Phase Control of Chaos

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Outline

1. Introduction.
2. Phase control of chaos.
3. Phase control of escapes.
4. Phase control of a neuron model.
5. Conclusions.

1.1 Introduction: The phase control scheme

In absence of forcing: periodic dynamics

$$\dot{\vec{x}} = \vec{f}(\vec{x})$$

$$+F\cos(\omega t)$$

Chaotic dynamics

$$+\varepsilon\cos(r\omega t+\boxed{\varphi})$$

We tune the phase φ in search of the desired behavior of the system, for $\varepsilon \ll 1$ and different values of r.

Scheme proposed in Qu *et al*. PRL **74** 1736 (1995)

1.2. Applications.

We show that this scheme can be used:

- To tame chaotic behavior.

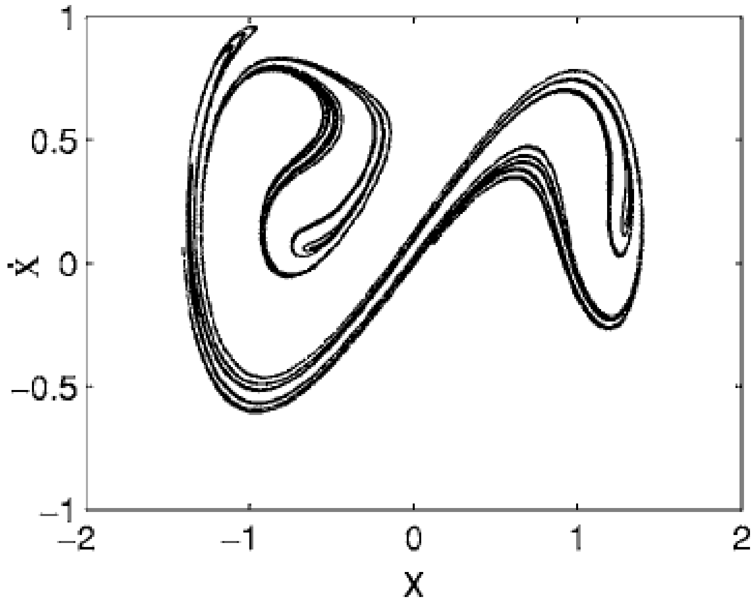
- To avoid escapes in a system with chaos and divergences to infinity (boundary crisis).

- To control the dynamics in a Neuron Model.

2.1. Phase control of chaos

We use the paradigmatic Duffing oscillator:

$$\ddot{x} + \delta\dot{x} - x + x^3 = F\cos(t)$$



Lyapunov exponent

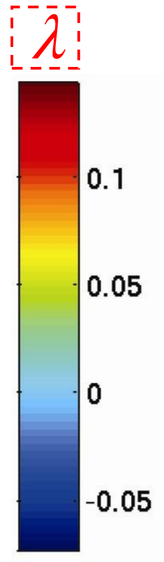
$$\lambda \approx 0.14$$

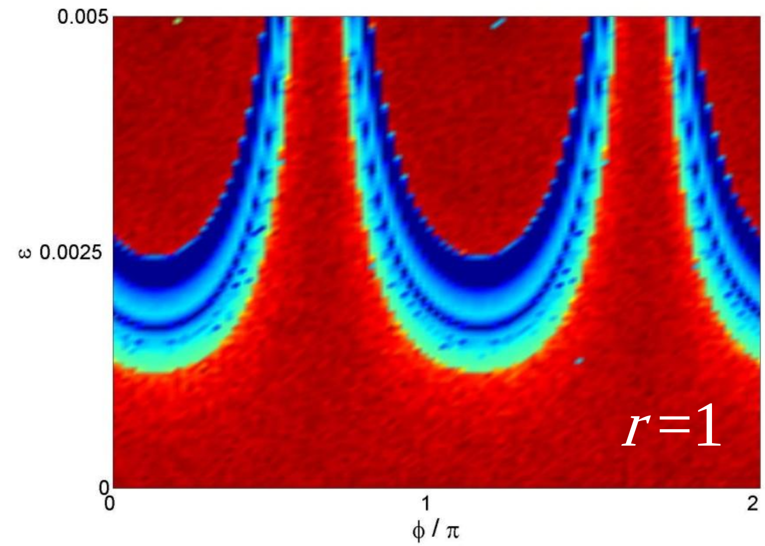
Goal: suppress the chaotic behavior.

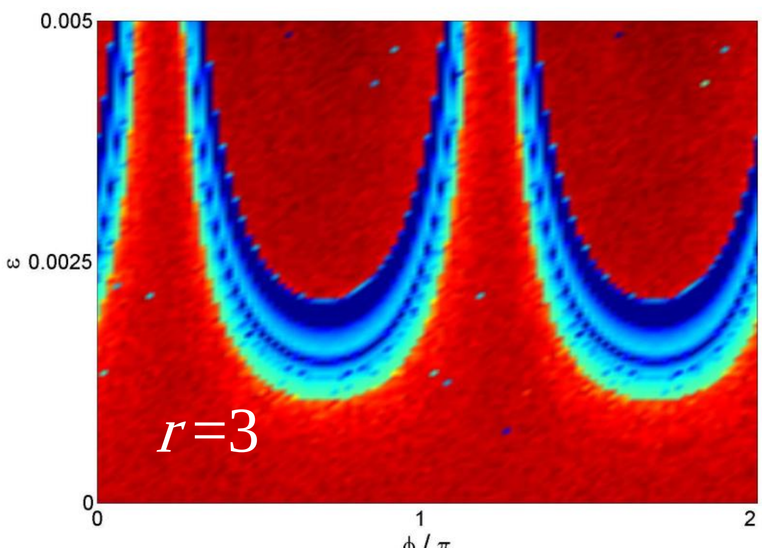
$$\ddot{x} + \delta\dot{x} - x + \boxed{(1 + \epsilon\cos(rt + \phi))}x^3 = F\cos(t)$$

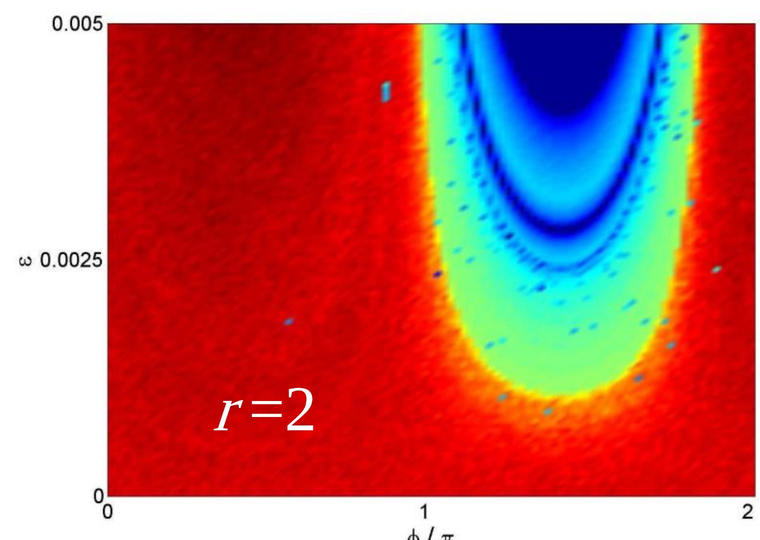
2.2. Numerical results: λ as a function of ε and φ

We compute the Lyapunov exponent λ for different values of ε and φ, fixing r



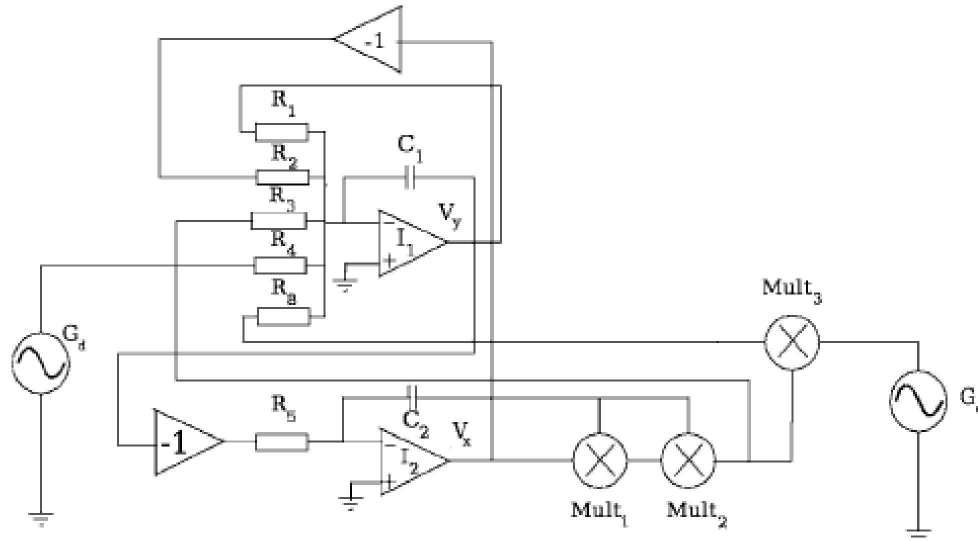




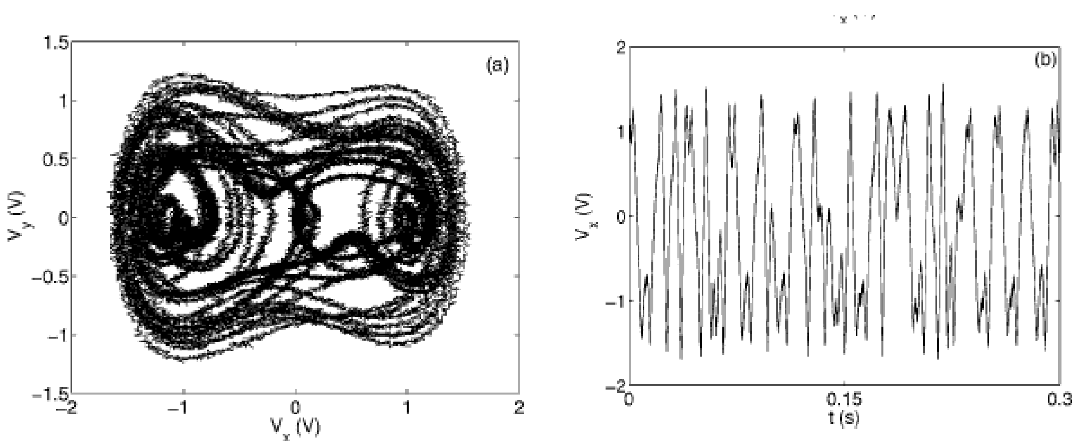


2.3. Experimental implementation on a circuit

Circuit that mimics the Duffing oscillator

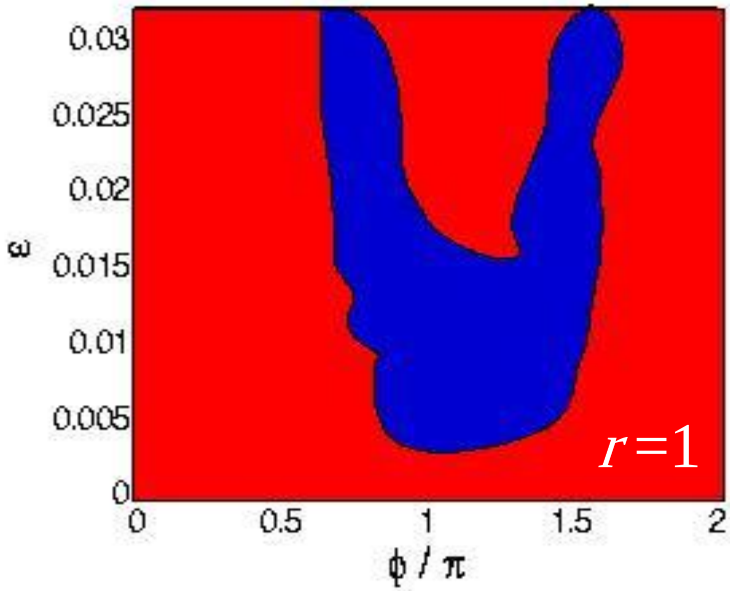


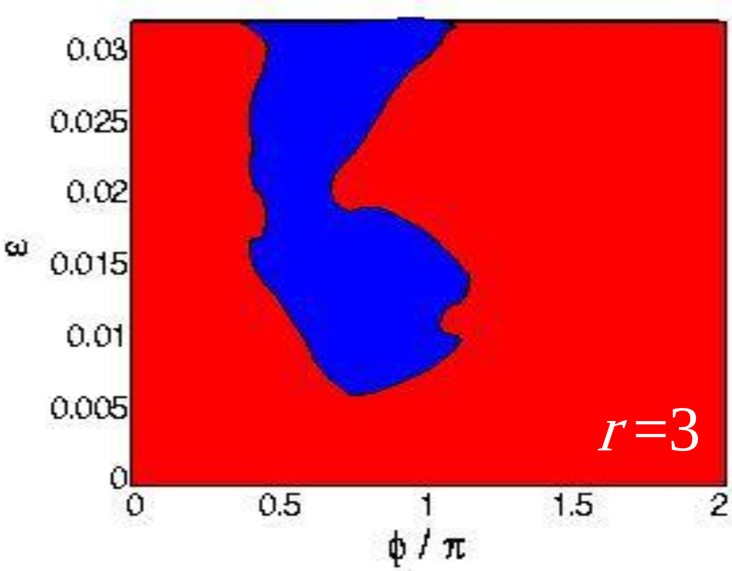
Behavior

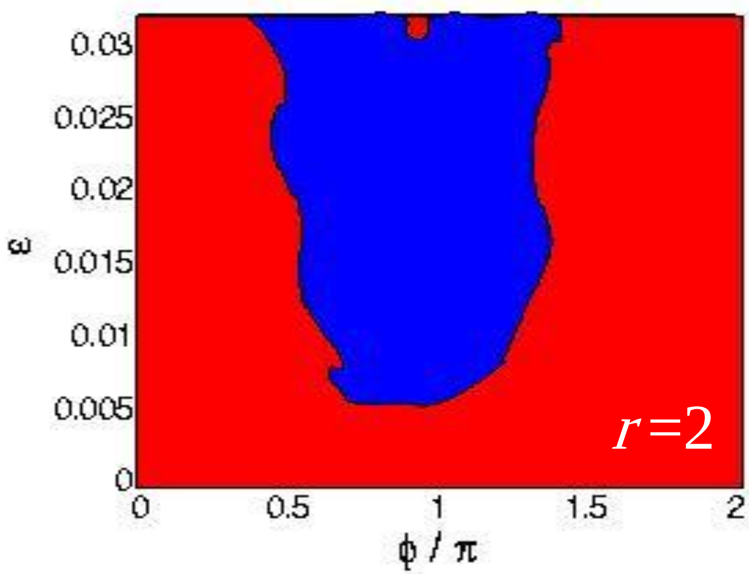


2.4. Experimental results.

These diagrams show the zones of Chaos ( ) and Periodic behavior ( ) for different values of ε and φ, fixing r



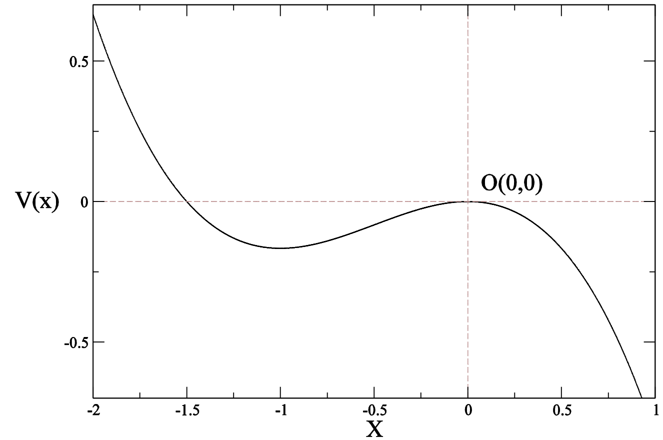




3.1 Escapes in a Helmhotz oscillator (I)

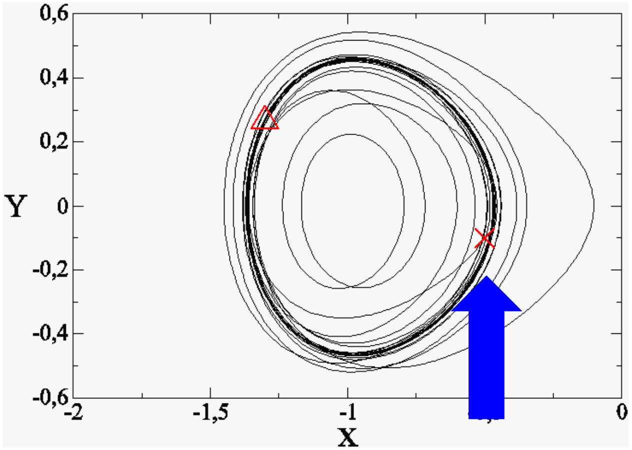
Cubic potential

$$V(x) = -\frac{x^2}{2} - \frac{x^3}{3}$$

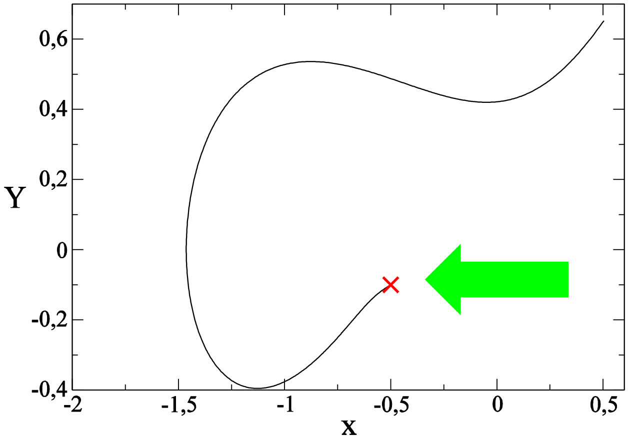


$$\ddot{x} + 0.1\dot{x} - x - x^2 = F\cos t.$$

$F = 0.12$



$F = F_c = 0.21$



3.2 Implementation of the phase control scheme

- Aim: avoiding escapes for *F=0,21*.

- Implementation of the phase control method:

$$\boxed{\ddot{x} + 0.1\dot{x} - x - (1 + \epsilon\cos(t + \phi))x^2 = 0.21cost}$$

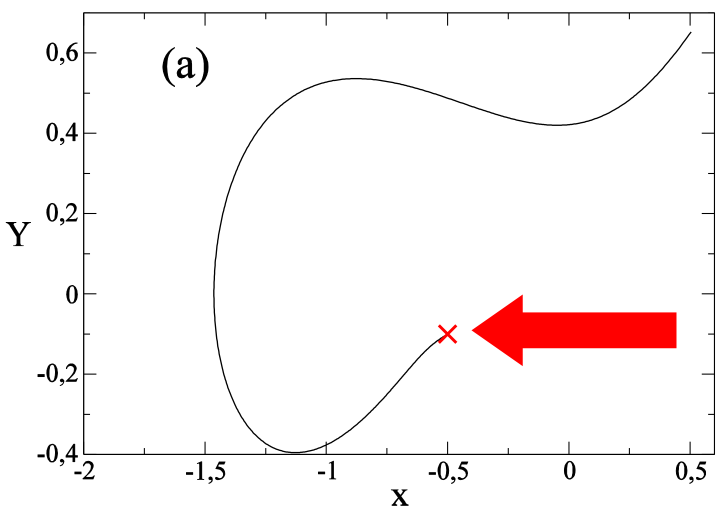
Perturbation amplitude (fixed)

Phase

3.3 Numerical evidence of control of escapes (I)

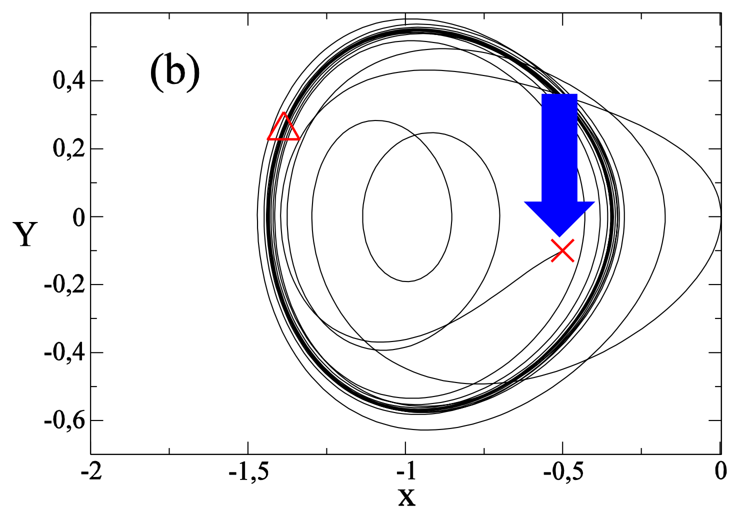
$$\boxed{F = F_c = 0.21}$$

Diverging trajectory



$$\boxed{\varepsilon = 0}$$

Controlled trajectory



$\varepsilon = 0.05$ and $\phi = \pi$

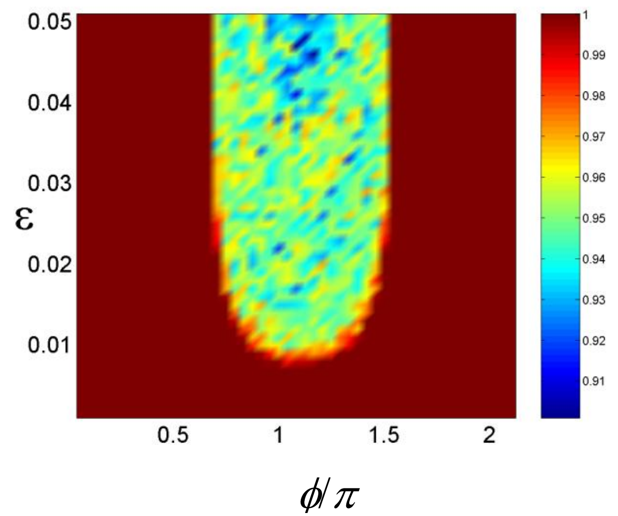
3.3 Numerical evidence of control of escapes (II)

$$F = F_c = 0.21$$

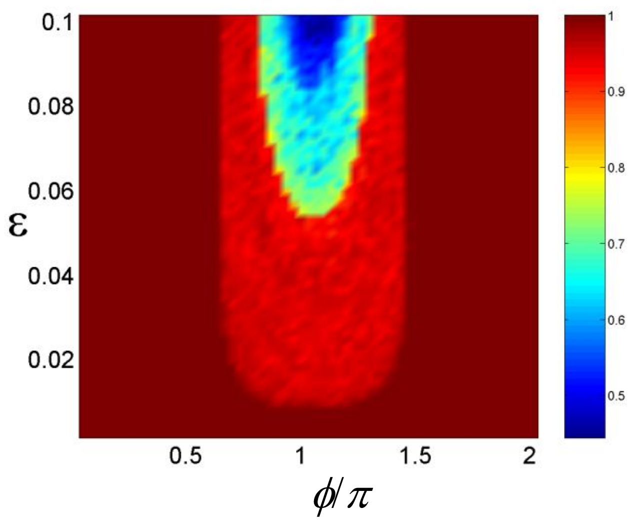
$$\phi_{optimo} = \pi$$

Escape rates for ε and φ

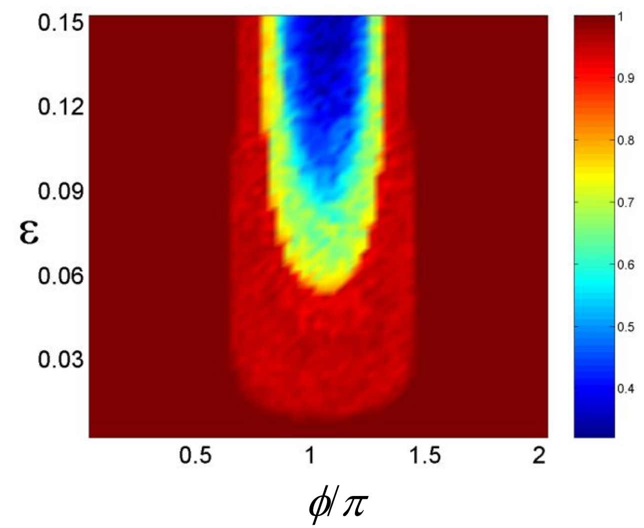
$$\varepsilon = 0.05$$



$$\varepsilon = 0.1$$



$$\varepsilon = 0.15$$



3.4 Experimental implementation (I)

$$\ddot{x} + 0.1\dot{x} - x - (1 + \epsilon\cos(t + \Phi))x^2 = 0.21\cos t$$

$x \propto V_x$

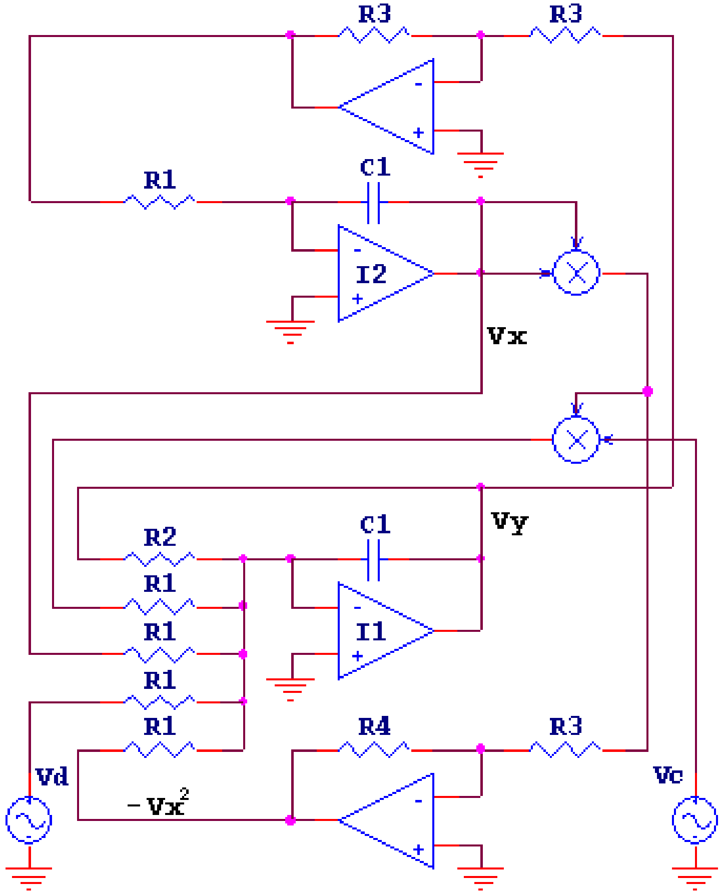
$\dot{x} \propto V_y$

$R_1 = 100\,K\Omega, R_2 = 1\,M\Omega, R_3 = 2\,K\Omega$

$R_4 = 5\,K\Omega \;\; y \;\; C_1 = 10\,nF$

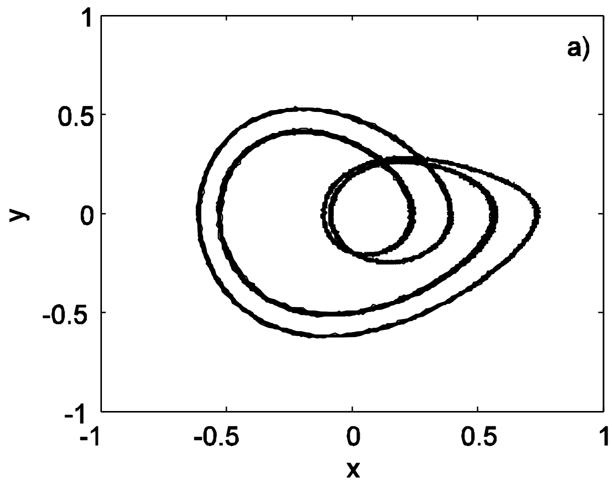
$V_d \approx 200\,mV \;,\; V_c \approx 30\;mV$

$\varepsilon \approx 0.03$

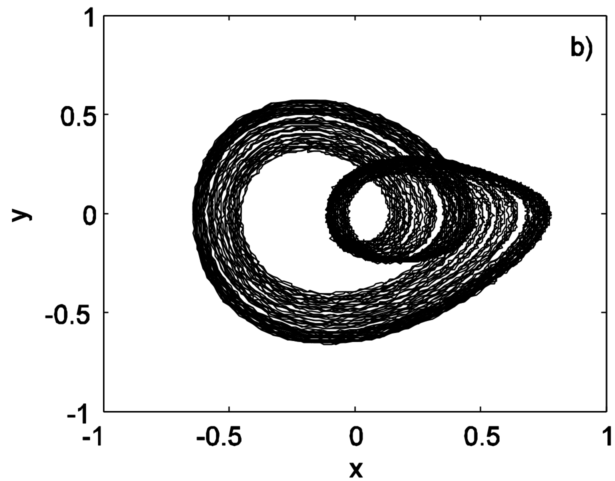


3.4 Experimental implementation (II)

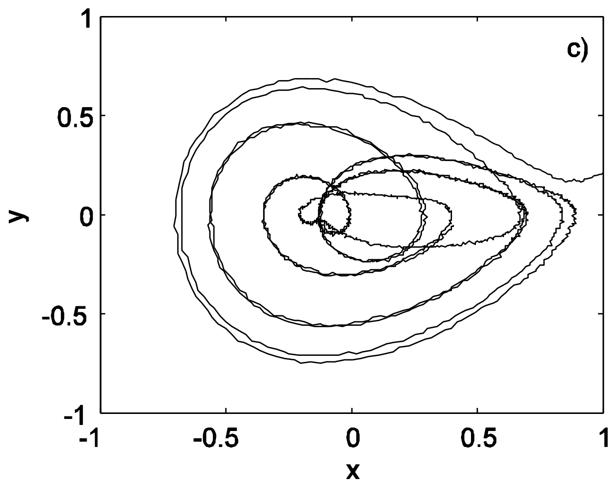
Periodic orbit



Chaotic orbit



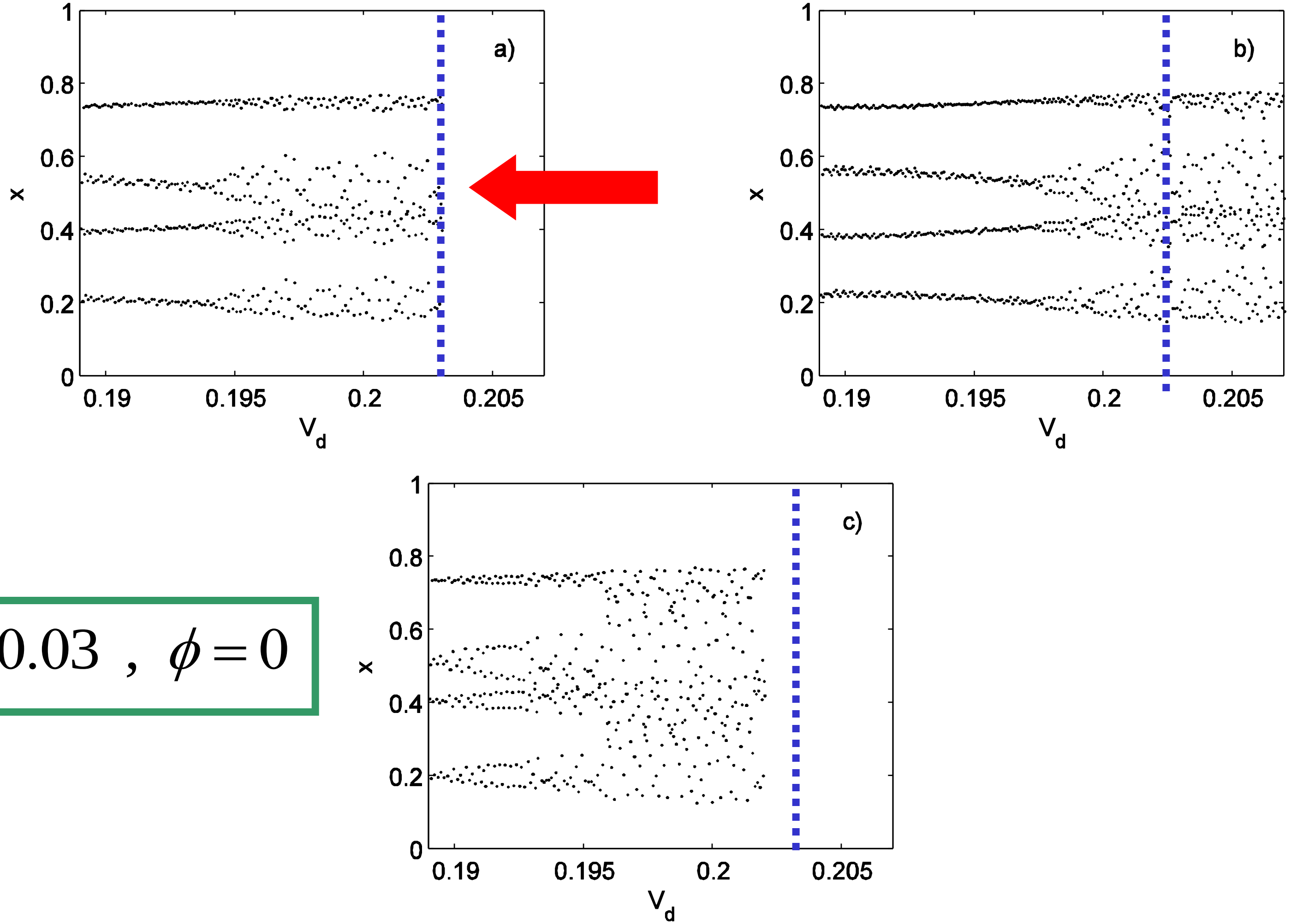
Diverging orbit



3.5 Experimental phase control of escapes

NO CONTROL

$\varepsilon = 0.03 \;,\; \phi = \pi$



$\varepsilon = 0.03 \;,\; \phi = 0$

4.1 A neuron model

$$\boxed{\begin{aligned}\frac{du_1}{dt} &= c(-v_1 + u_1 - (u_1^3/3) + Imp) \\ \frac{dv_1}{dt} &= u_1 - bv_1 + a,\end{aligned}}$$

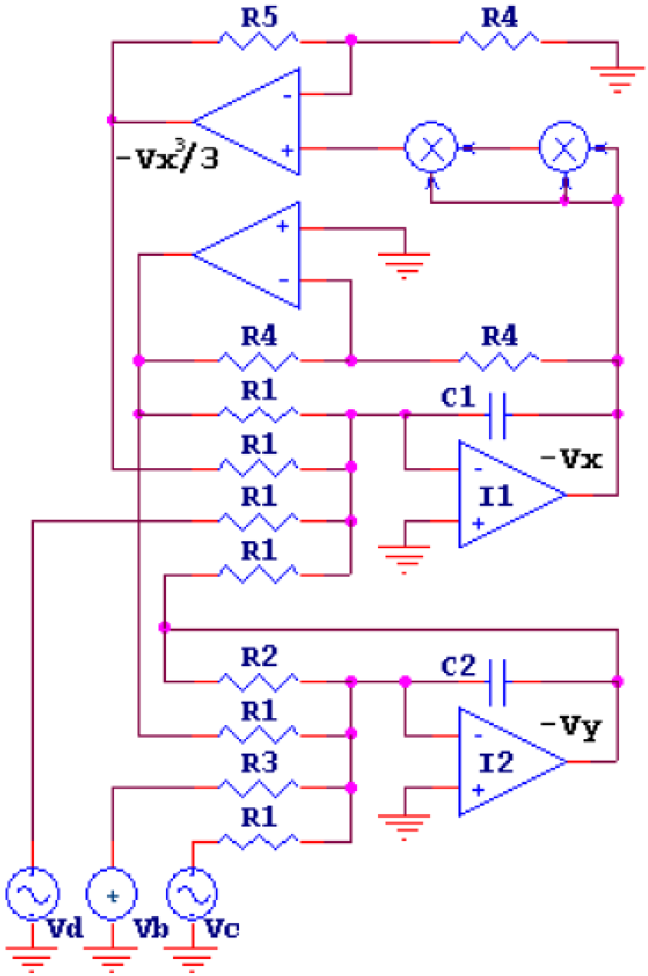
FitzHug-Nagumo

$$\boxed{Imp = A(1+\delta)\sin(\tfrac{2\pi}{T}t)}$$

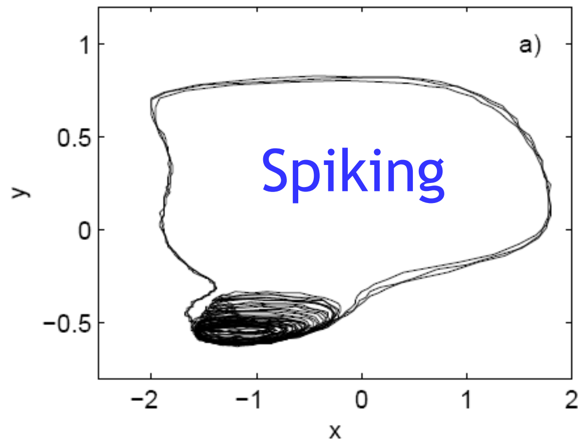
$$\boxed{\delta = \epsilon\sin(\tfrac{2\pi r}{T}t + \Phi)}$$

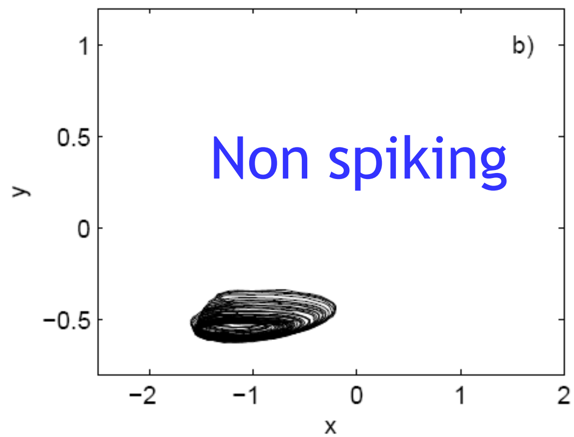
Phase control
(modulation amplitude)

4.2 Experimental implementation



Two typical regimes:

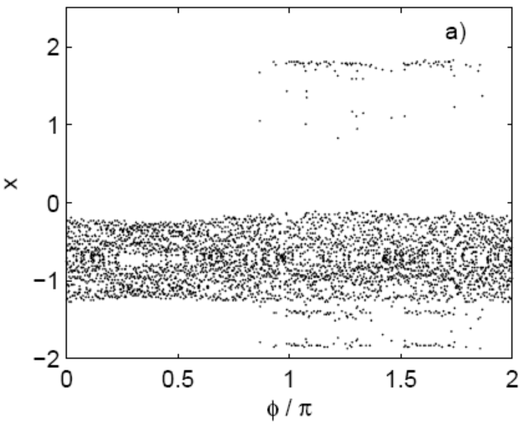


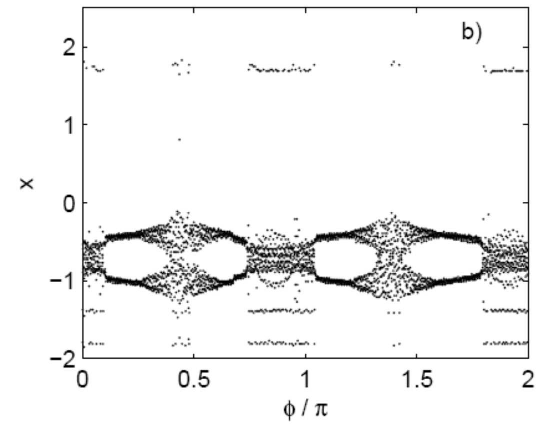


4.3 Experimental results

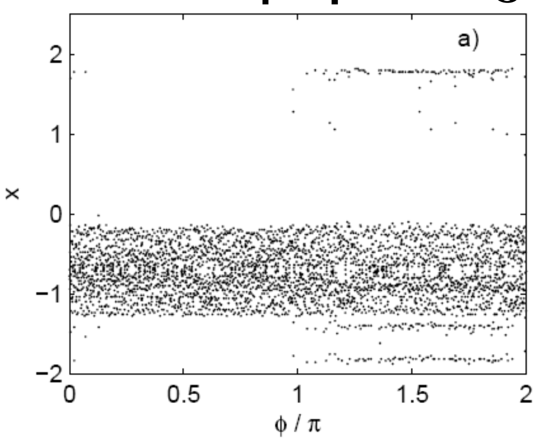
r=1 ε=0.02 r=1/2

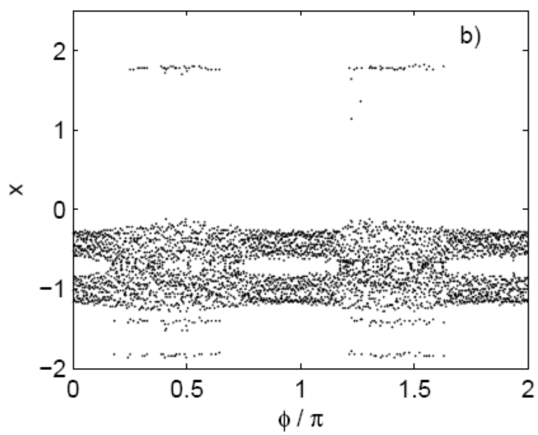
Perturbing the spiking regime





r=1 ε=0.02 r=1/2



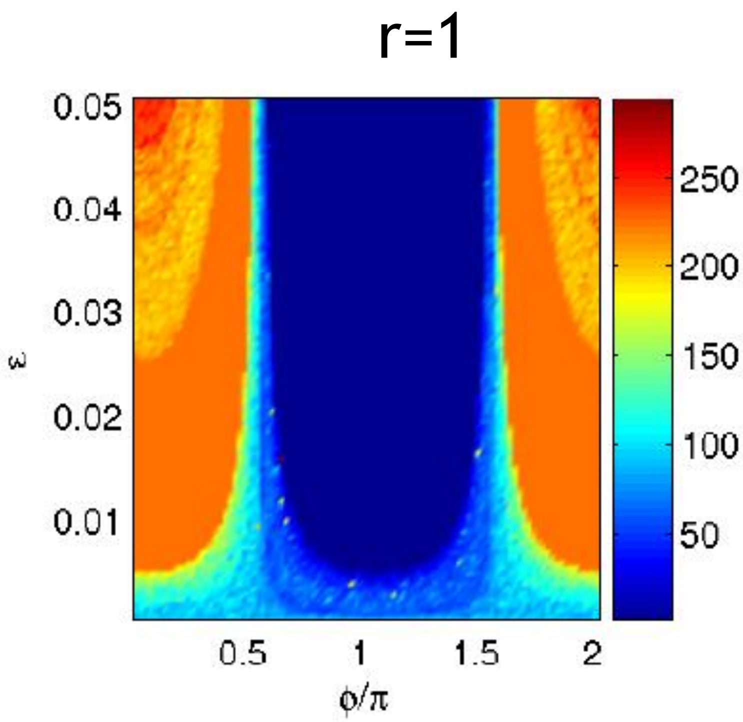


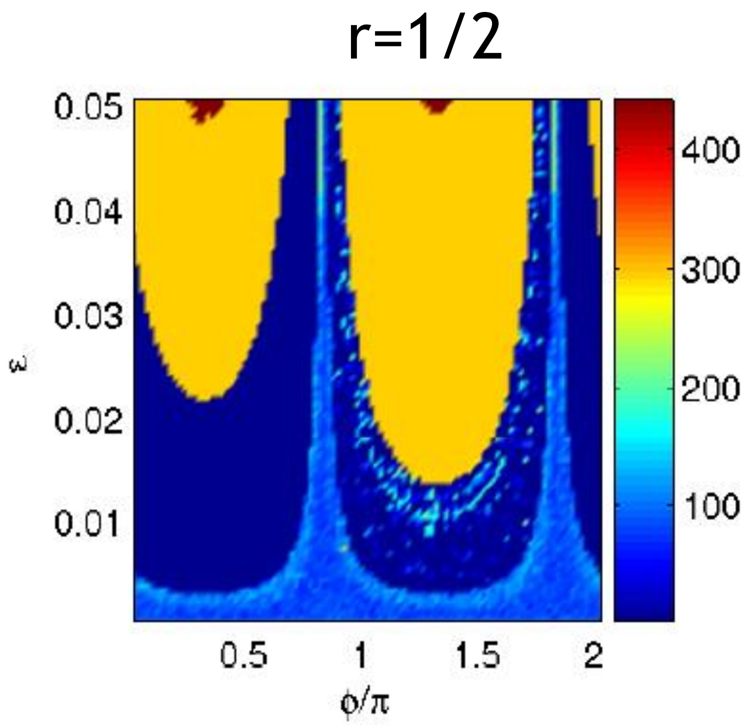
Perturbing the non spiking regime

The phase affects the global dynamics

4.4 Some numerical results

Average number of spikes when applying a perturbation to the spiking regime





High (low) number implies bursting (non bursting) regime

5. Conclusions

- We have shown both numerical and experimentally that the phase control scheme can tame chaotic behavior in the paradigmatic Duffing oscillator.

- We have shown that this method can also be applied to avoid escapes in the paradigmatic Helhmoltz oscillator.

- We have shown that it can change the global bursting dynamics on a neuron model.