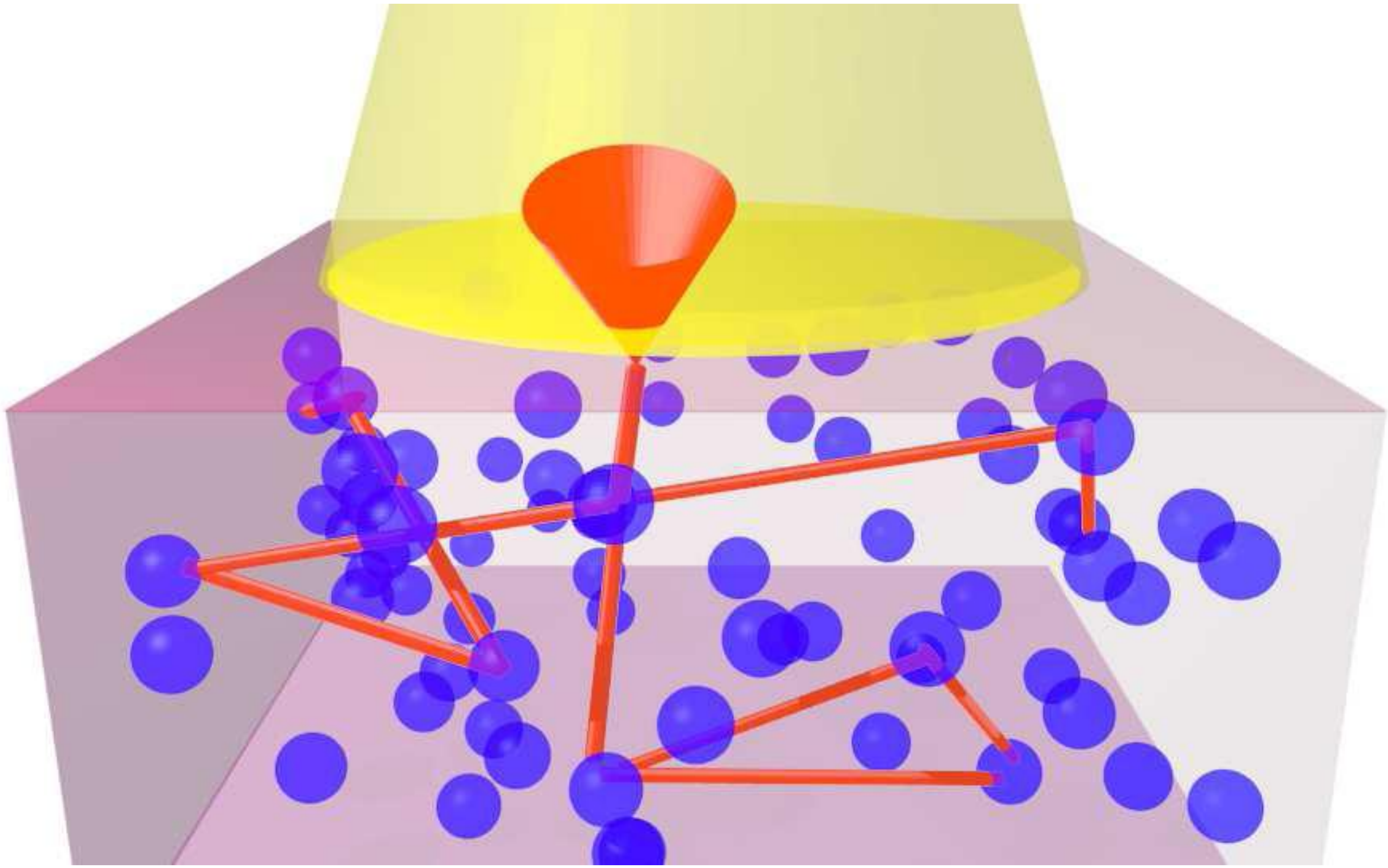
	Ruby Nd	Dye	He-Ne	Ar^+	CO_2	Semi-conduct.
τ_{sp} (sec)	10^{-3}	10^{-8}	10^{-8}	10^{-8}	$0.2^{(o)}$	10^{-9}
T_2 (sec)	10^{-12}	10^{-13}	10^{-8}	10^{-8}	$(4.4\times10^{7}/\text{torr})^{-1}$	10^{-14}
T_2^*(sec)	10^{-10}	10^{-13}	10^{-10}	$\frac{1}{3}10^{-10}$	$(3.14\times10^{8})^{-1}$	--
$\sigma(cm^2)$	3×10^{-20}	3×10^{-16}	10^{-12}	10^{-12}	10^{-15}	10^{-15}
$\rho(cm^{-3})$	10^{19}	10^{15}	10^{9}	10^{10}	10^{14}	10^{18}
$\alpha(cm^{-1})$	0.3	3	10^{-3}	10^{-2}	0.1	50
ℓ(cm)	10	0.1	$10^{3(*)}$	$10^{2(*)}$	10	3×10^{-2}

[Laser for $\exp(\alpha l)>1$]

Table 2

(0) The inversion decay time due to collisions is 0.4×10^{-2} s

(*) High reflectivity mirrors reduce the cavity length to 10 - 100 cm



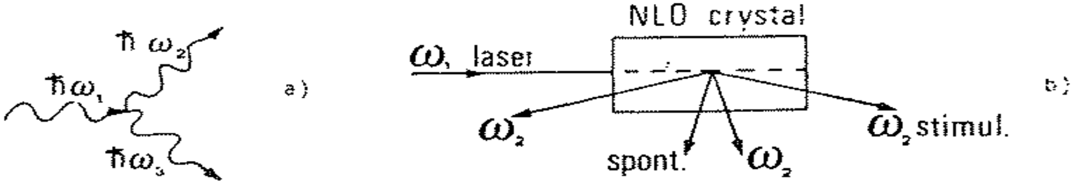
The RANDOM LASER- ***ZnO scatterers at random locations (blue) are optically pumped from above (wide yellow beam). The pumping yields an inversion of the atomic occupation number causing stimulated emission (orange light paths). The emitted***

intensity multiply scatters and concentrates . At the laser threshold the system experiences a phase transition and coherent intensity escapes the

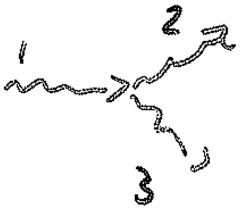
system through its surfaces (orange cone). The scatterers radius is 600nm, λ = 723nm .

2- NONLINEAR OPTICS

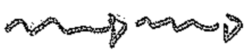
Spontaneous vs Stimulated



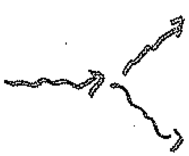
spont. ($\hbar\omega = E_n$) $\omega_1 = \omega_2 + \omega_3$



stimul. ($\hbar k = p$) $\vec{k}_1 = \vec{k}_2 + \vec{k}_3$



$P_i = \varepsilon_0 \chi^{(1)}_{ij} E_j$ ***Linear polarization***



$P_i = \varepsilon_0 \chi^{(2)}_{ijk} E_j E_k$ ***quadratic polarization***

(3 quanta involved)

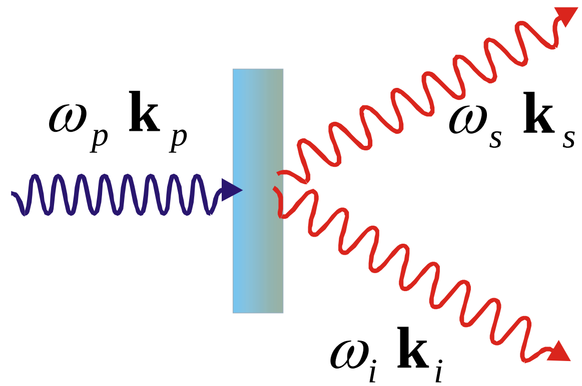
3- quanta NLO processes

Nature of the quanta		Name of the process
2	3	
light	molecular vibrations	Raman
light	optical phonons in solids	Raman
light	acoustical phonons in solids	Brillouin
light	sound waves in liquids	Brillouin
light	light	parametric conversion (sum or difference of frequency, second harmonic generation, etc.)

Parametric down conversion-

Entangled photons

Spontaneous Parametric down-conversion (SPDC)



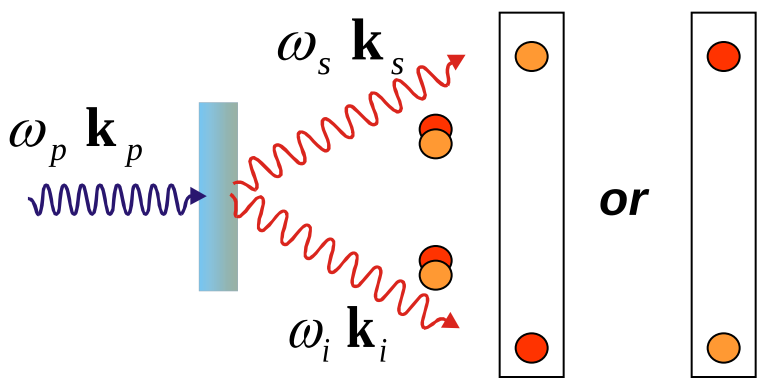
$\omega_p = \omega_i + \omega_s$

$k_p = k_i + k_s$

Energy and momentum conservation

$$|\psi\rangle = \left(|{\color{red}\bullet}\rangle_s |{\color{orange}\bullet}\rangle_i + |{\color{orange}\bullet}\rangle_s |{\color{red}\bullet}\rangle_i \right)$$

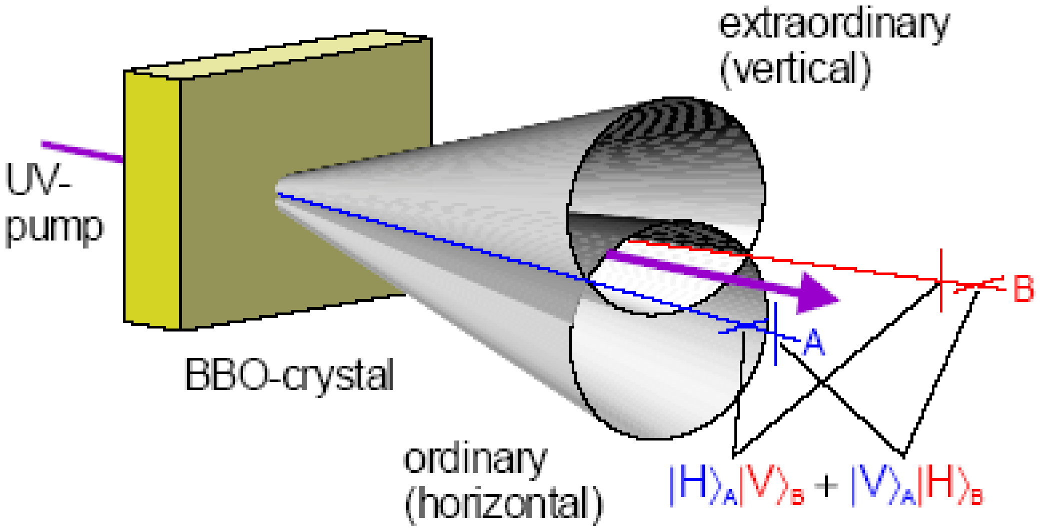
SPDC Entangled state



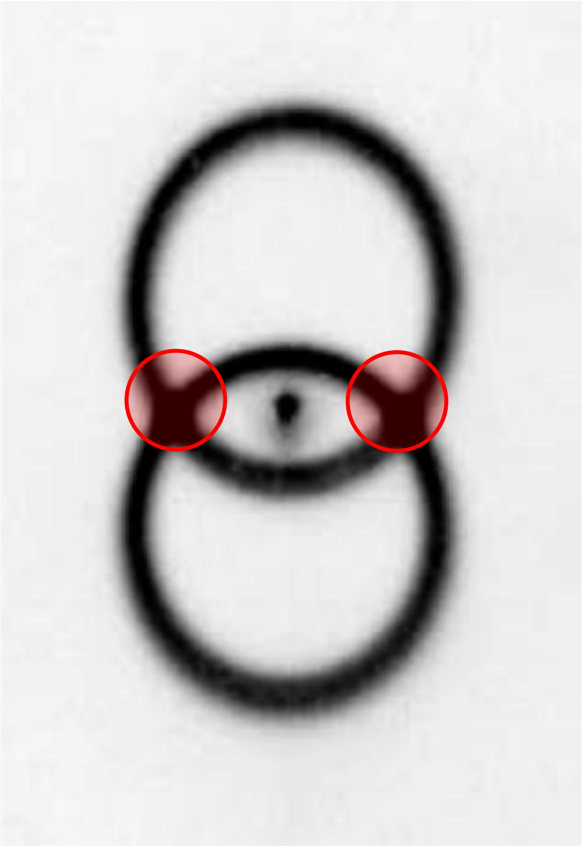
The properties of a single photon are not defined individually but are completely correlated to those of the other

Polarization Entanglement

- **_In type II crystals:_**



$$|\psi\rangle = \left(|H\rangle_1 |V\rangle_2 + |V\rangle_1 |H\rangle_2\right)$$



4 photon (self actions: self focusing
" defocusing
" phase modul. (broadening)
" amplit. " (steepening)

$$P_i = \chi^{(3)}_{ijkl} E_j^* E_k E_l$$

$$n = n_0 + n_2 |E|^2$$

3- COHERENCE,

PHOTON STATISTICS,

LASER PHASE TRANSITION

-Young interferometer

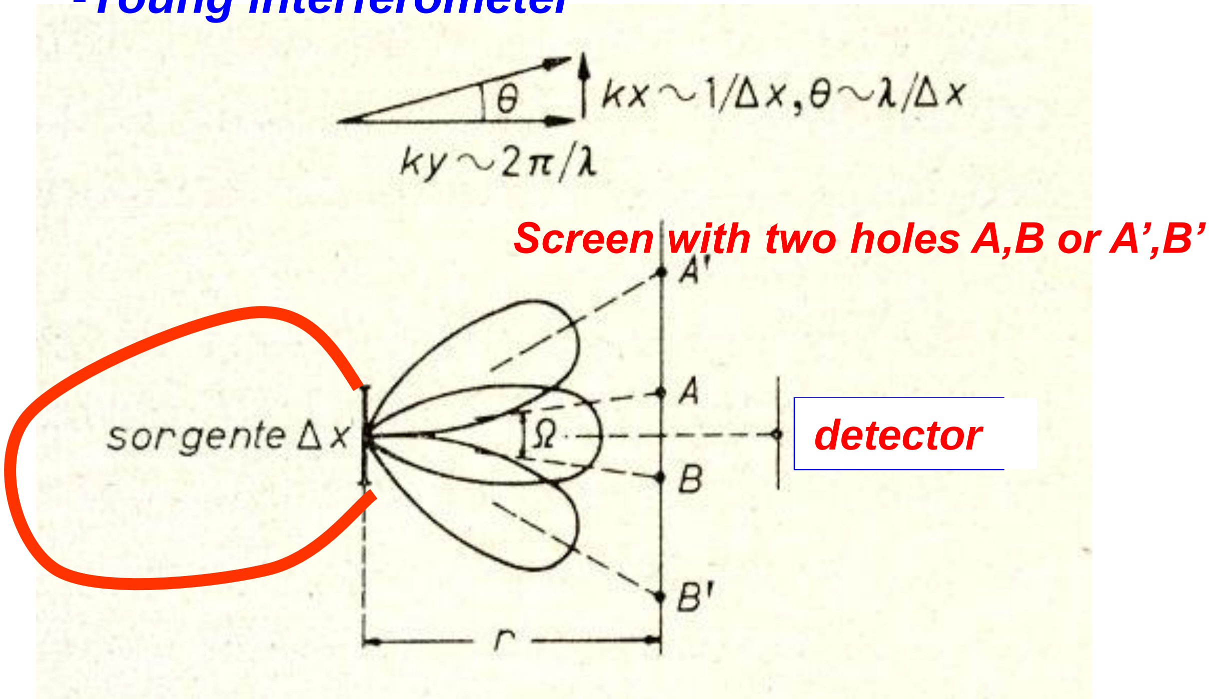


Photo-current proportional to square modulus of field

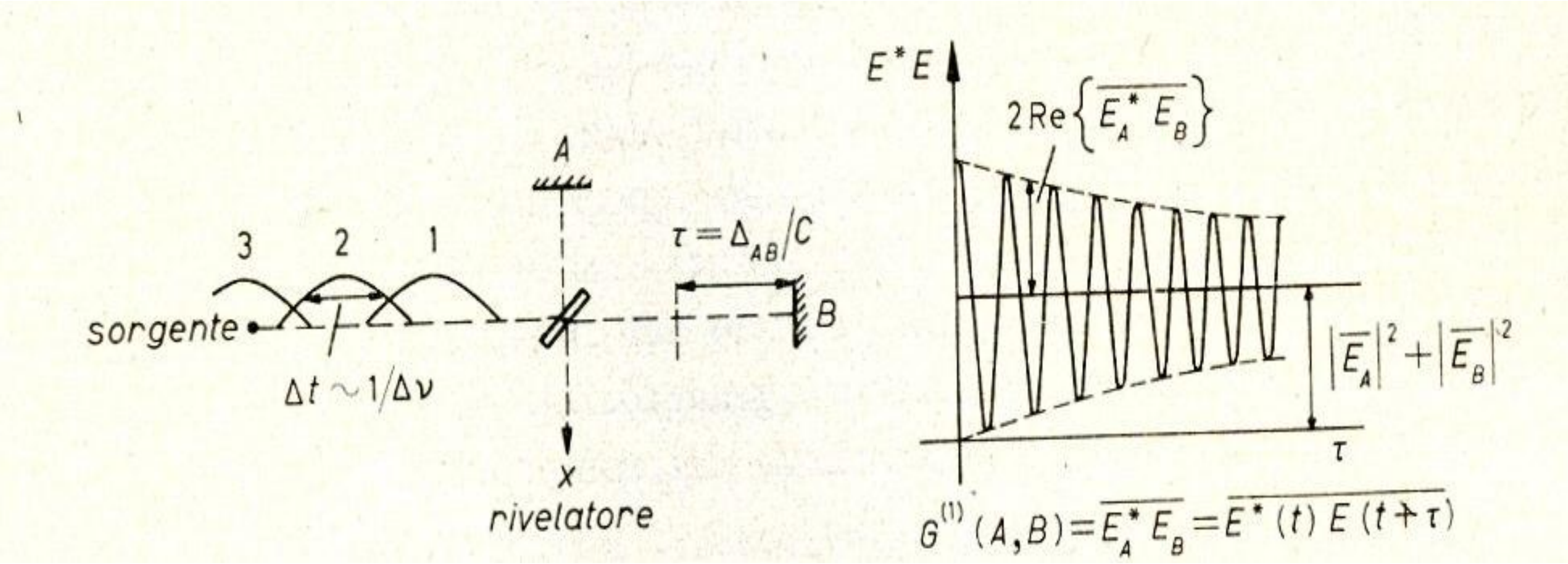
$$<|E_1 + E_2|^2> . \longrightarrow \quad I_1 = |E_1|^2 \quad I_2 = |E_2|^2 \quad < E^*_1 E_2 >$$

1-st order correlation function $G^{(1)}(1,2) = < E^*_1 E_2 >$

Coherence area

$$S_{AB} = \frac{\lambda^2 \cdot r^2}{(\Delta x)^2}$$

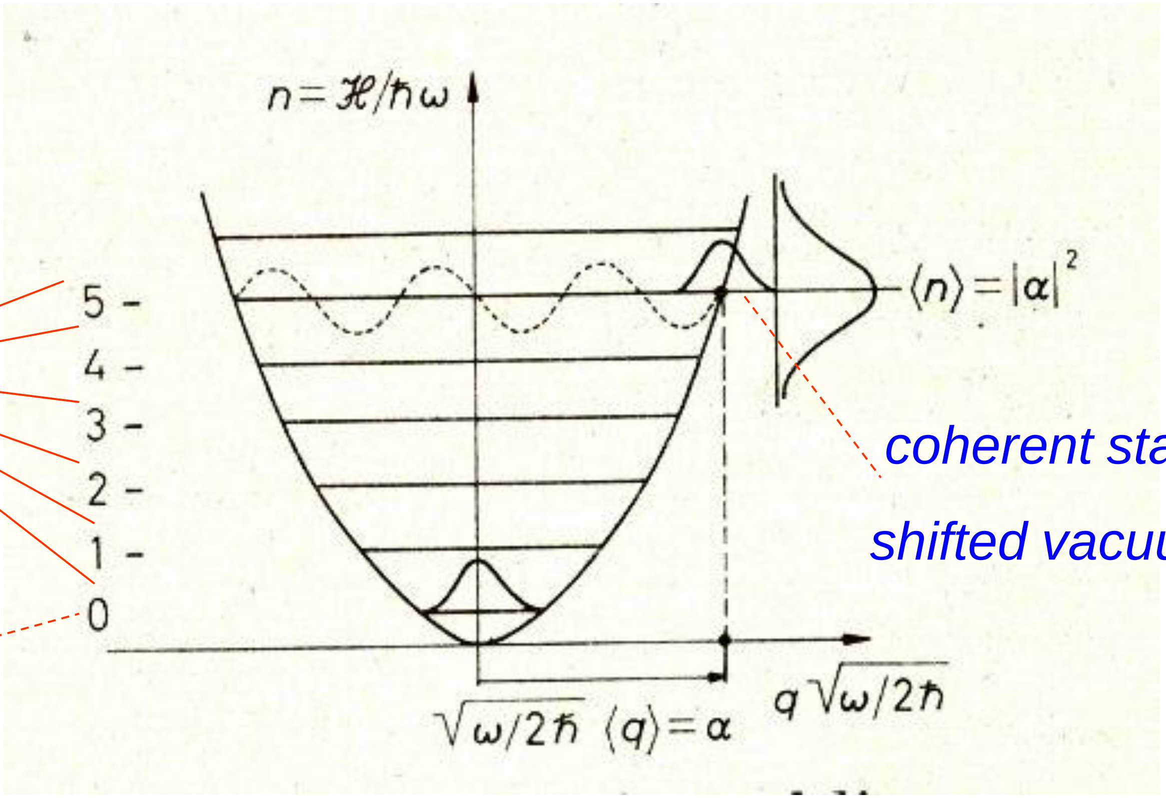
Michelson Interferometer



Harmonic Oscillator: energy H versus coordinate q

n-photon states

vacuum



coherent state = shifted vacuum

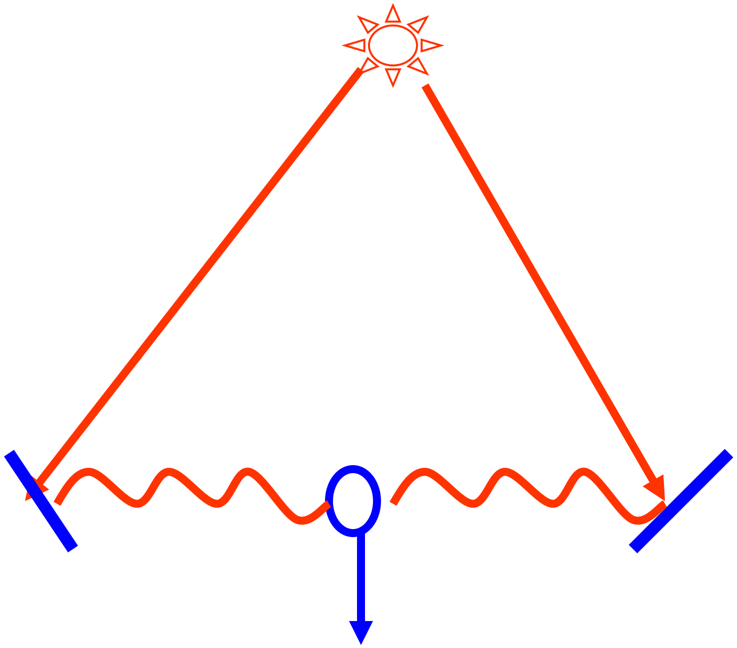
$$\Delta q \Delta p = (n + 1/2)\hbar$$

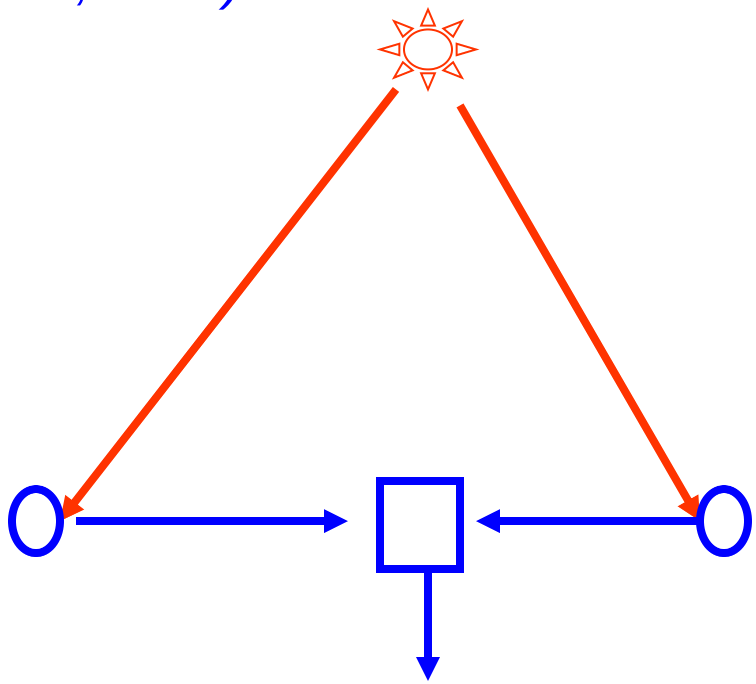
$$\Delta q \Delta p = 1/2\hbar$$

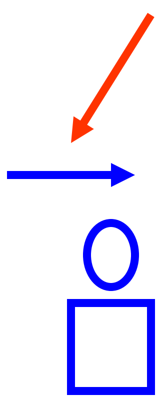
$$G^{(n)} = \left[G^{(1)}\right]^n,$$

Stellar Interferometers: ***field*** *(Michelson,1925)*

and ***intensity (****Hanbury-Brown &Twiss, 1956)*







optical field

electric signal

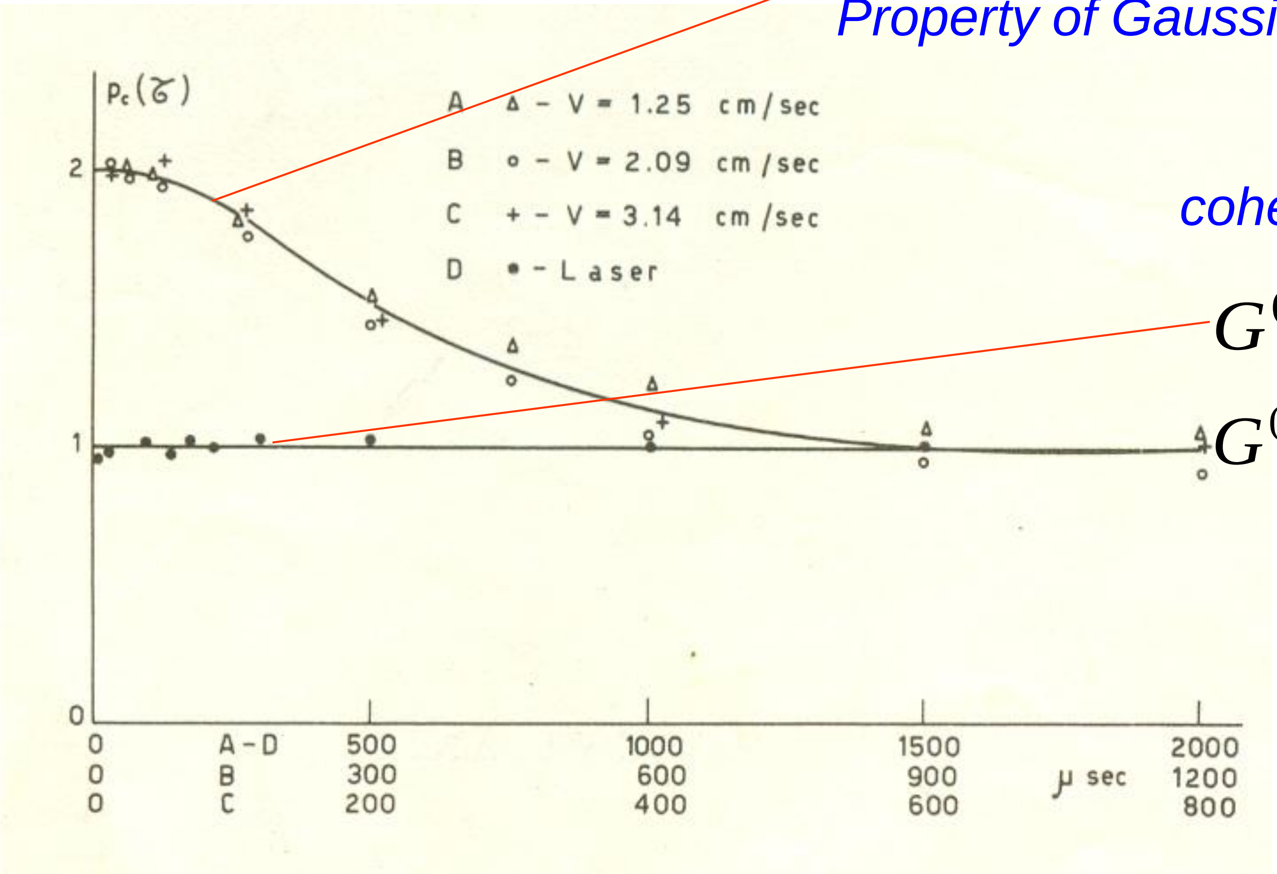
Photon detector

Electronic correlator

Hanbury-Brown & Twiss: intensity correlation

$$G^{(2)}(1,2) = <|E_1|^2 |E_2|^2> = I_1 I_2 + |G^{(1)}|^2$$

Property of Gaussian processes



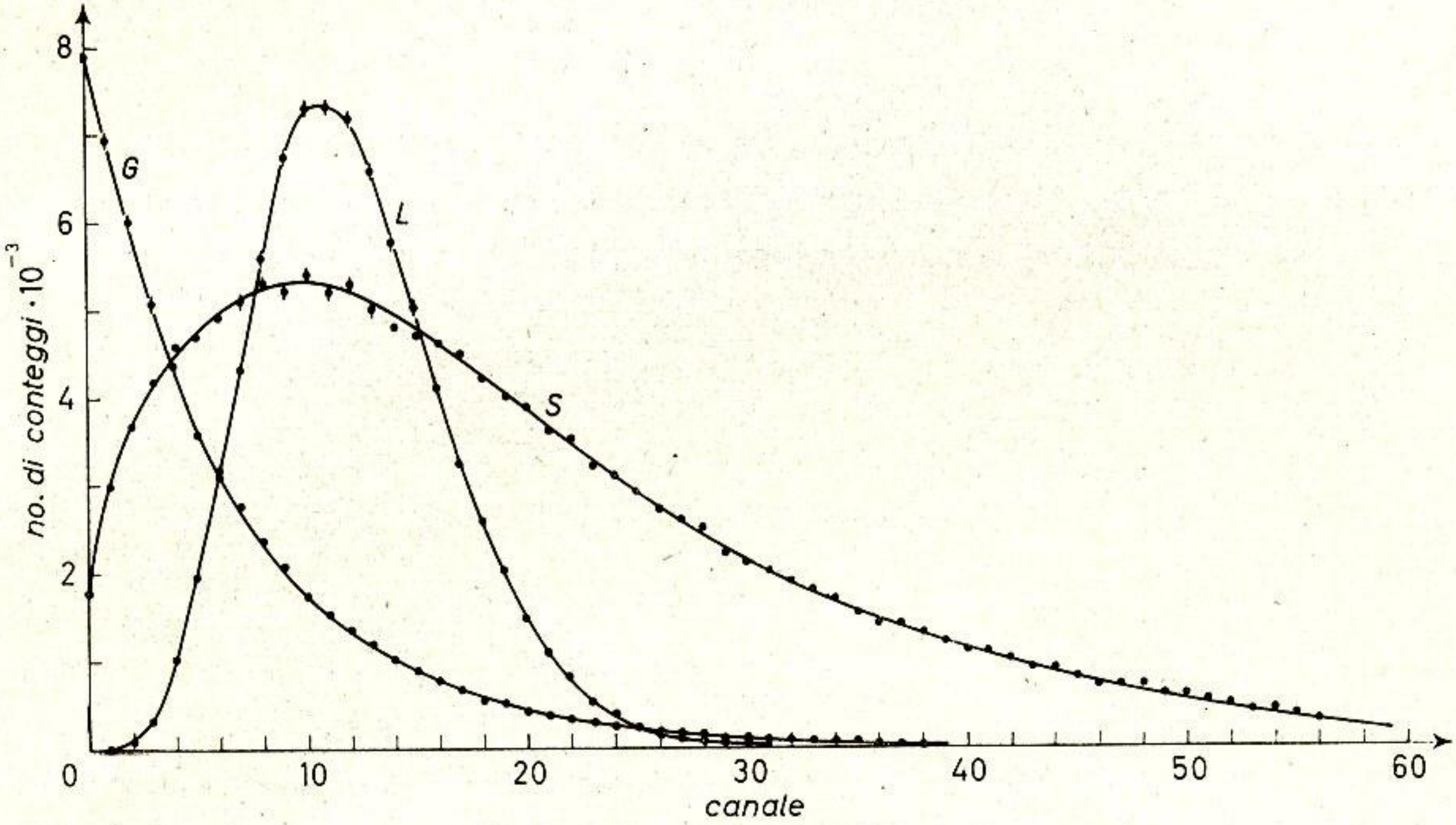
coherent state

$$G^{(2)} = \left[G^{(1)}\right]^2,$$

$$G^{(n)} = \left[G^{(1)}\right]^n,$$

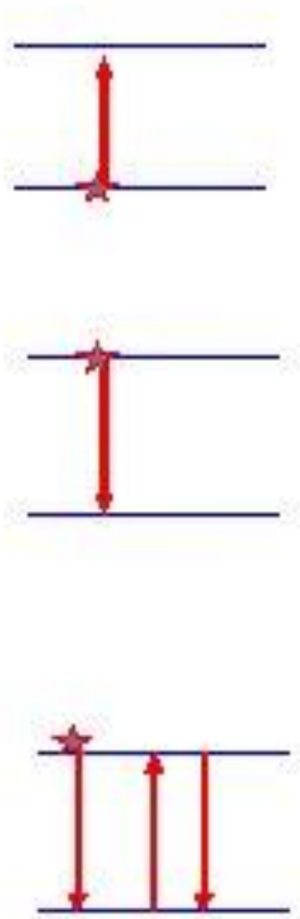
(** Arecchi,Gatti,Sona-1966**)

Statistics of photocounts(Arecchi, 1965)



L= laser; G= Gaussian light; S= superposition of L and G

P *=polarization; energy* $W = -P \cdot E$



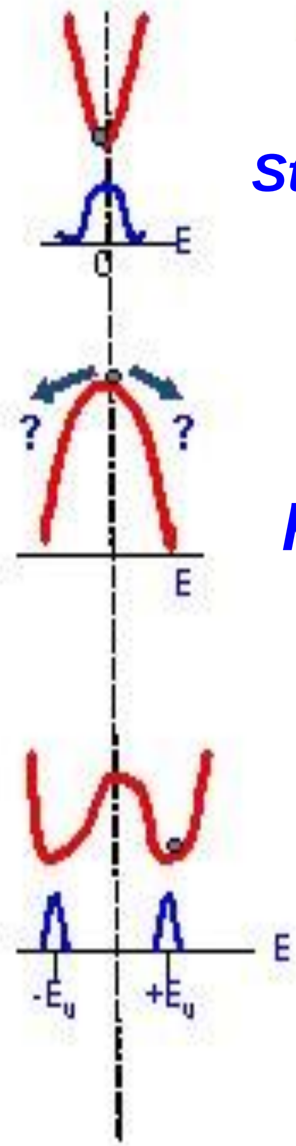
$P = -\alpha E$

$P = \alpha E$

$P = \alpha E - \beta E^3$

W(E)

p(E)



Stationary field probability

p(E)=exp(-W)

Energy curves

W(E) [red]

and

field probabilities

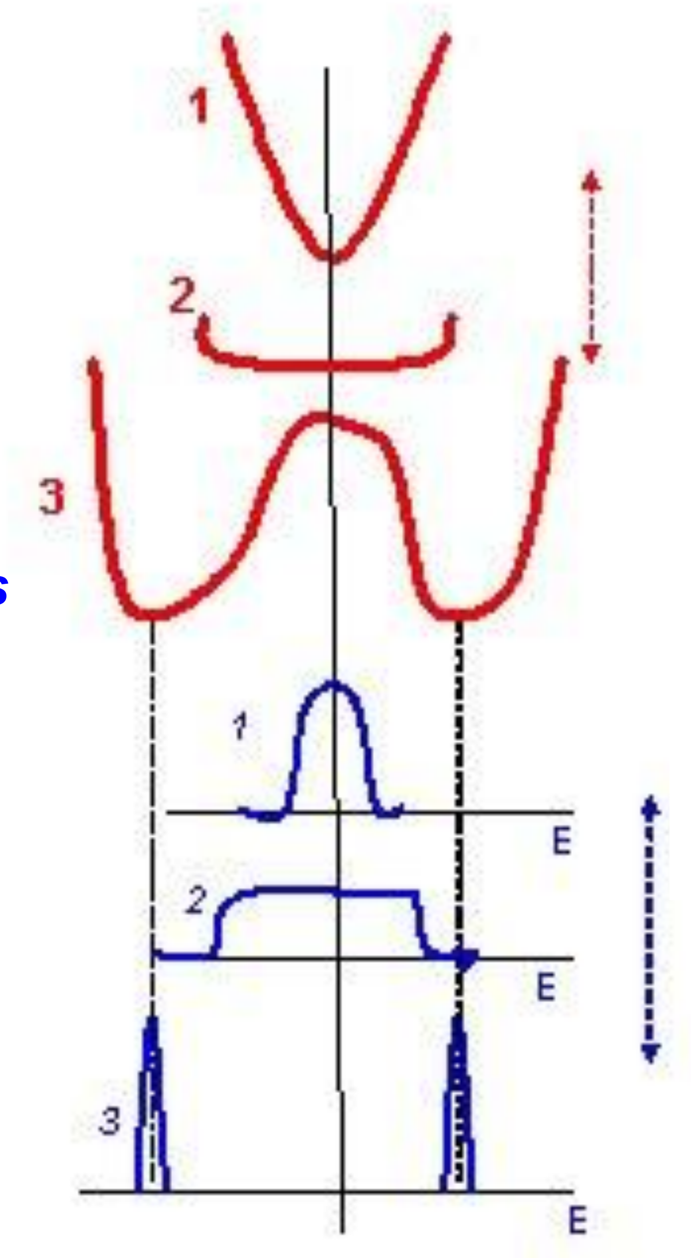
p(E) [blue]

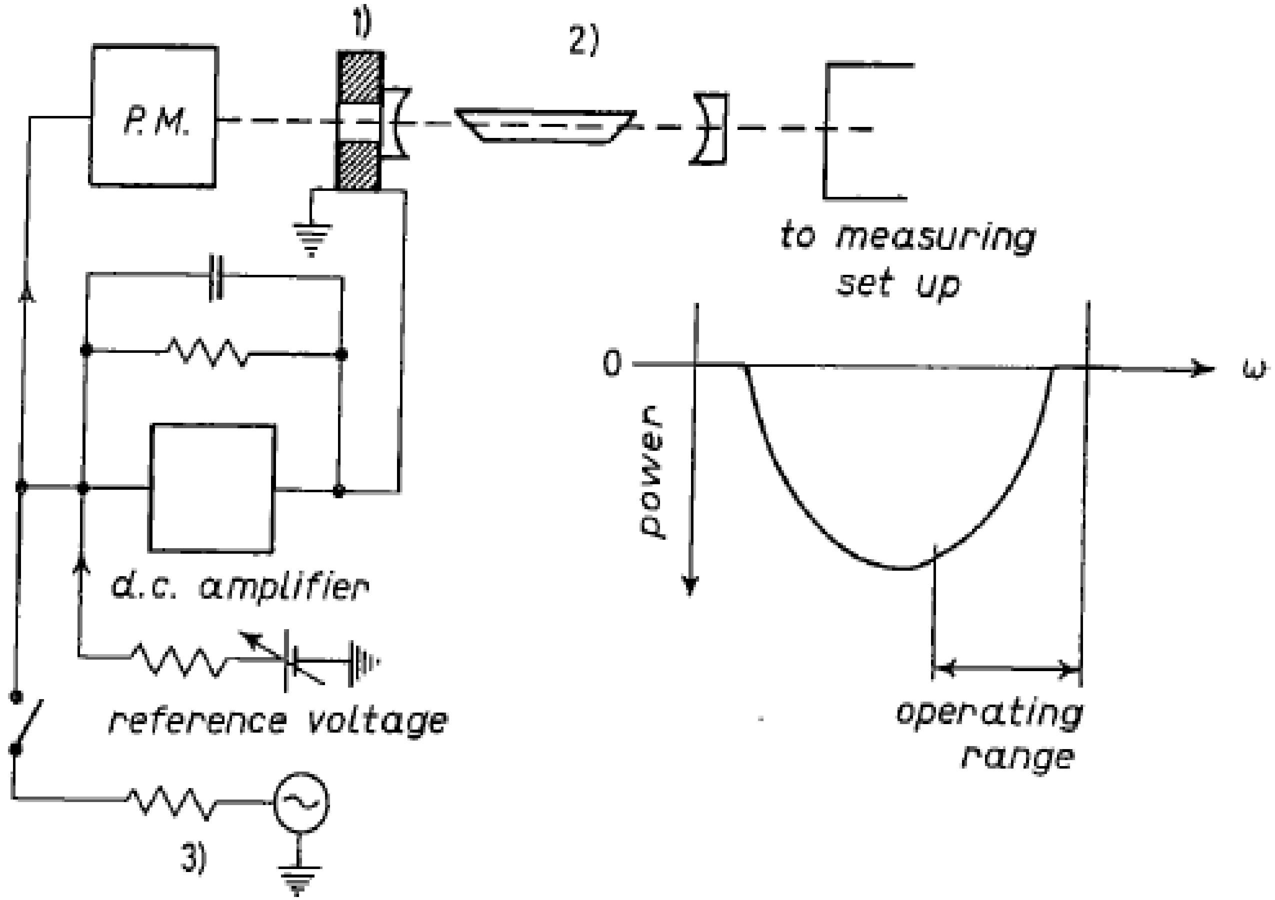
1-below

2-at

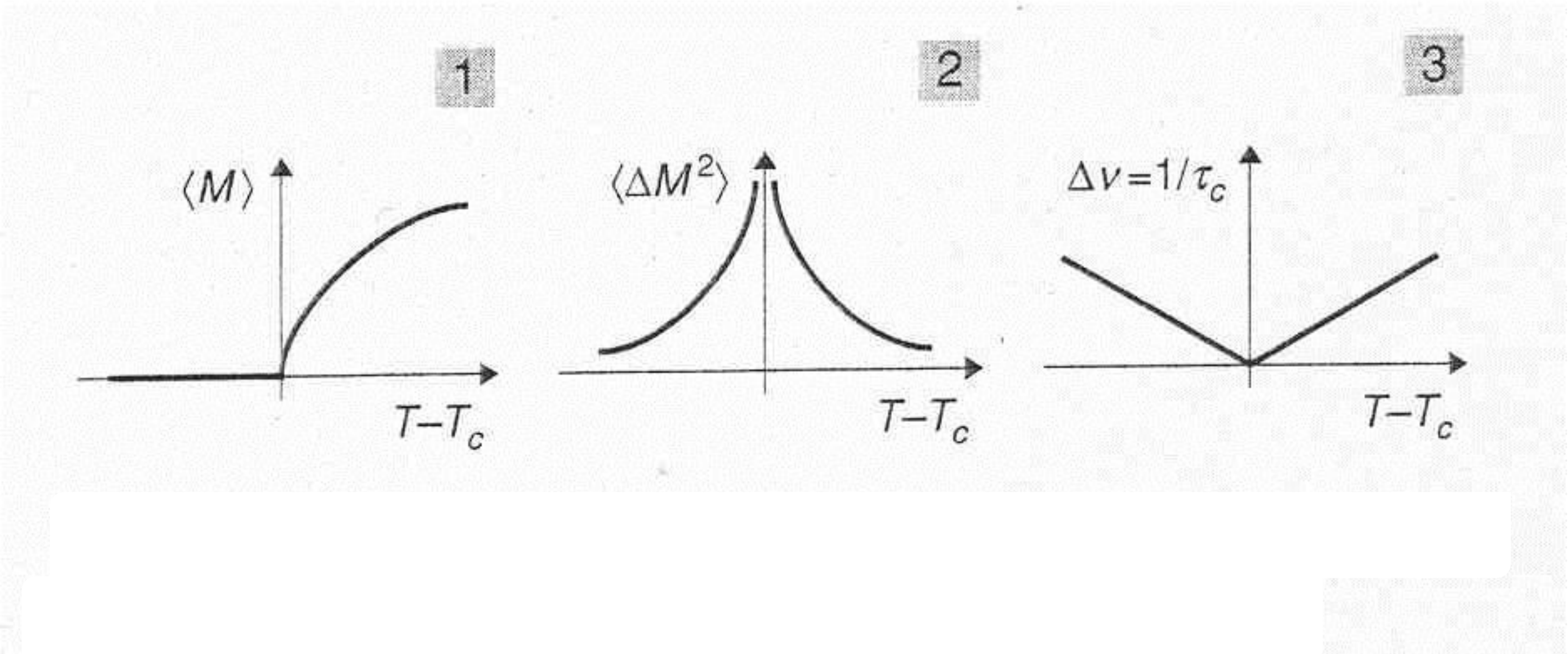
3-above

threshold





– Experimental set-up used for performing measurements of Gaussian noise superposed on a laser beam around threshold. 1) Piozoceramic disc; 2) laser length 20 cm, inner diameter 1 mm; 3) optional a.c. drive to control the operating range.

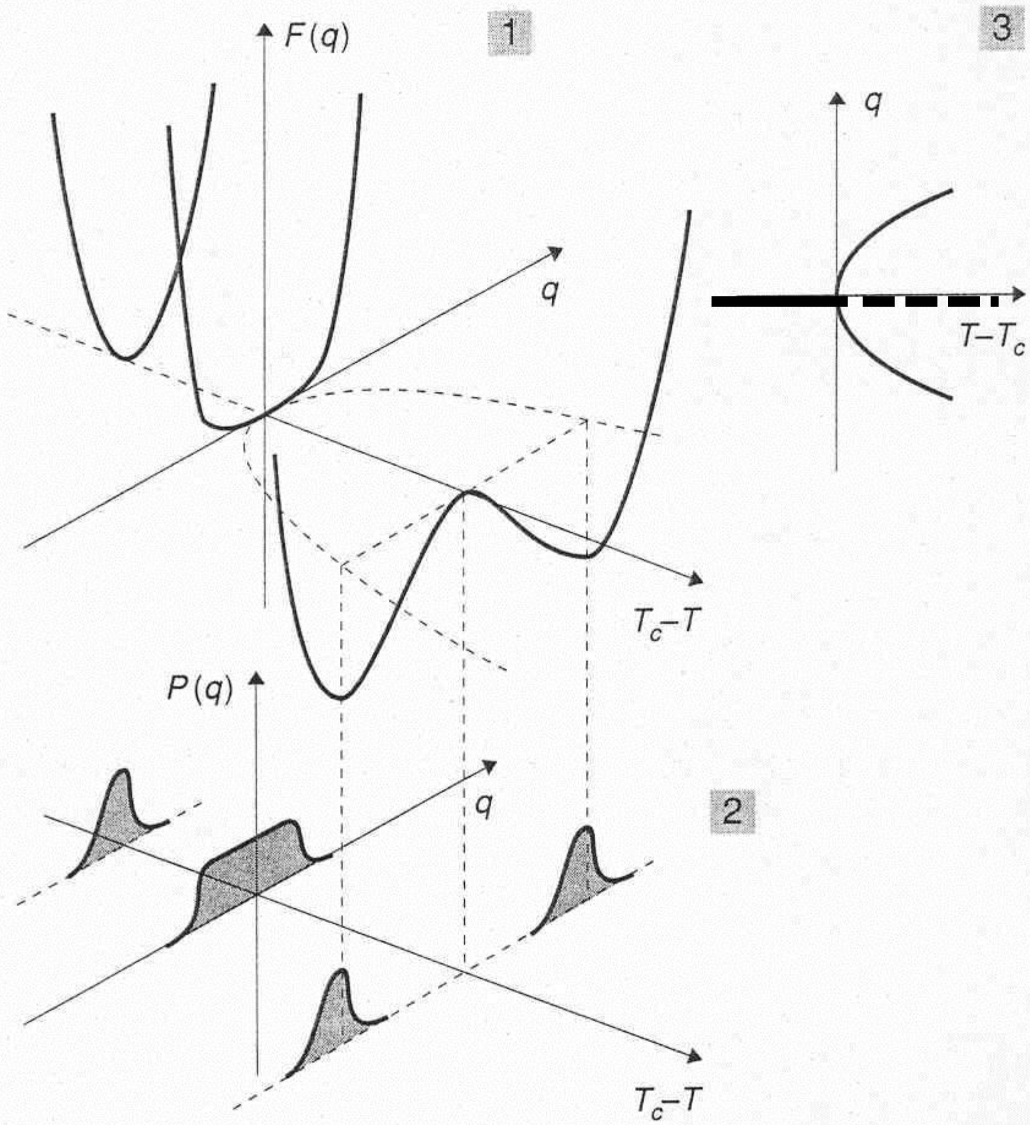


2nd order phase transition in magnetic material :

<M>=order parameter (total magnetization),

$\langle \Delta M^2 \rangle$ =fluctuations (diverge at Tc)

$\Delta\nu = 1/\tau_c$ =linewidth of fluctuations (zero at Tc =critical slow down)



Landau model

F = free energy,

q = order parameter,

T = temperature,

T_c= critical temperature.

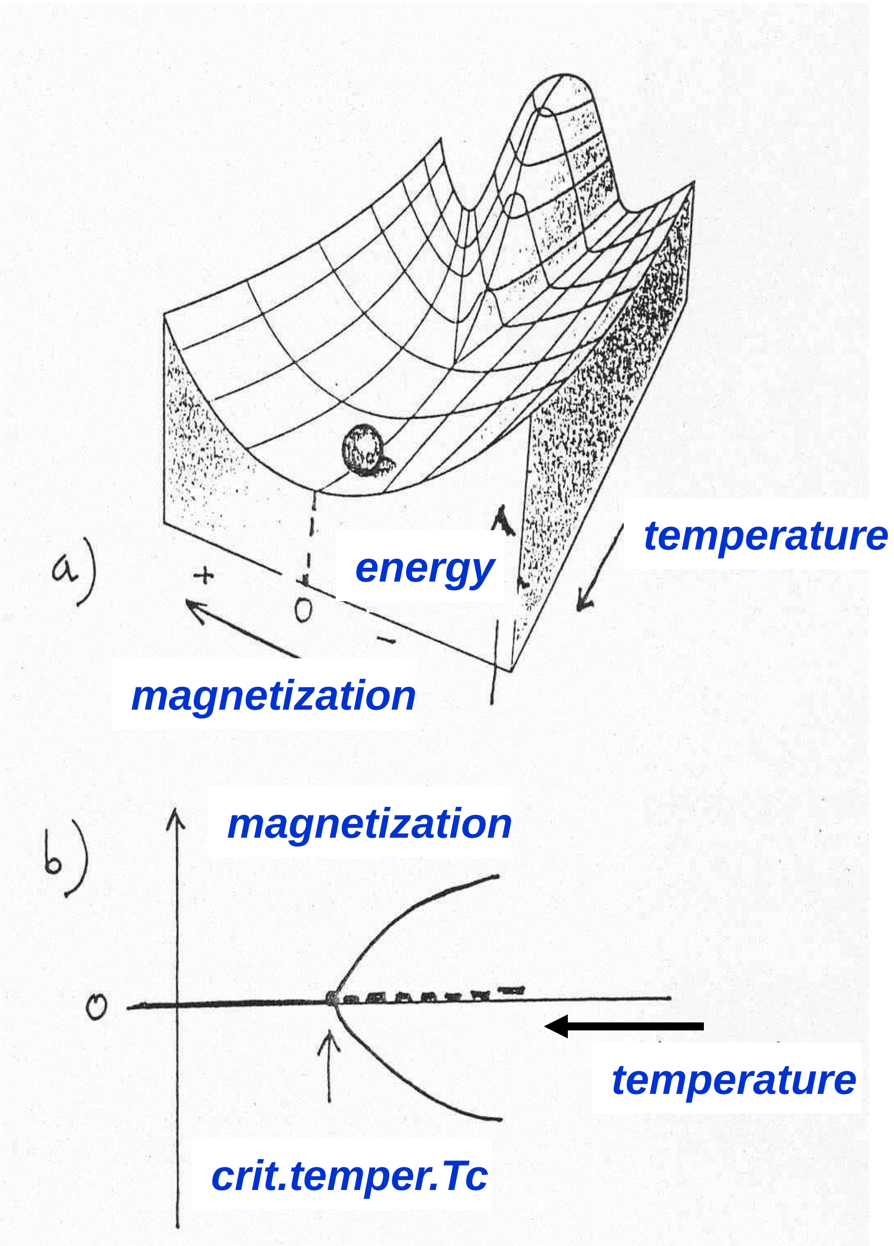
P(q)=equilibrium

probability distribution of q.

$$P(q) = N\exp\left[-F(q)/k_B T\right]$$

$$F = F_0(T) + \alpha q^2 + \beta q^4$$

$$\alpha = a(T - T_c)\ldots with\ldots a > 0$$

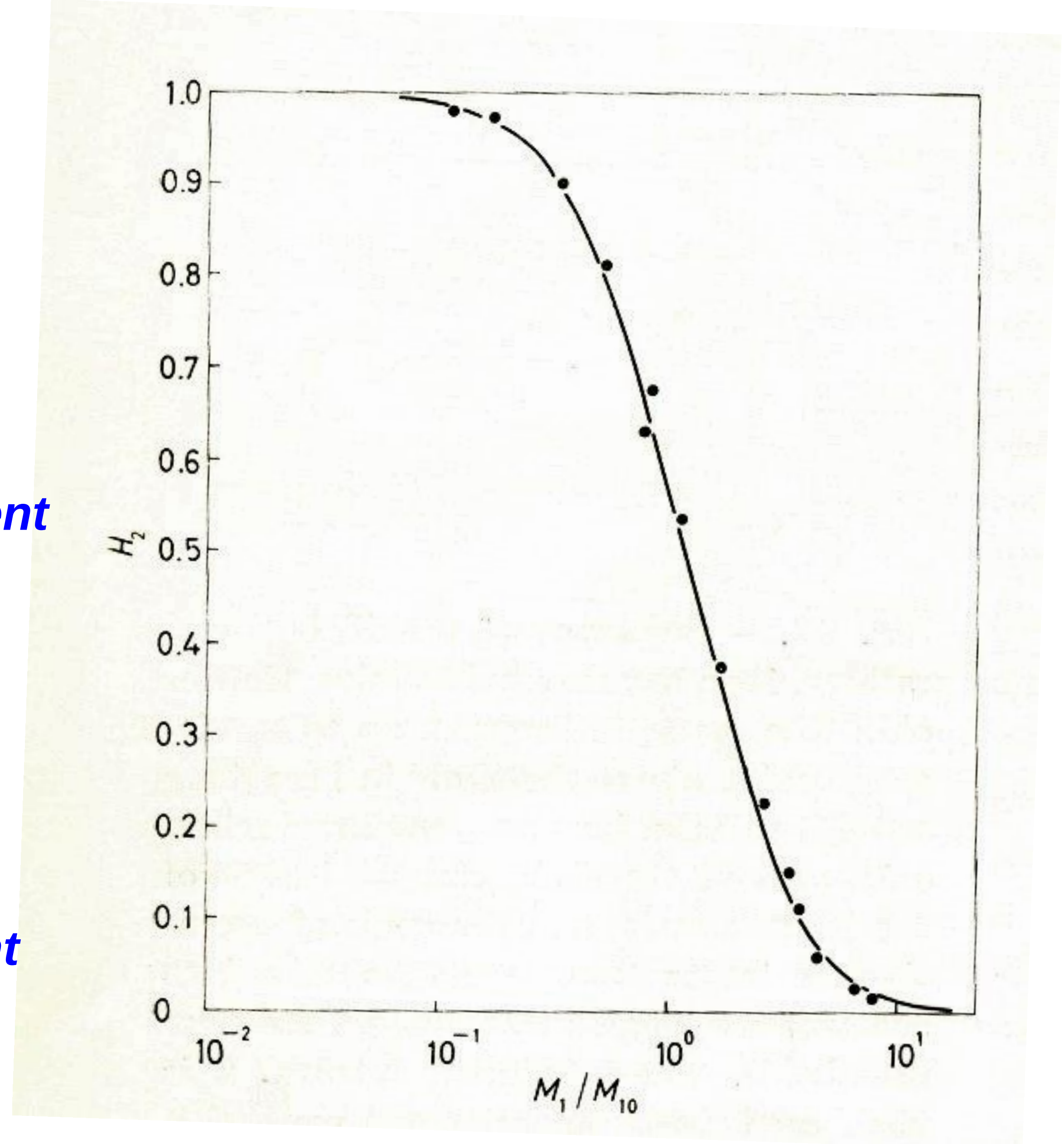

a)
+
0
−
energy
temperature
magnetization
b)
magnetization
0
temperature
crit.temper.Tc

H_2 is a combination of first & second moment of photon distribution versus the first moment M_1 normalized to its threshold value M_{10}

H_2=1 for Gaussian

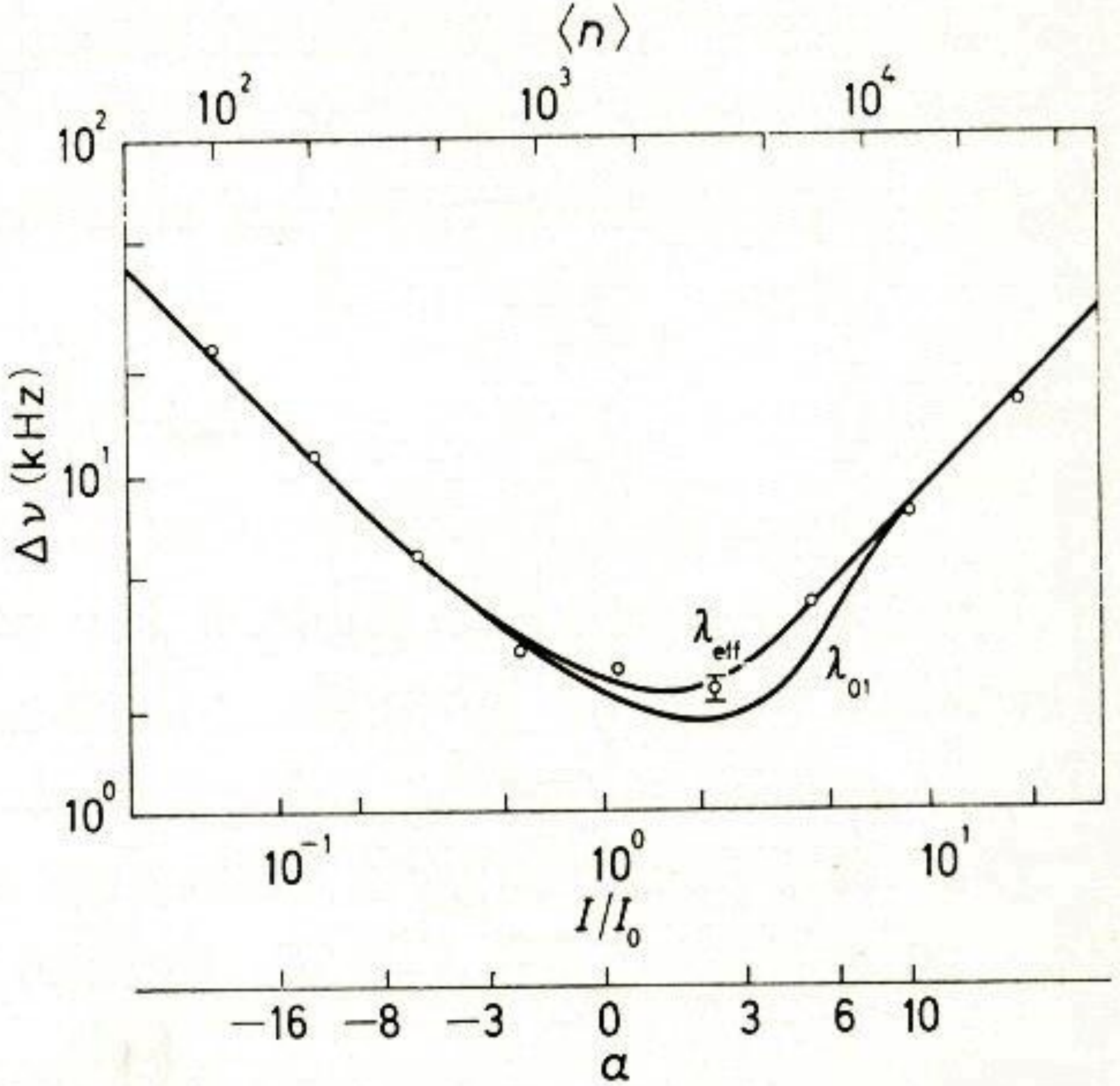
H_2=0 for coherent light

(Arecchi 1966)

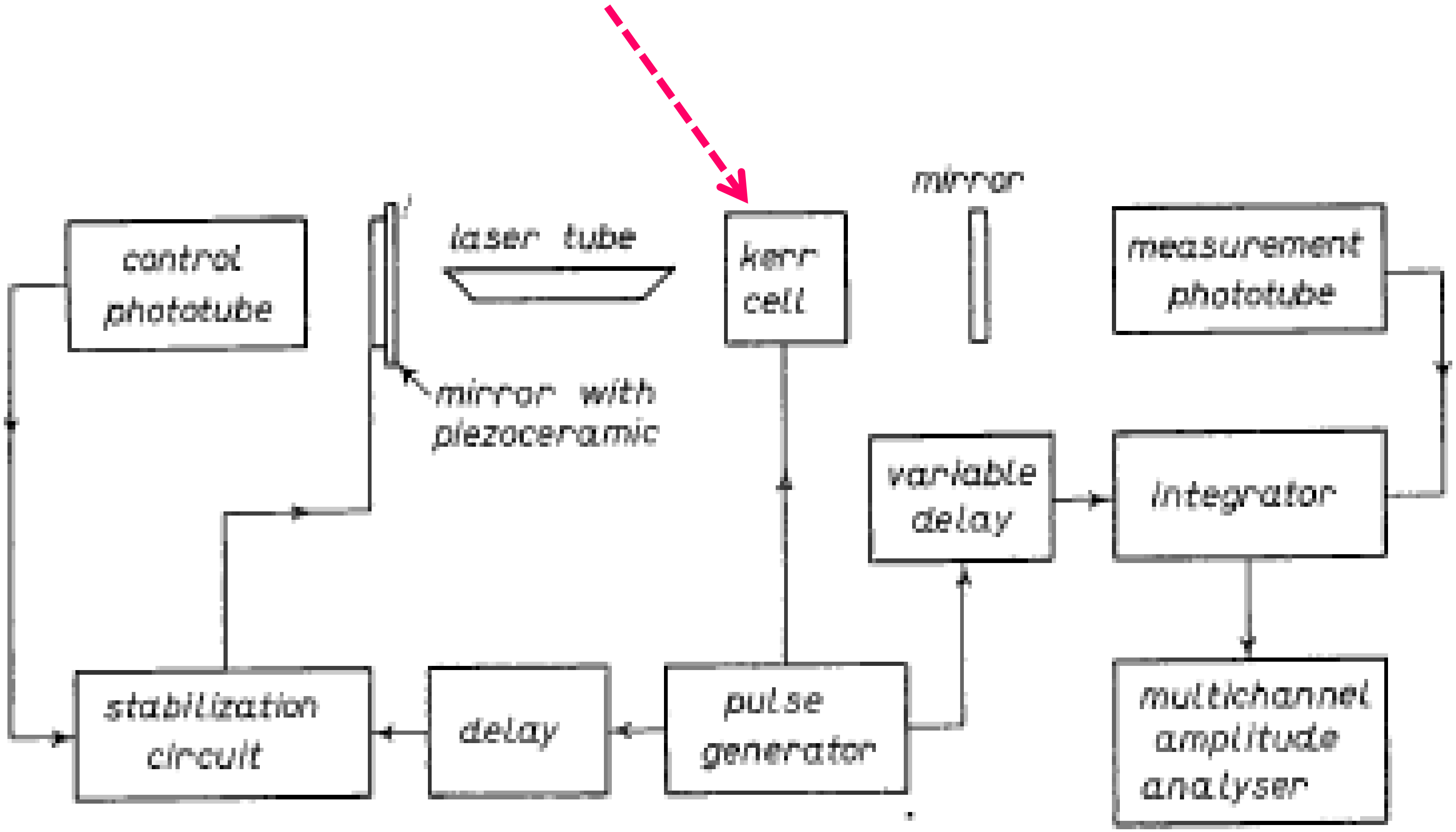


Frequency width of laser light fluctuations versus laser intensity normalized to threshold value

(Arecchi 1967)

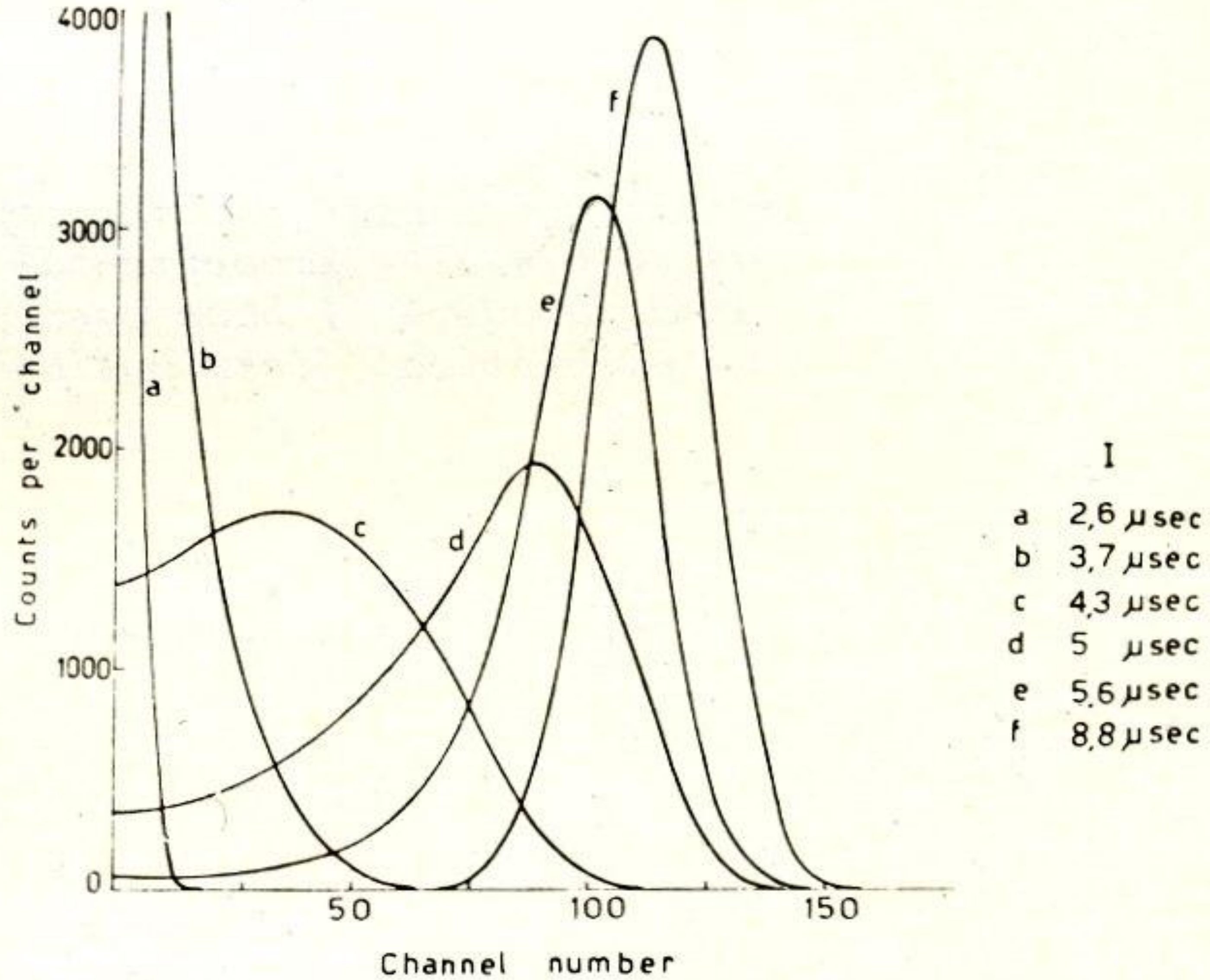


Laser transient statistics: high gain; losses switched from high to low via a Kerr shutter

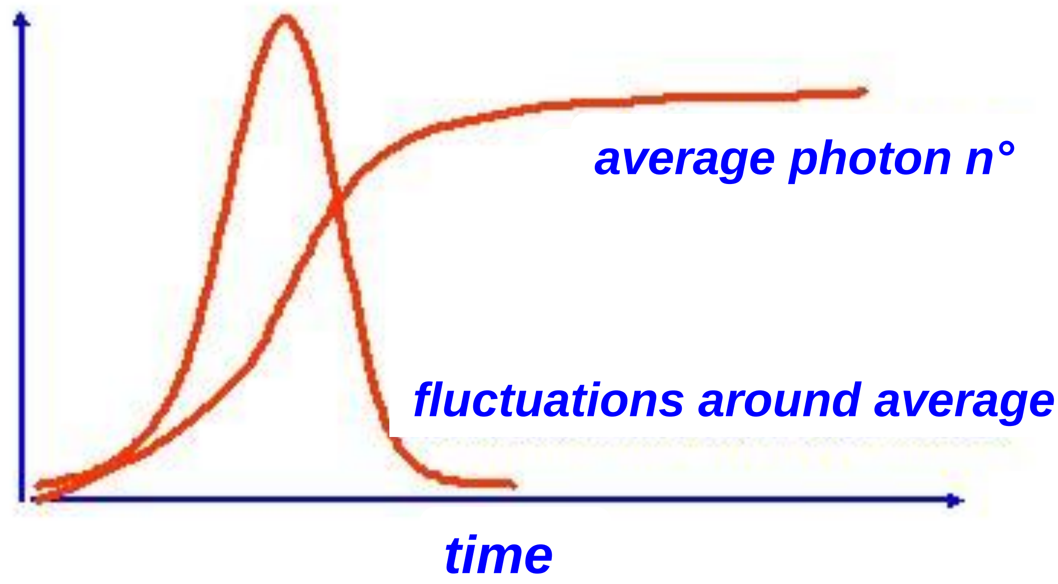


– Experimental set-up for the transient experiment.

TRANSIENT STATISTICS: **Photon statistics sampled over 50 nanosec windows at different times after t=0 (sudden onset of mirrors)**



Transient photon statistics



(Arecchi et al.1967)

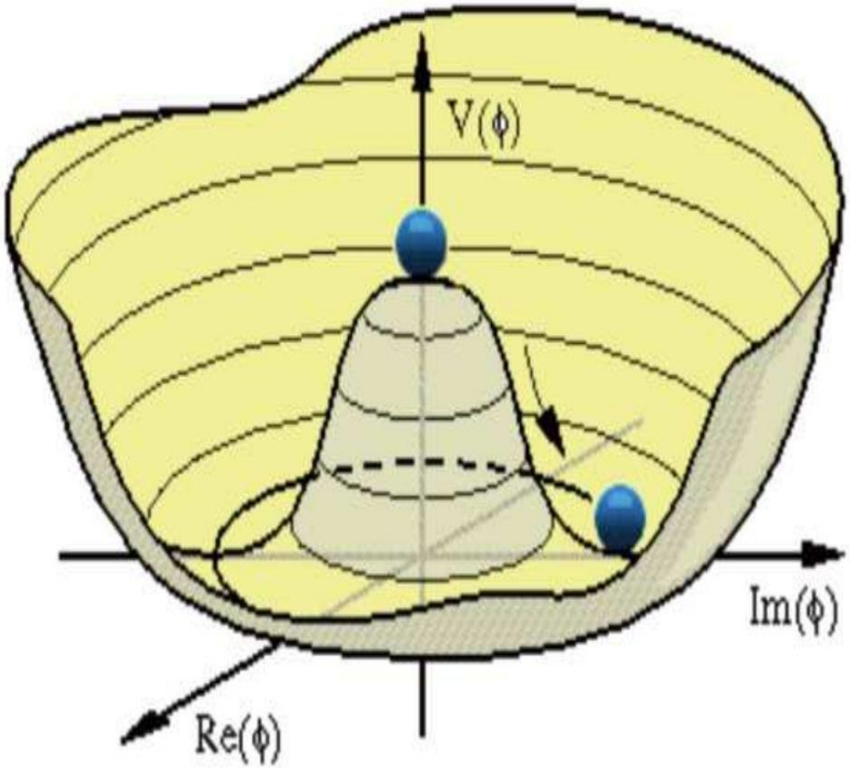
Higgs boson as laser with symmetry breaking in phase

Lowering temperature, parabola inverts and ball falls, but it is confined in a potential "mexican hat"; no tangential constrains = massless Goldstone boson

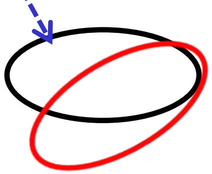
As a laser above threshold, it has definite intensity (radius of circle) but phase undefined (angular position Φ along circumference);

However from outside we can impose phase Φ_0 = minimum energy= MASS

.



Re Φ=x , Im Φ=y



Fields arising by symmetry breaking after Big-bang are massless ; in the Higgs field , tilting the circumference provides a mass (different from one particle to the other , as red and green) .