Chaos in Optics : role of the CO2 laser

R. Meucci

CNR- Istituto Nazionale di Ottica Applicata, Largo E. Fermi 6, 50125 Firenze, Italy and
Dept. of Physics, University of Firenze.

Firenze, 04 October 2012

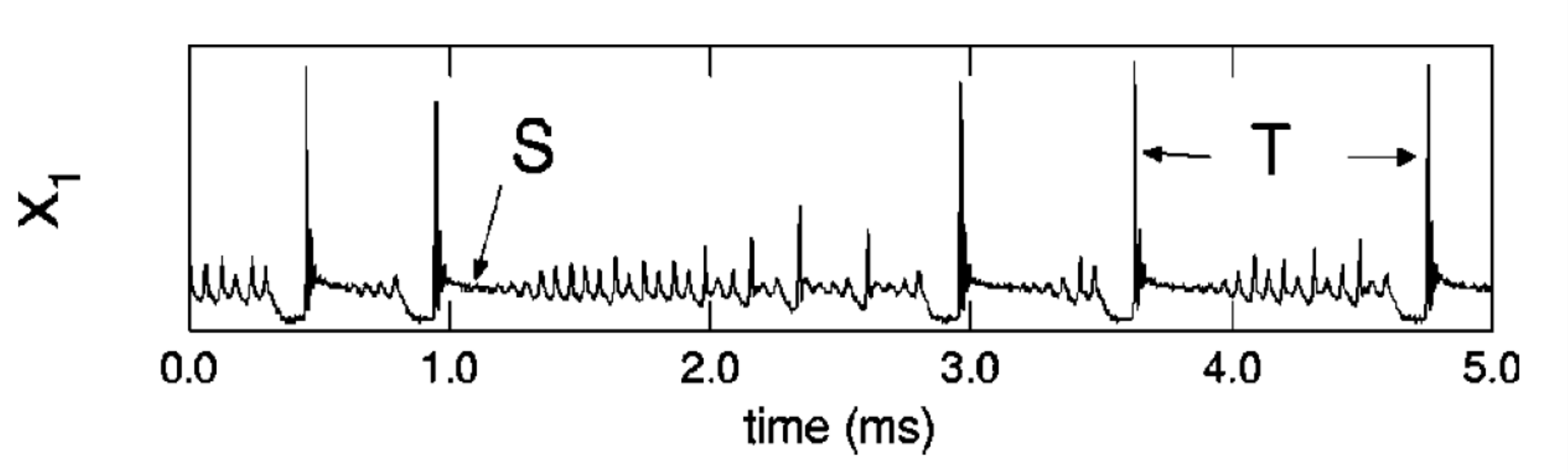
Outline of presentation

1. Homoclinic Chaos (HC) : a key study for investigating noise effects

2. Coherence Resonance, Stochastic Resonance and Enhanced Phase Synchronization in HC

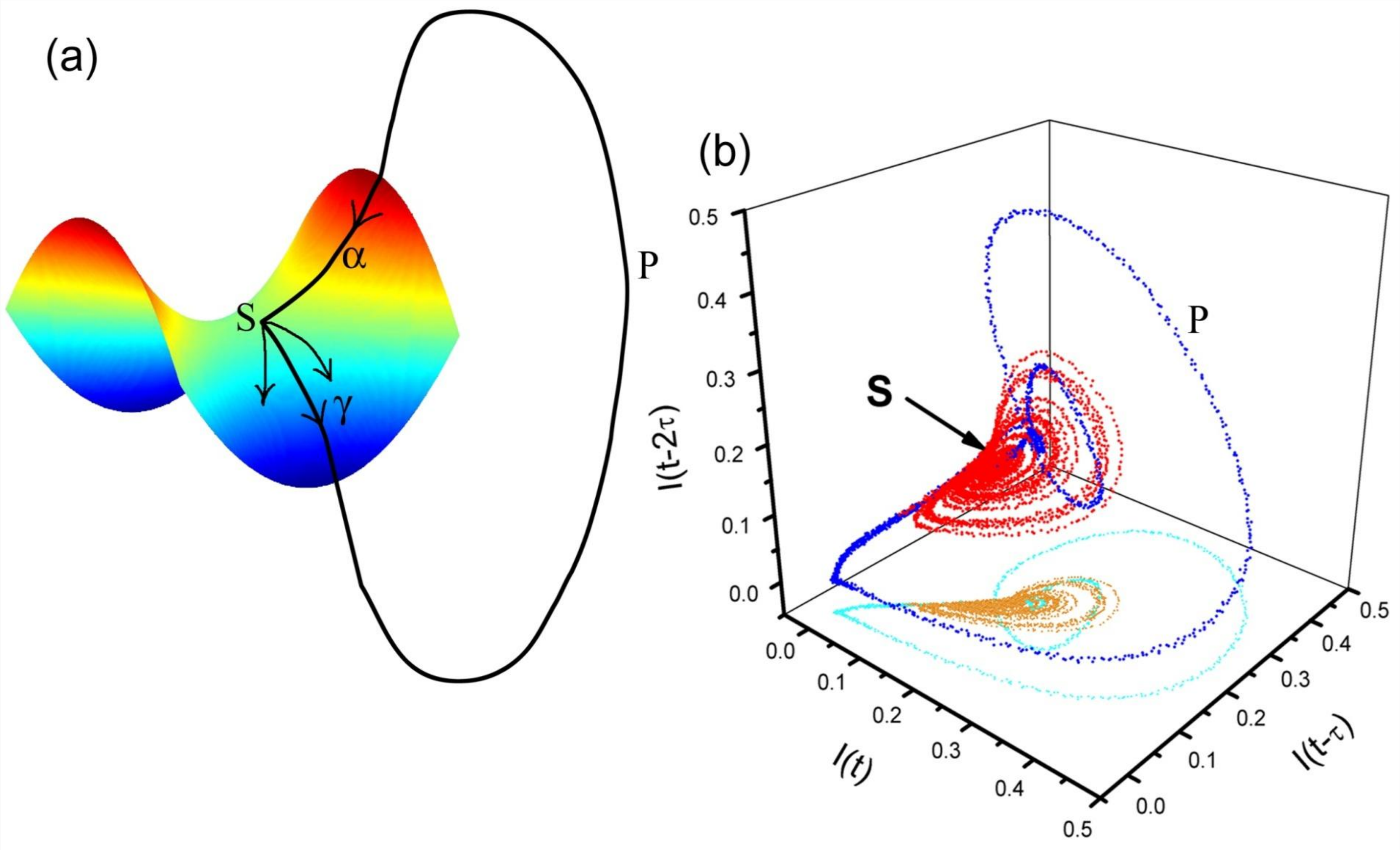
3. Control and synchronization of laser bursting

Time series of the laser intensity without external signal and noise

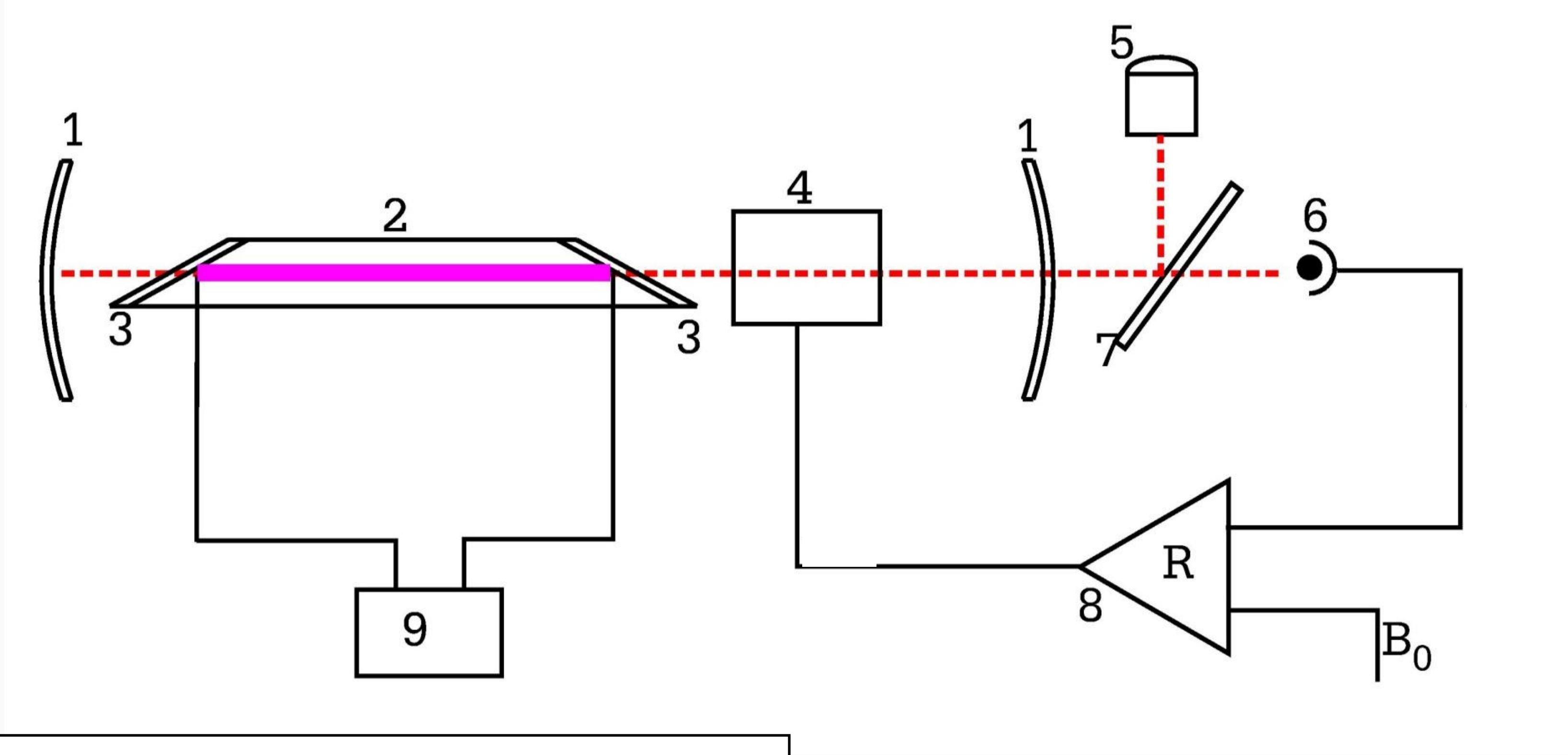


T interspike interval (ISI)
S saddle point

Stable and unstable manifolds at the saddle focus S



CO_2 laser with feedback



1- Laser mirror
2- CO2 laser tube
3- Brewster window
4- Electro-optic modulator
5- Power meter
6- Detector
7- Beam Splitter
8- Amplifier
9- Power supply

Control parameters: R and B_0

Equations and parameters for the CO2 laser- 2D model

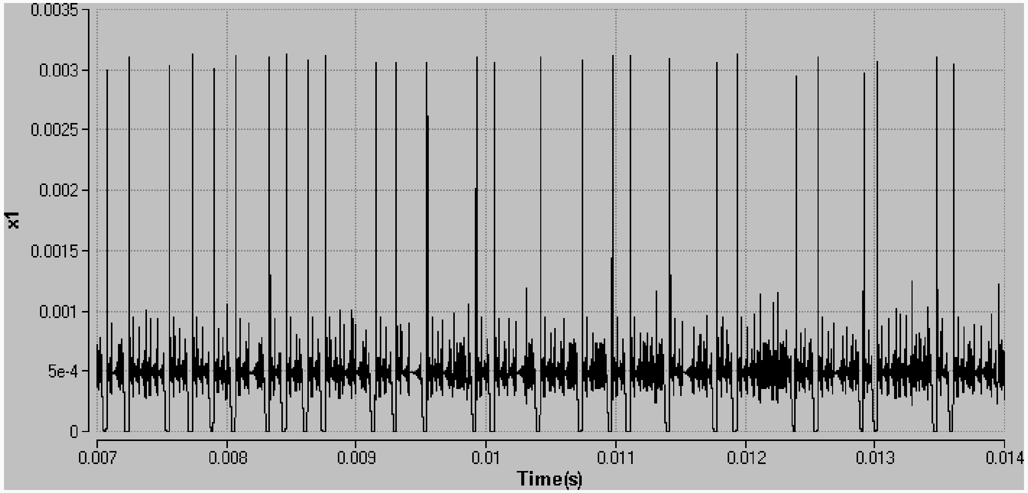
$$\dot{x}_1 = -k_0 x_1 (1 + k_1 \sin^2 x_3 + x_2)$$

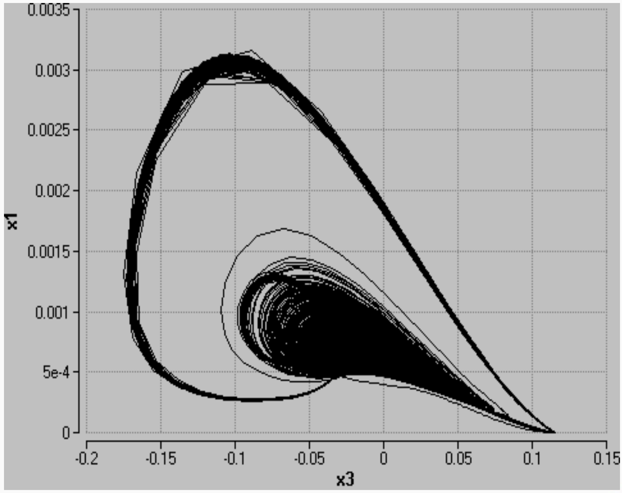
$$\dot{x}_2 = -\gamma(x_2 + k_0 \gamma^{-1} x_1 x_2 + p_0)$$

$$\dot{x}_3 = -\beta(x_3 \quad B_0 + R x_1) + ...$$

$\rightarrow D\xi(t)$

$\rightarrow \varepsilon \sin(\omega t)$





k0=2.0*10^7, k1=20, gamma=1.0*10^5, beta=1.0*10^6, p0=1.196, B0=0.1155, R=222

Equations and parameters for the CO2 laser- 6D model

$$\dot{x}_1 = k_0 x_1 (x_2 - 1 - k_1 \sin^2 x_6)$$

$$\dot{x}_2 = -\gamma_1 x_2 + g x_3 - 2 k_0 x_1 x_2 + p_0$$

$$\dot{x}_3 = -\gamma_1 x_3 + g x_2 + x_5 + p_0$$

$$\dot{x}_4 = -\gamma_2 x_4 + g x_5 + z x_2 + z p_0$$

$$\dot{x}_5 = -\gamma_2 x_5 + g x_4 + z x_3 + z p_0$$

$$\dot{x}_6 = \beta \left(-x_6 + B_0 - \frac{R x_1}{1 + \alpha x_1} \right) + D \boxed{\xi(t)}$$

noise source

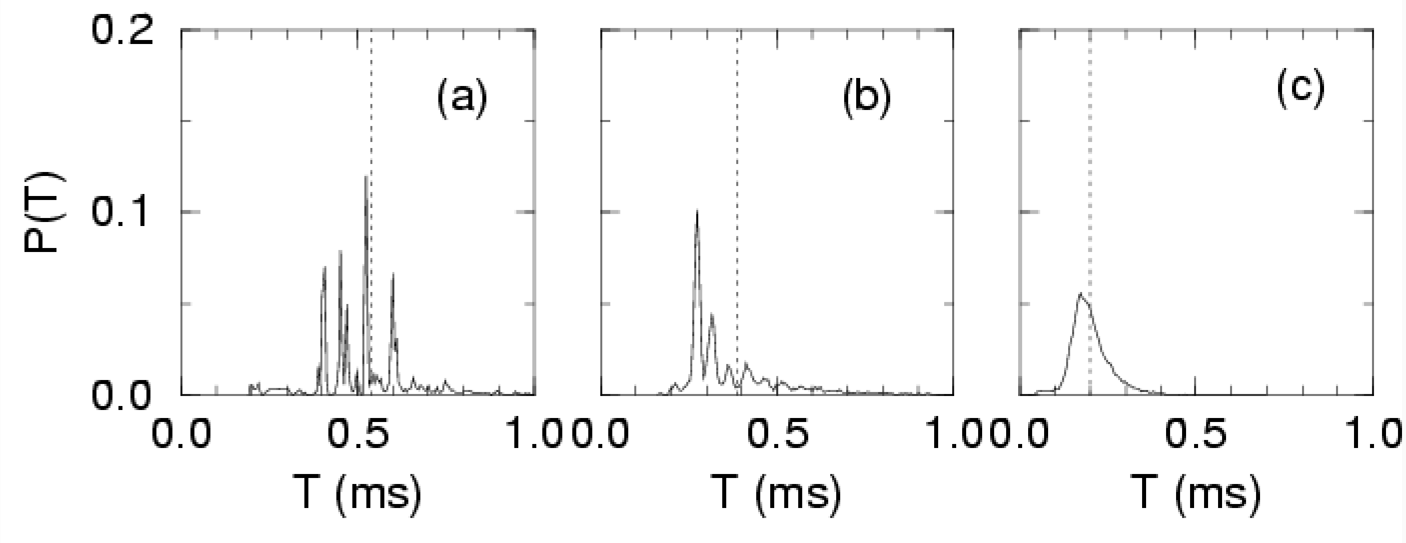
$k_0 = 28.5714,$
$k_1 = 4.5556,$
$\gamma_1 = 10.0643,$
$\gamma_2 = 1.0643,$
$g = 0.05,$

$p_0 = 0.016,$
$z = 10,$
$R = 160,$
$B_0 = 0.1031,$
$\beta = 0.4286,$
$\alpha = 32.8767$

A. N. Pisarchick et al., EPJD 13,385(2001)

Probabilty density of interspike intervals T

D=0 D=0.0005 D=0.01

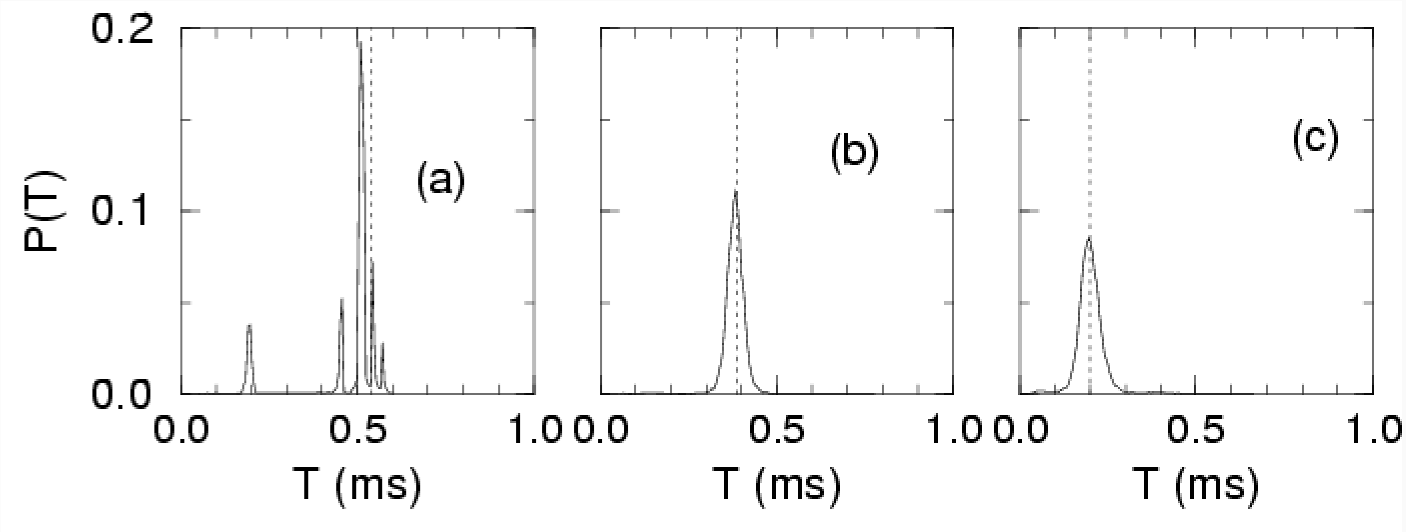


(without external forcing: A=0)

C. S. Zhou et al., PRE 67, 066220 (2003)

Probabilty density of interspike intervals T

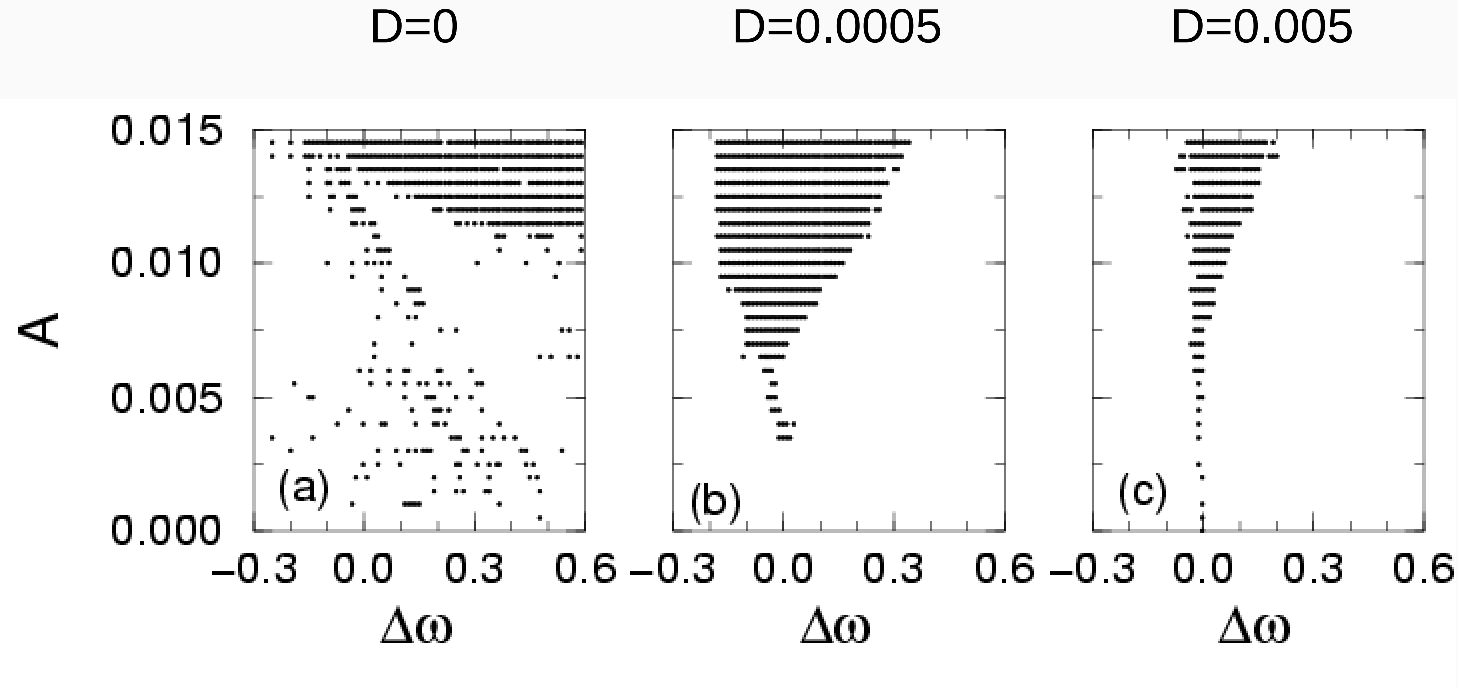
D=0 D=0.0005 D=0.01



with forcing A=0.01 applied at the average interspike interval $T_0(D)$

C. S. Zhou et al., PRE 67, 066220 (2003)

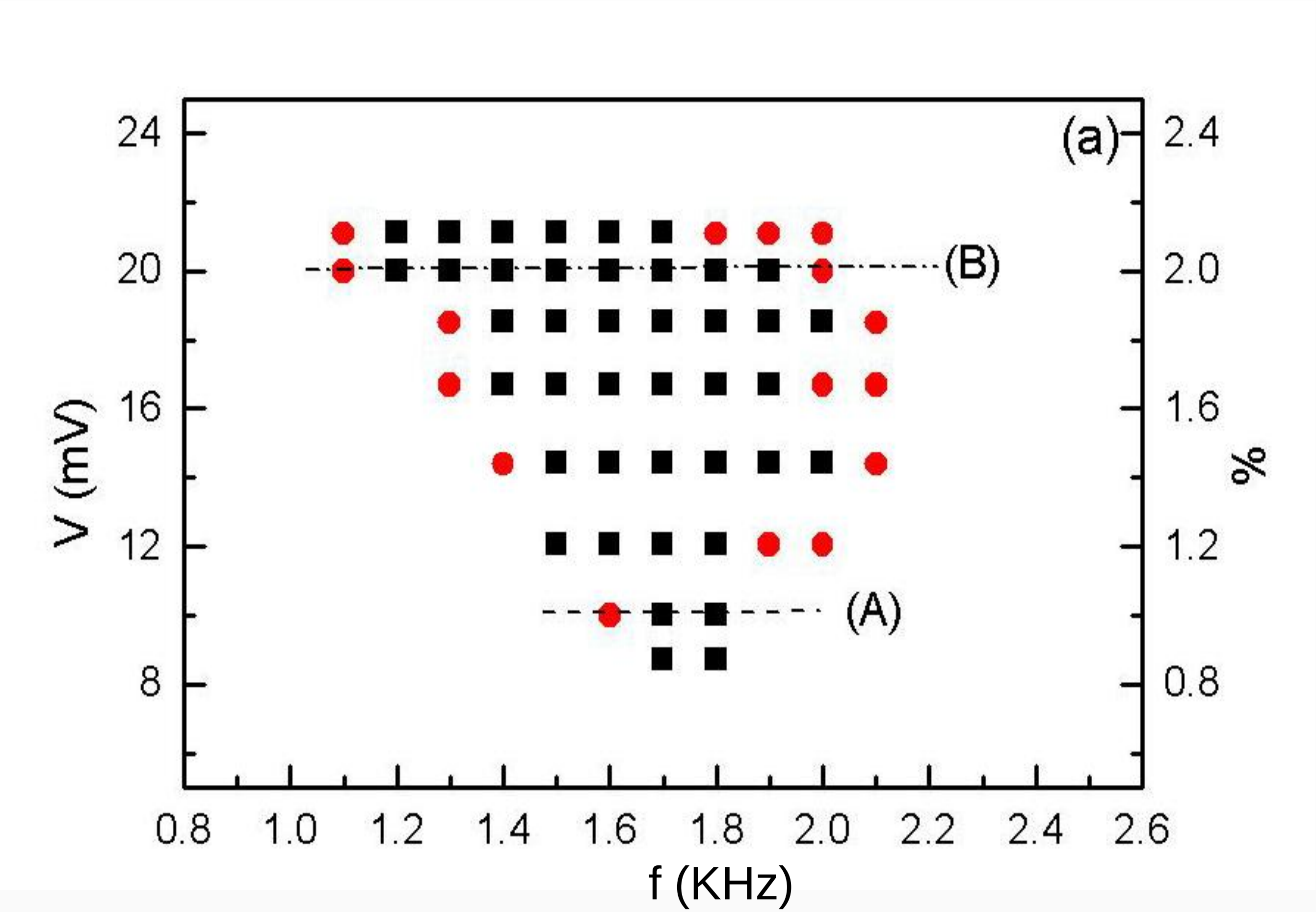
Synchronization region of the laser model



$$\Delta\omega = \left[f_e - f_0(D) \right] / f_0(D)$$

(a dot is plotted when $\Delta\Omega \leq 0.003$)

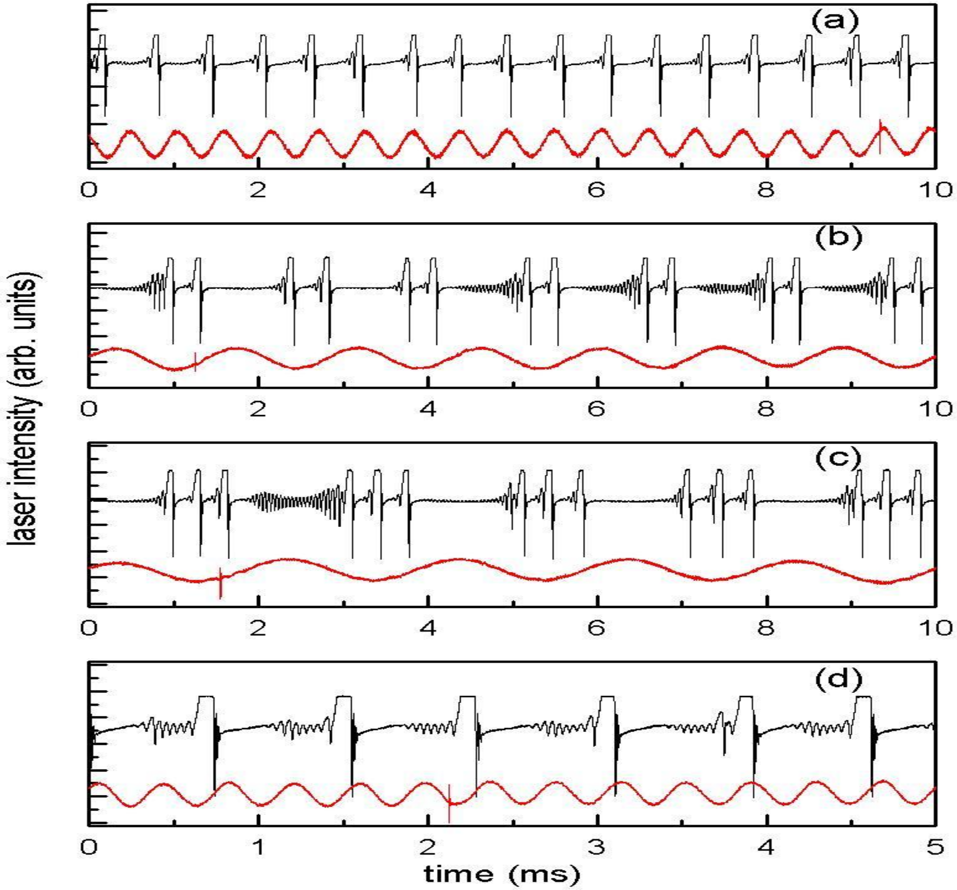
1:1 synchronization region (EXP.)



● phase slips

E. Allaria et al, PRL 86,791(2001)

Evidence of different regimes of synchronization



1:1
ν_{ext}=1.6 kHz

1:2
ν_{ext}=0.7 kHz

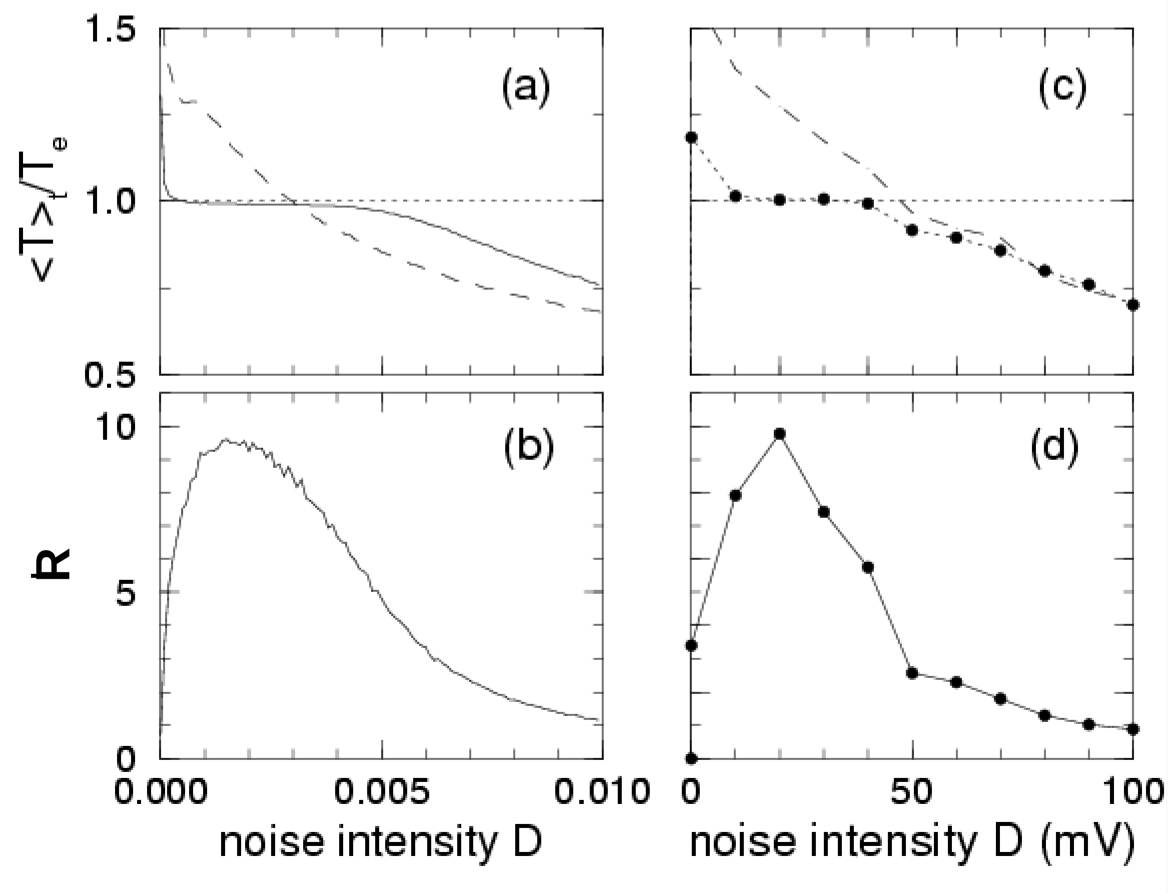
1: 3
ν_{ext}=0.5 kHz

2:1
ν_{ext}=2.6 kHz

E. Allaria et al., PRL 86,791(2001)

Stochastic resonance for a fixed driving period

Model Experiment



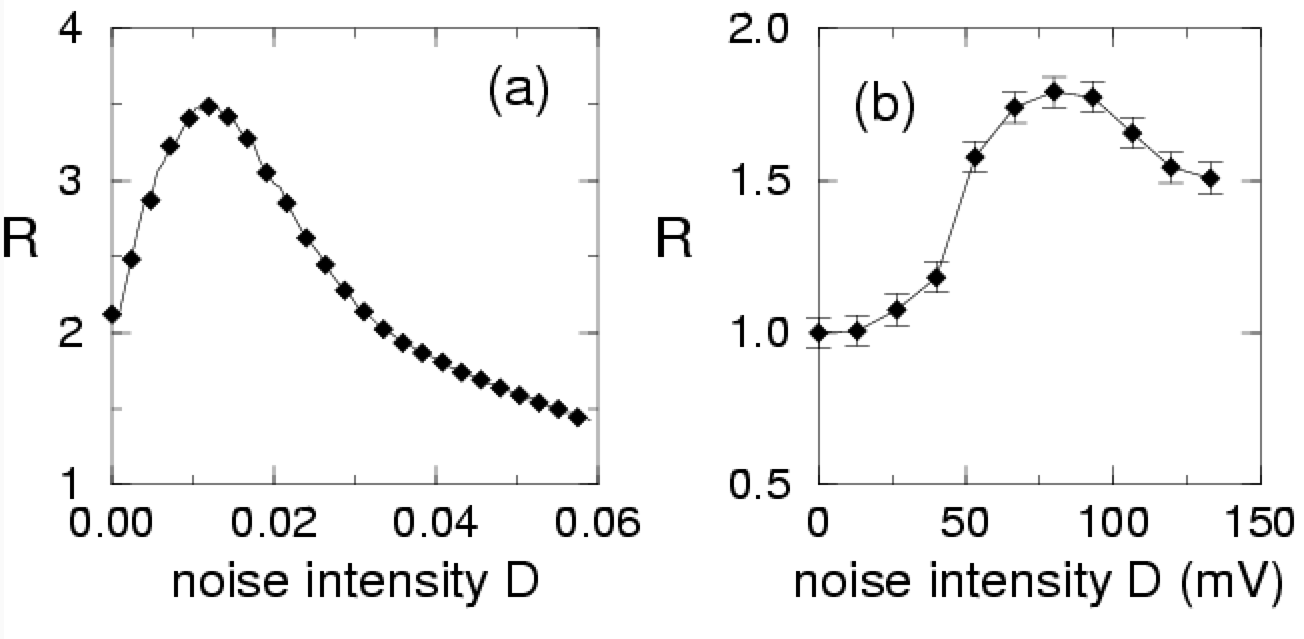
$R = Te/\sigma_T$

$A=0.01$; $T_e=0.3$ms $A=0.01$; $Te=1.12$ms

C. S. Zhou et al., PRE 67, 015205(R) (2003)

Coherence resonance

Model Experiment

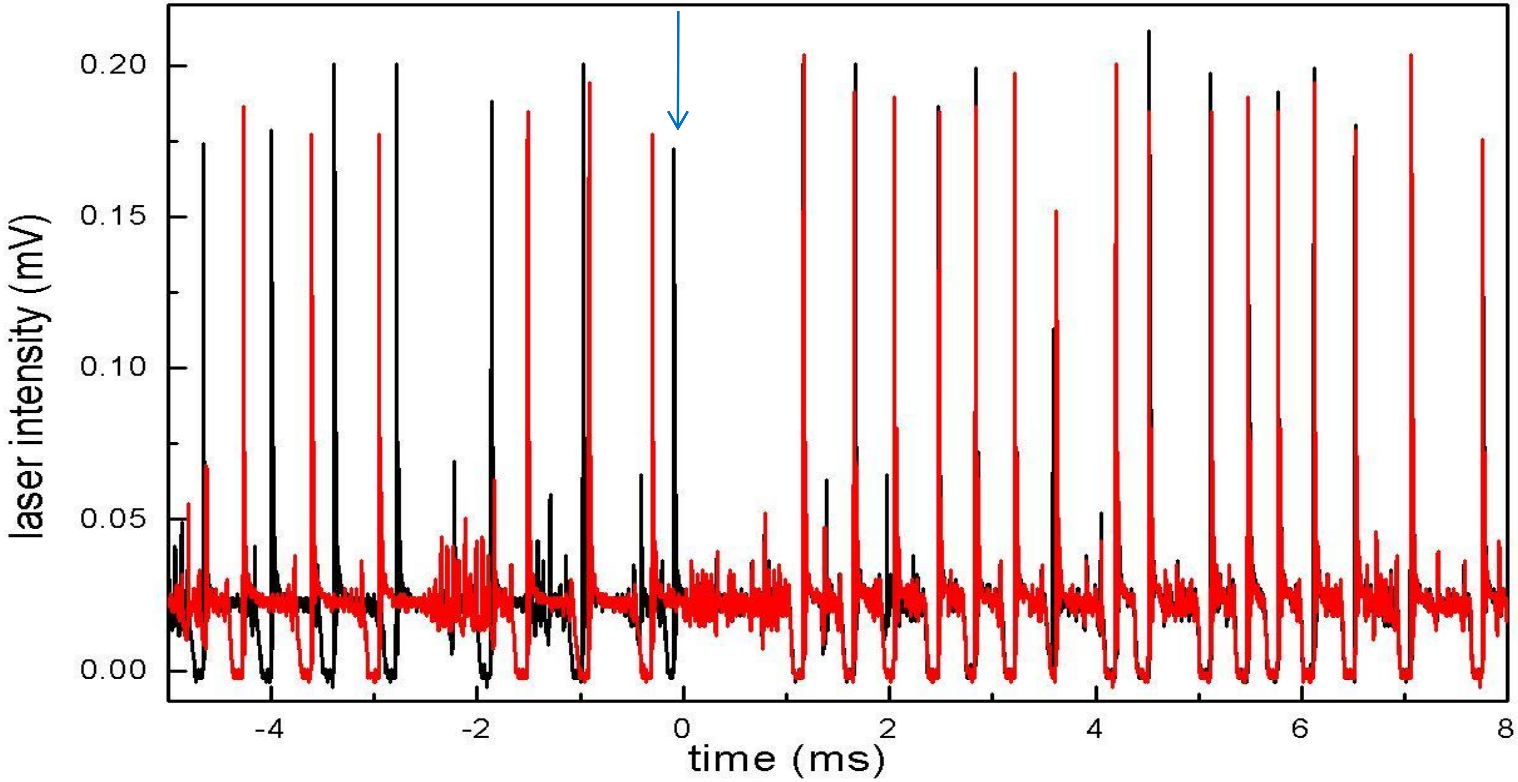


$R=T_0(D)/\sigma_T$

C. S. Zhou et al., PRE 67, 066220 (2003)

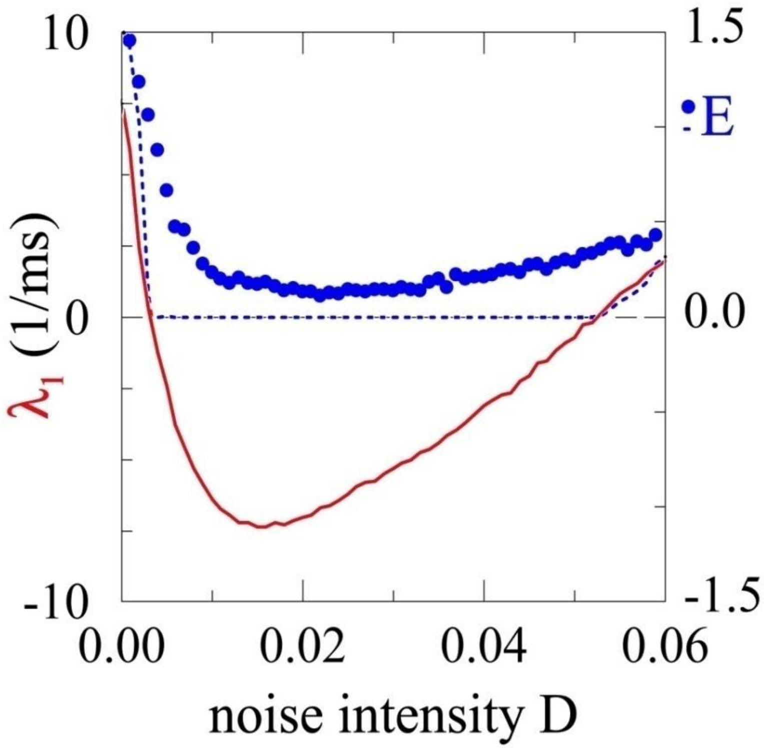
Evidence of Noise Induced Synchronization

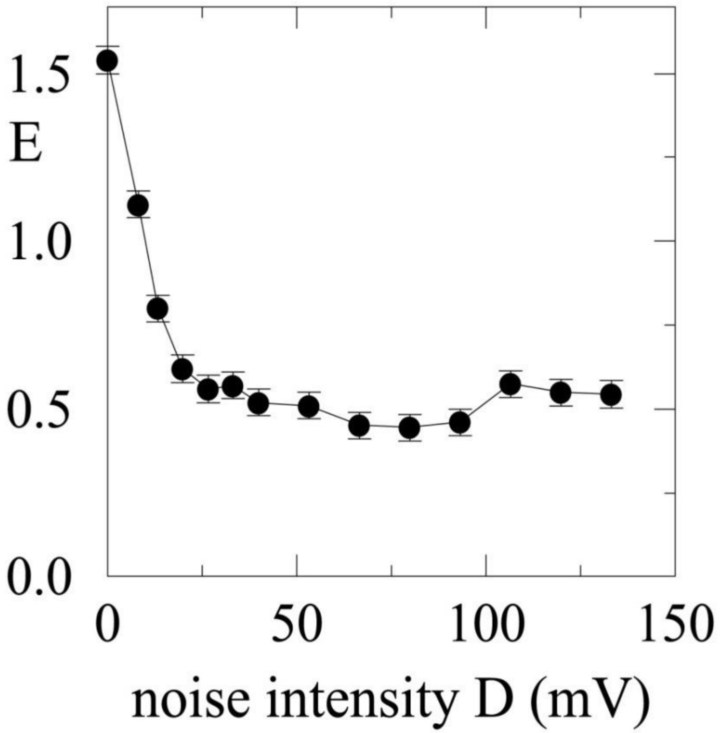
Start of the common noise signal



C. S. Zhou et al., PRE 67,066220 (2003)

Evidence of Noise Induced Synchronization





Numerical results:

a) Largest Lyapunov Exponent (λ_1) and Synchronization Error (E) for a system without (-) and with (●) intrinsic noise

Experimental results

$$E=\langle|x_1-y_1|\rangle/\langle|x_1-\langle x_1\rangle|\rangle$$

Chaotic spiking and incomplete homoclinic scenarios in semiconductor devices with optoelectronic feedback

More details will be given by Kais et al.

Kais Al-Naimee, Francesco Marino, Marzena Ciszak, Riccardo Meucci, F. Tito Arecchi,
NJP **11**, 073022 (2009)

Control and synchronization of bursting in a coupled CO_2 laser system

Intermittent bursting and switching

Interior crisis: a chaotic attractor touches an unstable periodic orbit and suddenly expands

Crisis-induced intermittency: spontaneous jumps between the unstable orbit and the chaotic attractor (bursting)

[Theory]

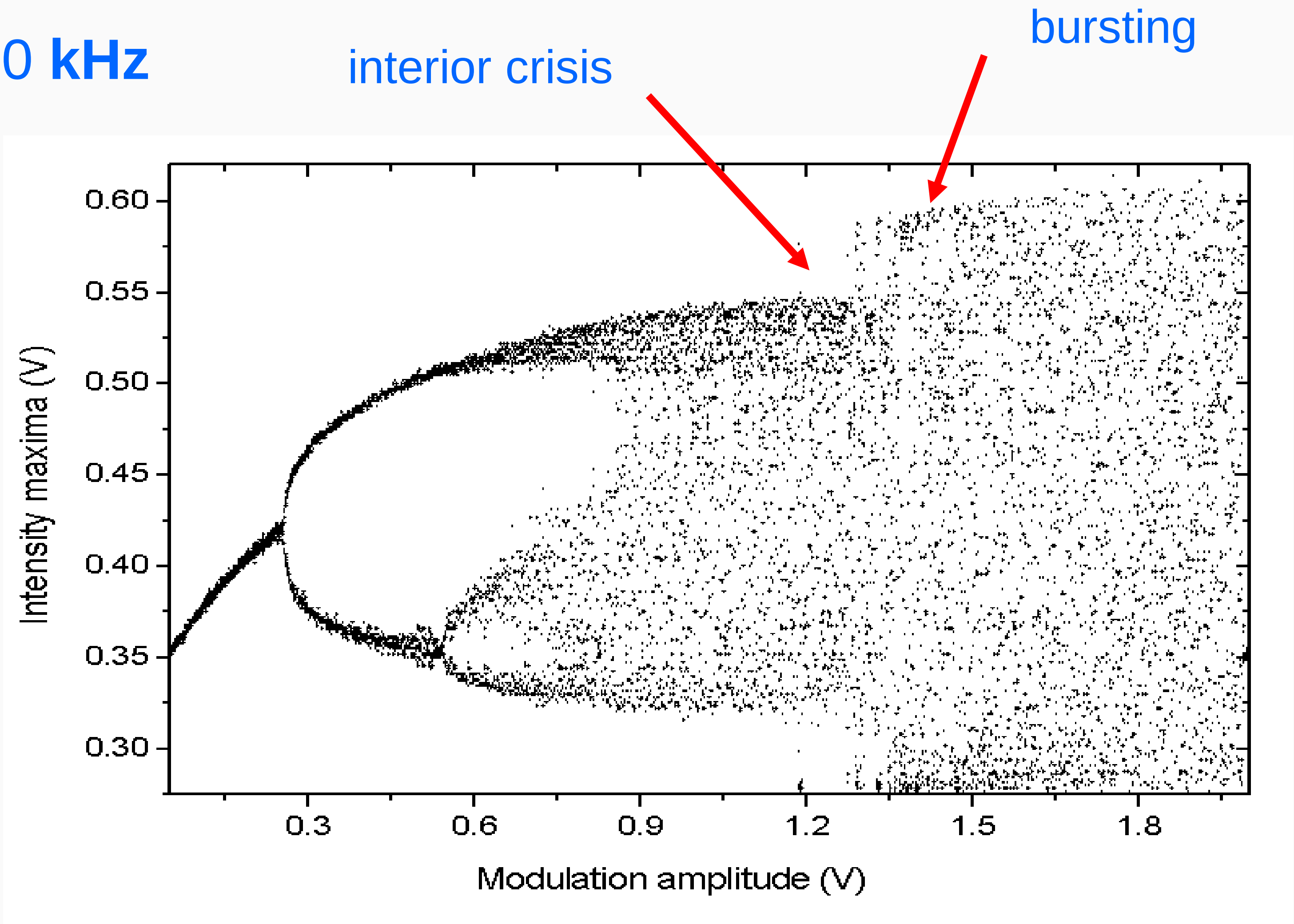
C. Grebogi, E. Ott, and J. A. Yorke, Phys. Rev. Lett. **48**, 1507 (1982)

C. Grebogi, E. Ott, and J. A. Yorke, Physica D **7**, 181 (1983)

C. Grebogi, E. Ott., F. Romeiras, and J. A. Yorke, Phys. Rev. A **36**, 5365 (1987)

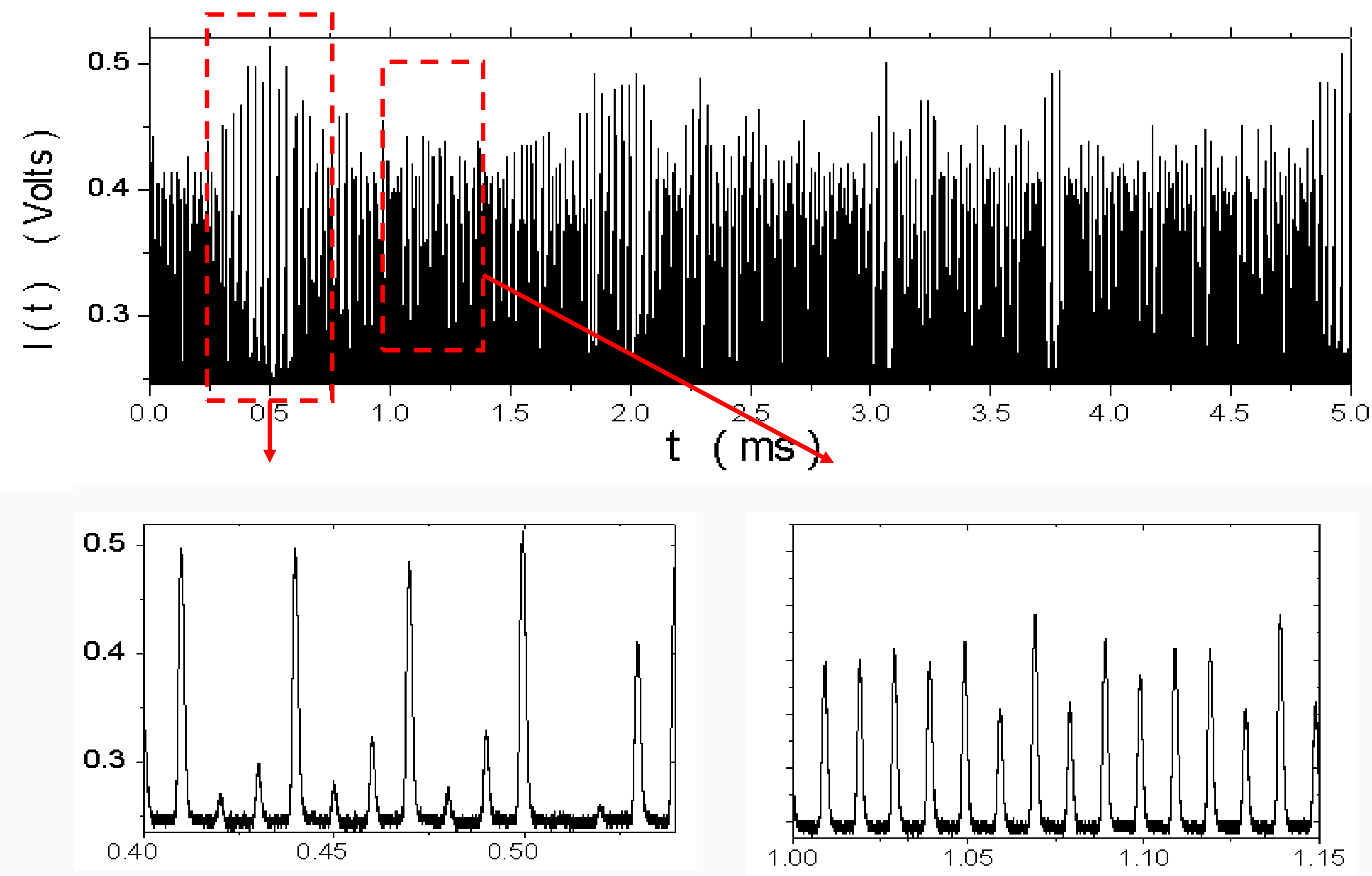
Bifurcation diagram

f*=100 kHz

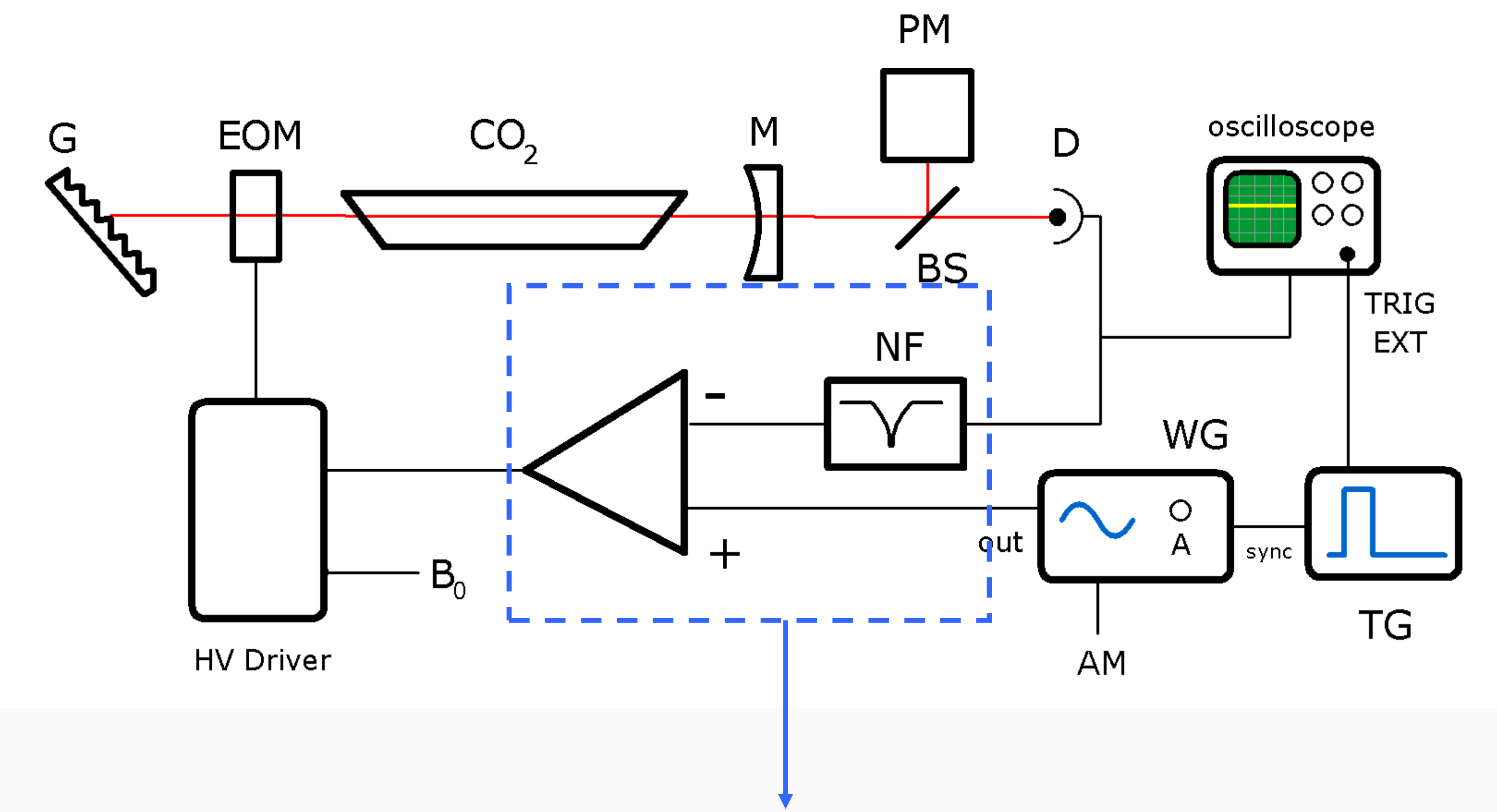

interior crisis

bursting

Temporal series (*bursting*)



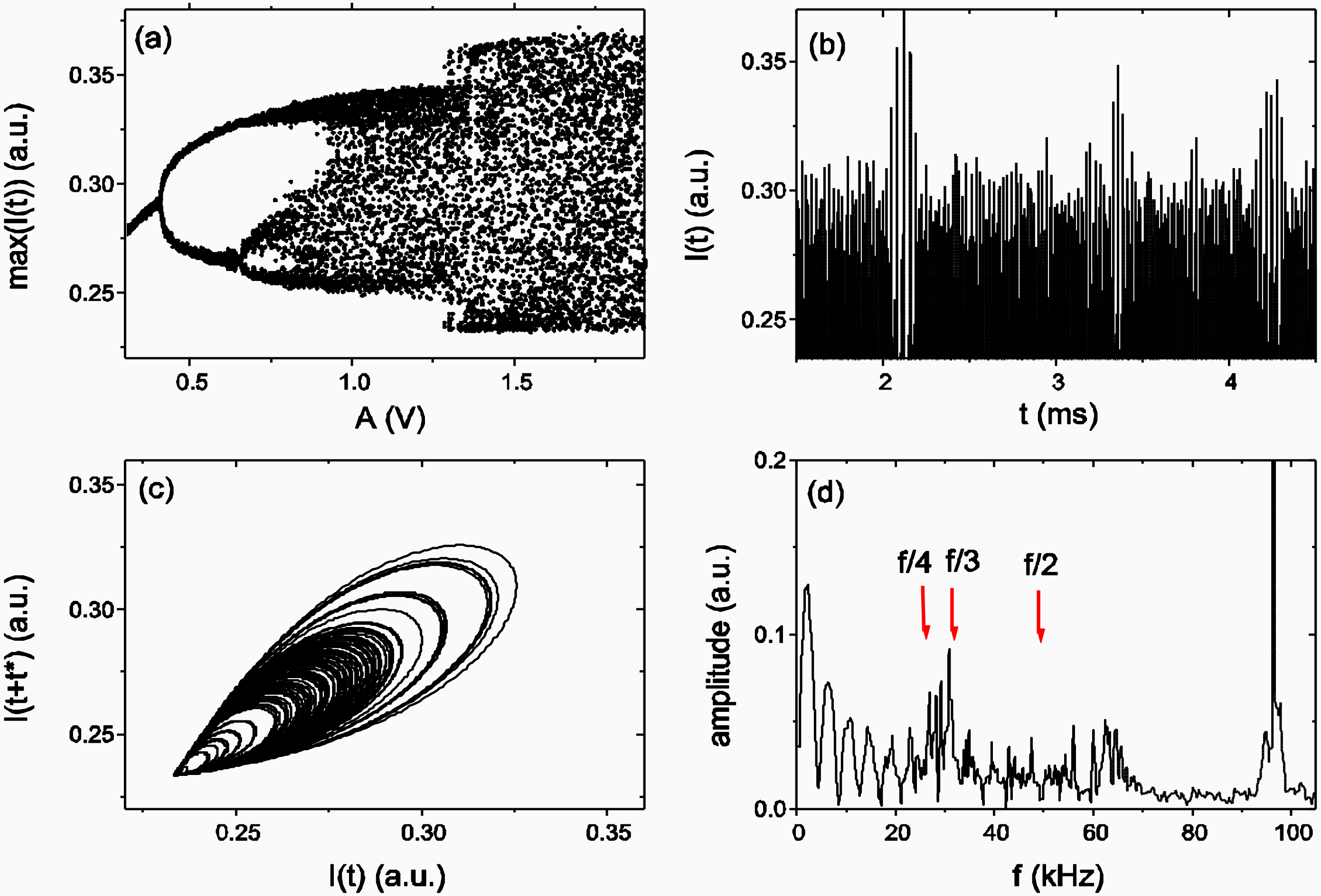
Experimental setup



control

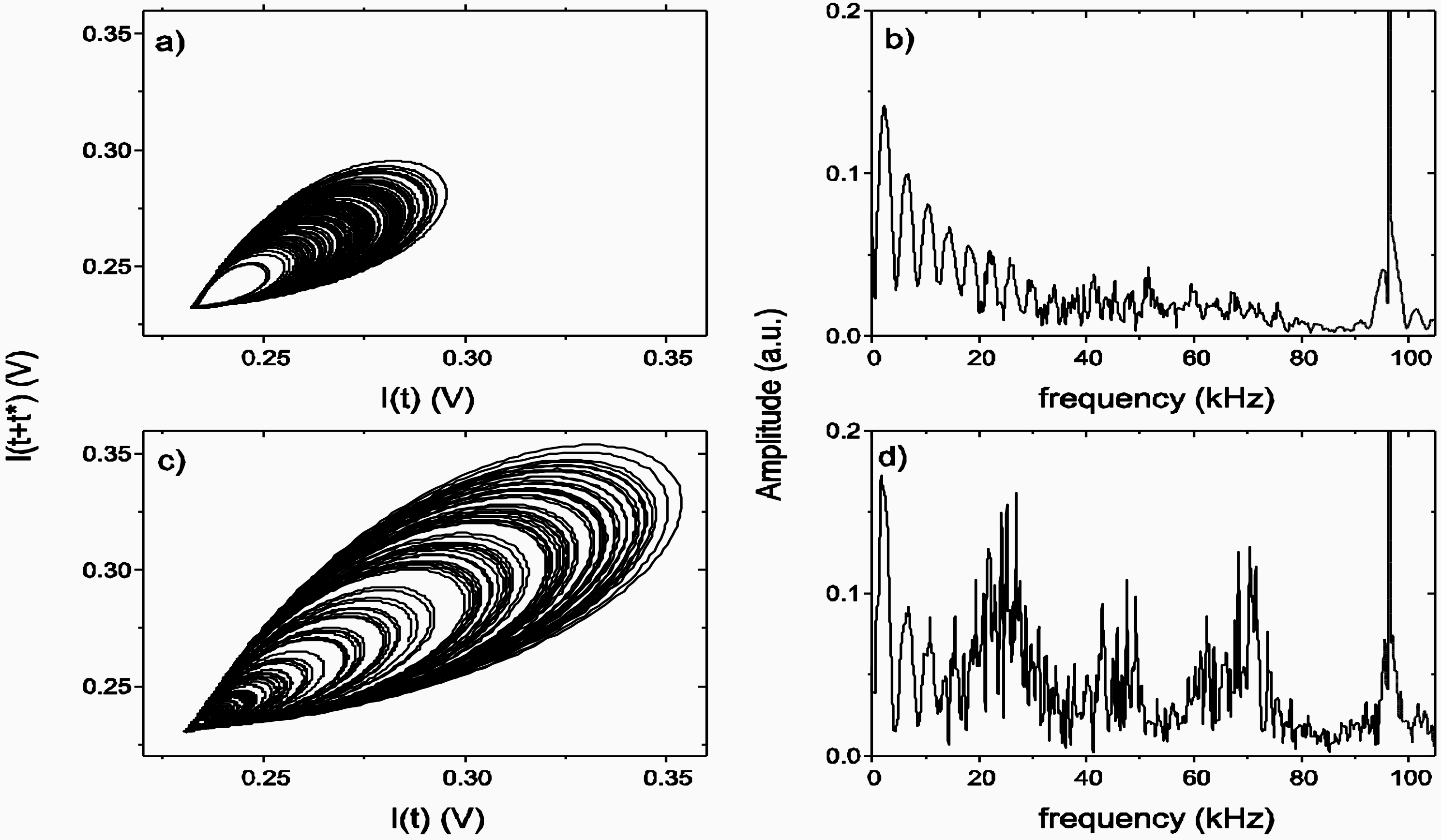
Phys. Rev. Lett. 95, 184101 (2005)

Bursting near an interior crisis



Phys. Rev. Lett. 95, 184101 (2005)

Control and enhancing of bursting



Phys. Rev. Lett. 95, 184101 (2005)

Synchronization experiments in CO_2 lasers

CO_2 lasers with saturable absorber

T. Sugawara, M. Tachikawa , T. Tsukamoto and T. Shimizu, Phys. Rev. Lett. **72**, 3502 (1994)

Y. Liu, J. R. Rios Leite, Phys. Lett. A **191**, 134 (1994)

Y. Liu, P. C. de Oliveira, M. B. Danailov, and J. R. Rios Leite, Phys. Rev. A **50**, 3464 (1994)

Phase synchronization with external signal

E. Allaria, F.T. Arecchi, A. Di Garbo, and R. Meucci, Phys. Rev. Lett. **86**, 791 (2001)

Noise–enhanced synchronization of homoclinic chaos

C. S. Zhou, J. Kurths, E. Allaria, S. Boccaletti, R. Meucci and F. T. Arecchi Phys. Rev. E **67**, 015205 (2003)

Bidirectional and master-slave synchronization configuration

R. Meucci, F.Salvadori, M. V. Ivanchenko, K. Al Naimee, C. Zhou, F. T. Arecchi, S. Boccaletti, and J. Kurths, Phys. Rev. E **74,** 066207 (2006)

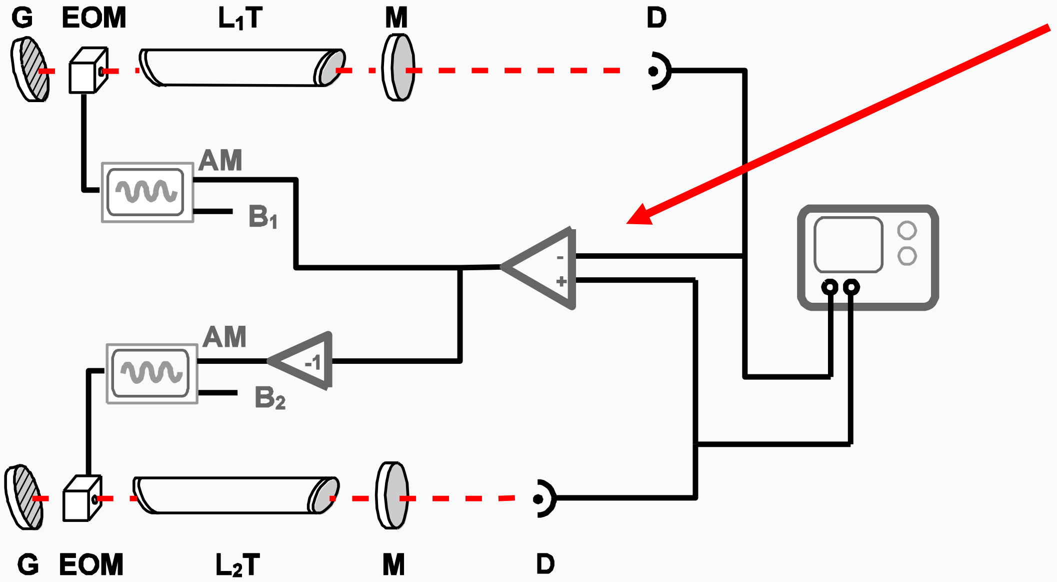
Bidirectionally coupled modulated lasers

Experimental setup

$F_1(t)=A_1[1+ \varepsilon(y_1-x_1)]\sin(2\pi f t)+B_1$

Coupling

ε

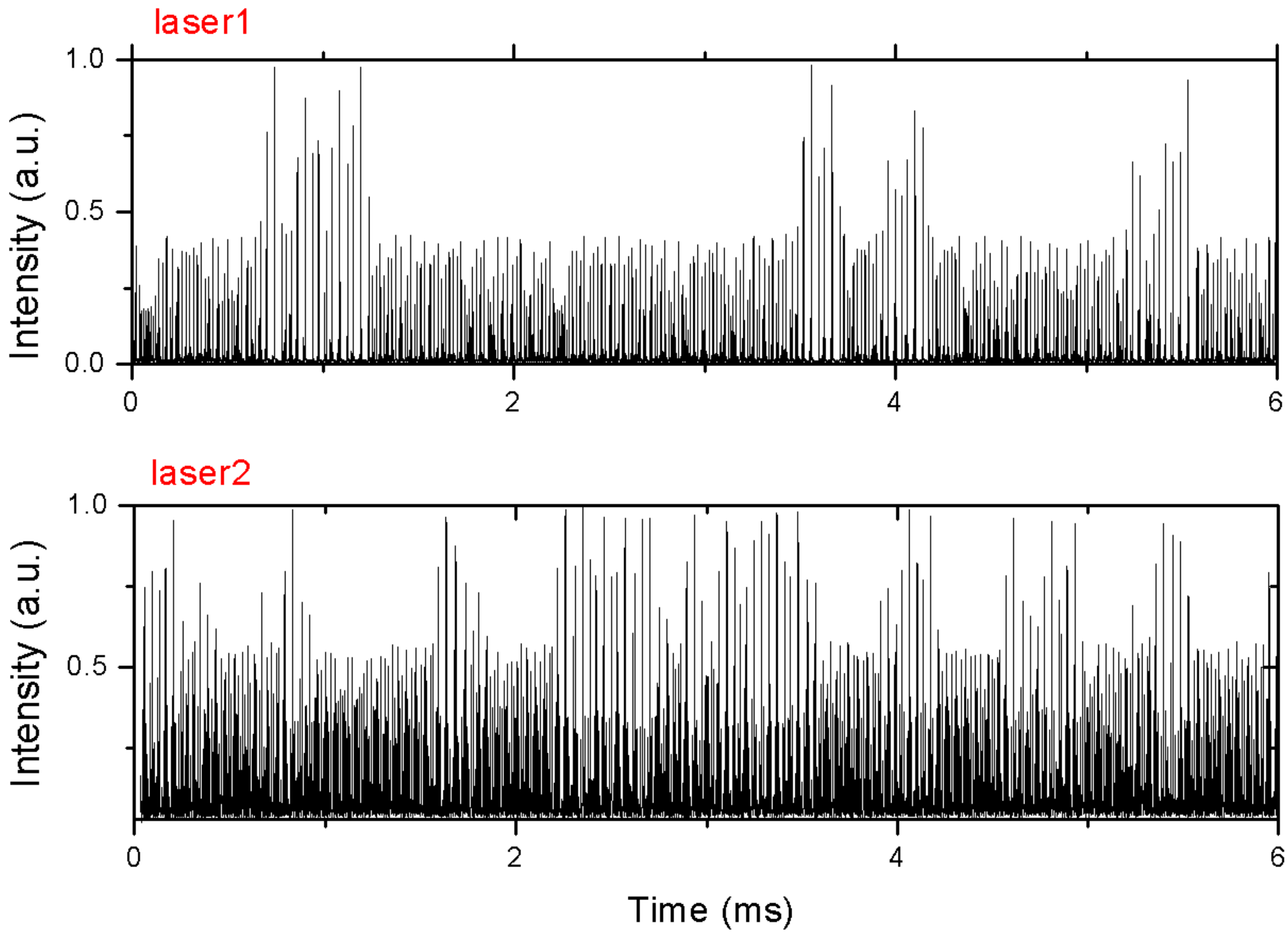


$F_2(t)=A2[1+ \varepsilon(x_1-y_1)]\sin(2\pi f t)+B_2$

Phys. Rev. E **74**, 066207 (2006)

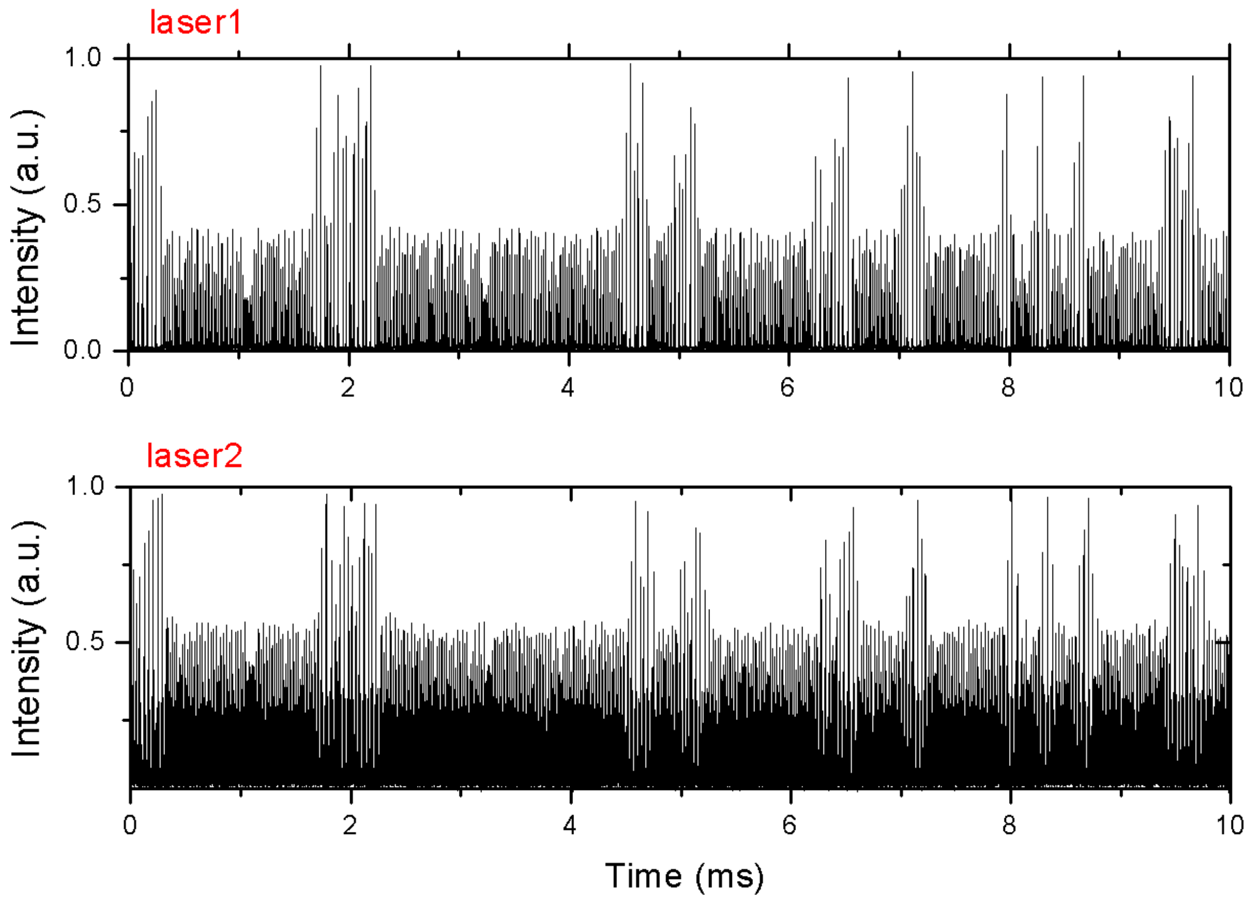
Time evolution of the lasers

without coupling (ε=0)



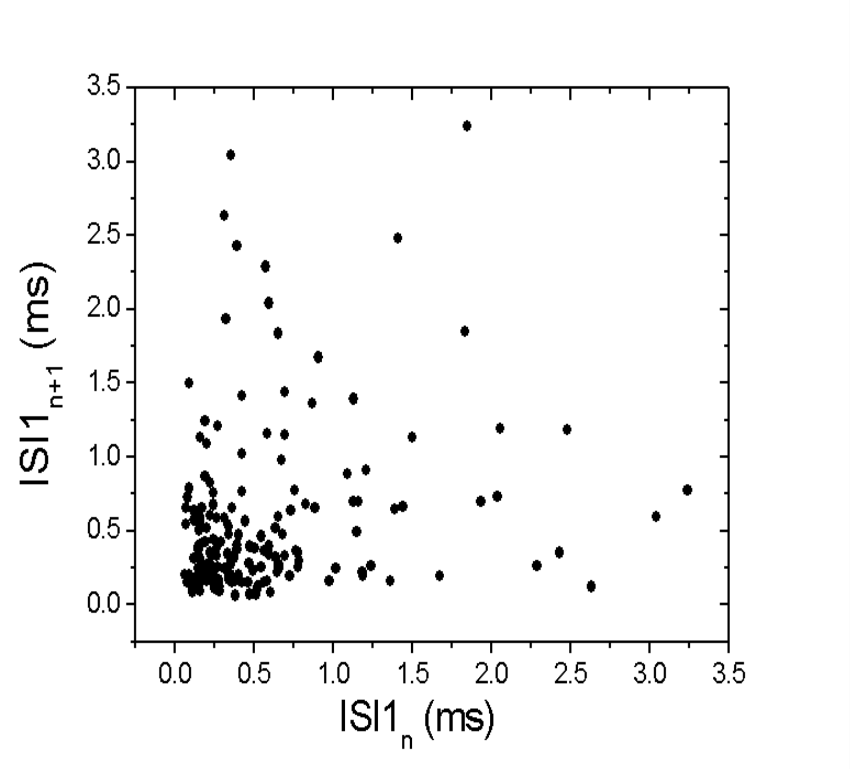
Time evolution of the lasers

with coupling (ε=150)

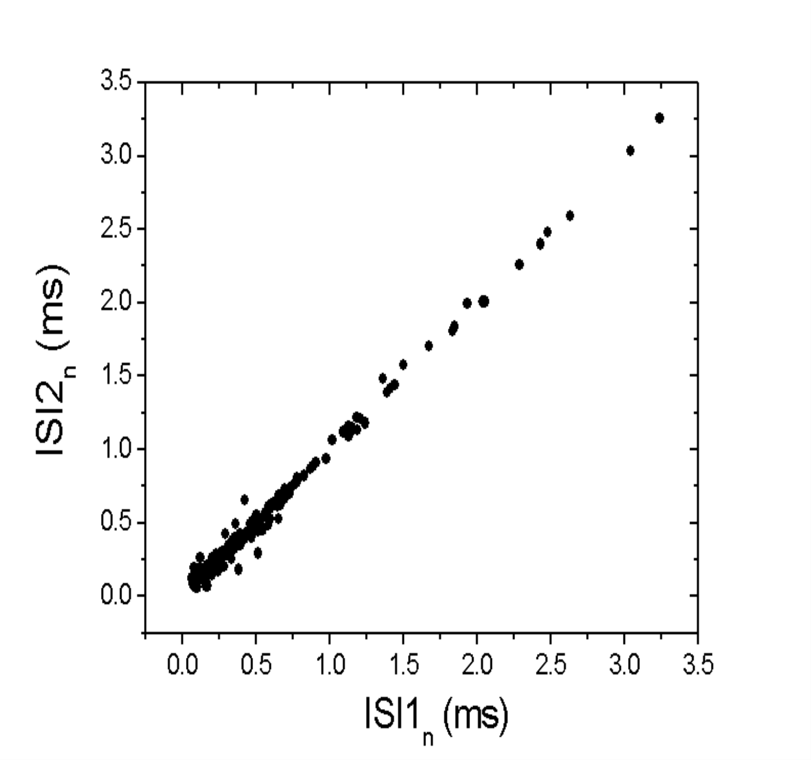


ISI distributions

Auto-correlation (single laser)

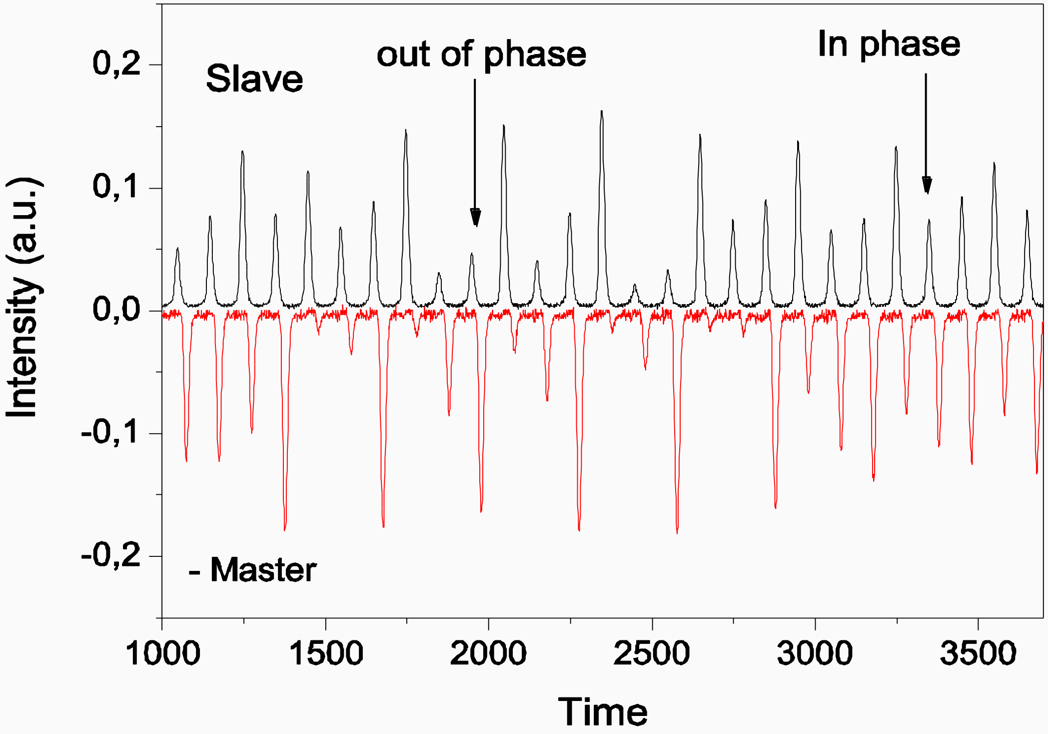


Cross-correlation (two lasers)



Master-slave synchronization

Evidence of phase and anti-phase behavior



Bursting chaotic systems exhibit local structures

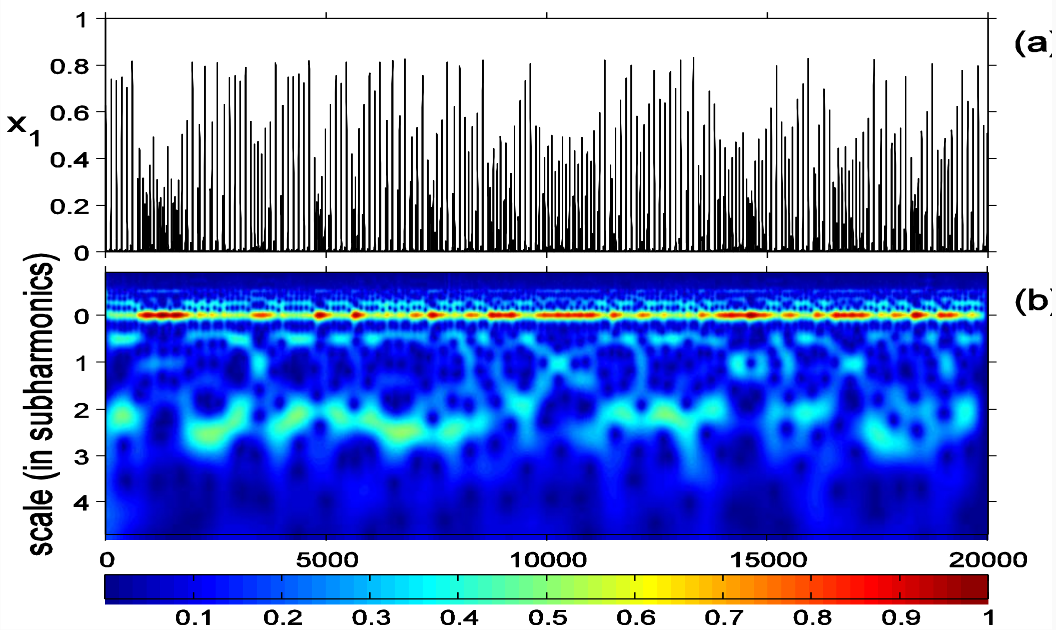
Multiscale analysis by CWT

$$W_{\psi}x(\sigma,\tau)=\frac{1}{\sigma}\int_{R}\psi^{*}\left(\frac{t-\tau}{\sigma}\right)x(t)dt$$

Where $\psi(t)$ is the mother wavelet translated by τ and dilated by σ .

The scale variable σ is the inverse of frequency

Multiscale analysis by *CWT*



The continuous horizontal line (normalized to be scale 0) in the CWT corresponds to the system's intrinsic frequency *f* which is caused by the periodic forcing. On larger scales, patches occur at those times where bursts appear (higher subharmonics).

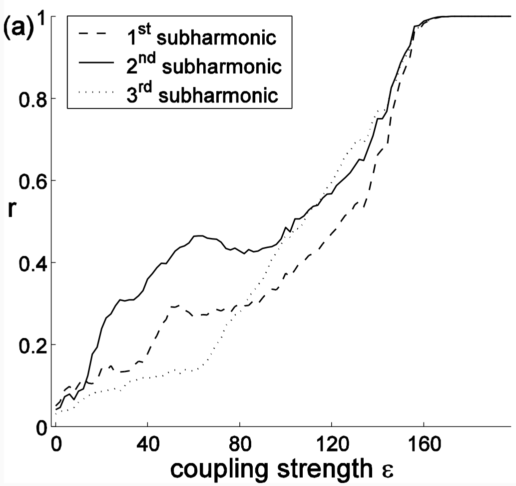
A. Bergner, R. Meucci, K. A. Al-Naimee, M. C. Romano, M. Theil, J. Kurths and F. T. Arecchi,
Phys. Rev. E **78**, 016211 (2008)

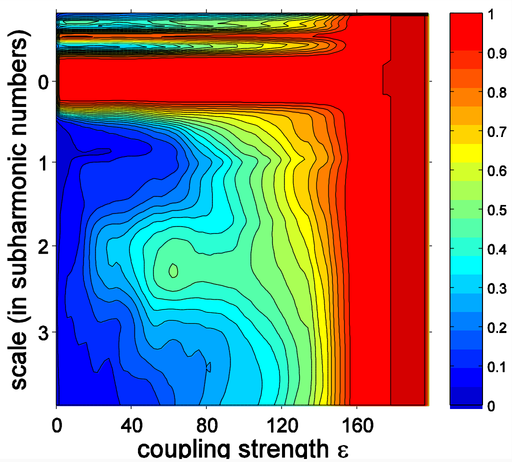
Measuring synchronization and coherency

Mean Resultant Length

$$r_{xy}(\sigma) = \left|\left\langle e^{i\Delta\phi_{xy}(\sigma,t)} \right\rangle\right|$$

(the phase differences at time t are extracted from the argument of CWT)

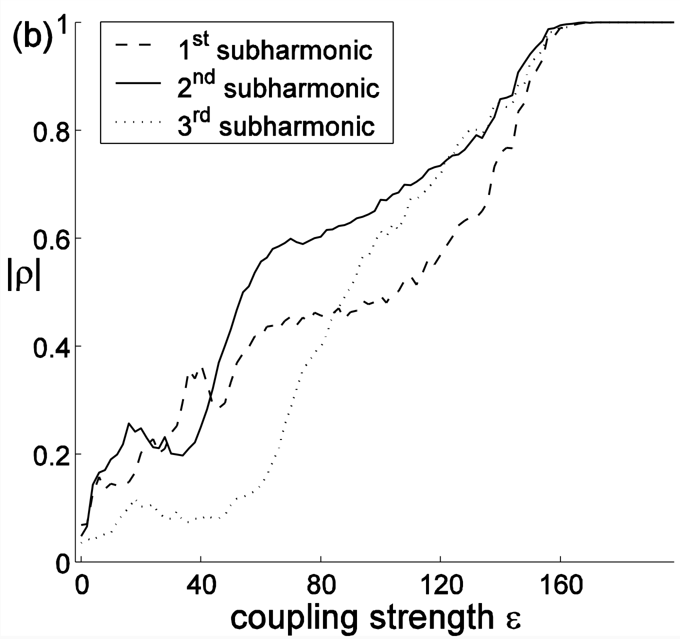


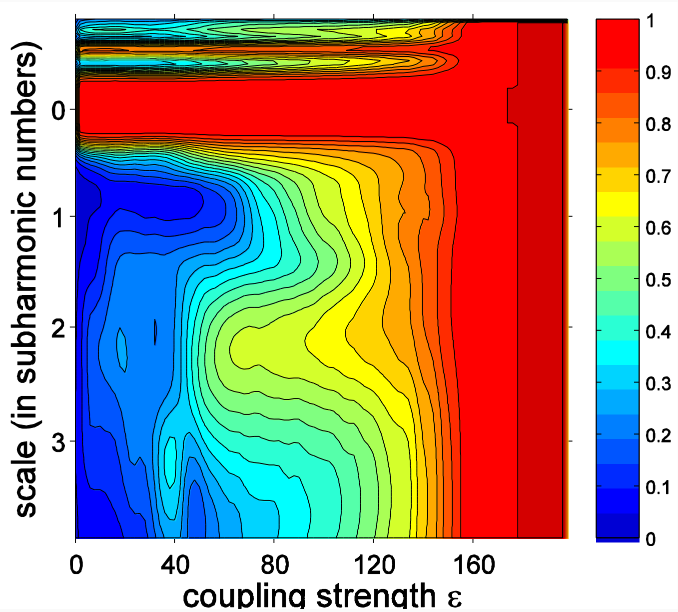


Measuring synchronization and coherency

Cross Correlation Coefficient

$$\rho_{xy}(\sigma) = \frac{\langle x(\sigma,t)\, y^{*}(\sigma,t) \rangle}{\sqrt{\langle |x(\sigma,t)|^{2} \rangle \langle |y(\sigma,t)|^{2} \rangle}}$$





The modulus of ρ ,called ***coherence ,***
measures the orthogonality between two functions

Main results in bursting synchronization

- Synchronization between modulated lasers in *bidirectional* and *master-slave* configurations (phase and anti-phase synchronization, possible applications for coding)
- CWT resolves different local strutures of multi-time scale systems.

Conclusions

- Role of noise in Homoclinic Chaos

It reduces the average ISI value and its fluctuations

As a result, noise enhances phase synchronization and the laser displays both CR and SR

- Synchronization of bursting regime and possible implications for neuroscience

Chaos in optics. Scholarpedia, 3(9):4104(2008)

Chaos in lasers. Scholarpedia, 3(9):7066(2008)

Acknowledgements

E. Allaria (Trieste, Italy)
K. Al- Naimee (Firenze, Italy and Baghdad, Iraq)
S. Abdalah (Firenze , Italy)
W. Gadomski (Warsaw, Poland)
J. Kurths (Potsdam, Germany)
I. Leyva (Madrid, Spain)
I. P. Marino (Madrid, Spain)
A. Pisarchick (Leon, Mexico)
J. Roversi (Campinas, Brazil)
M. Sanjuan (Madrid, Spain)
S. Zambrano (Madrid, Spain)
C. Zhou (Potsdam, Germany)
M. Ciszak (Firenze, Italy)
F. Marino (Firenze, Italy)