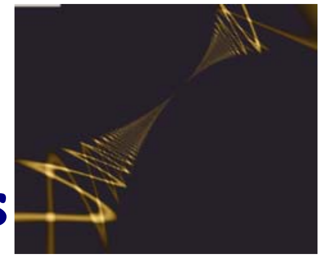
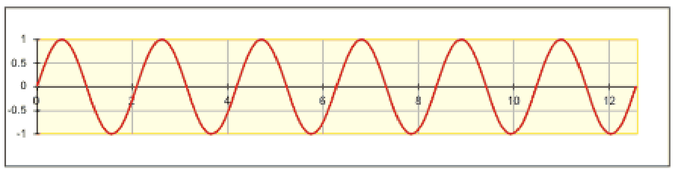
Lecture 8

Oscillations and Waves

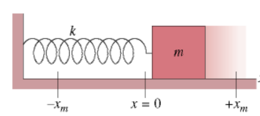


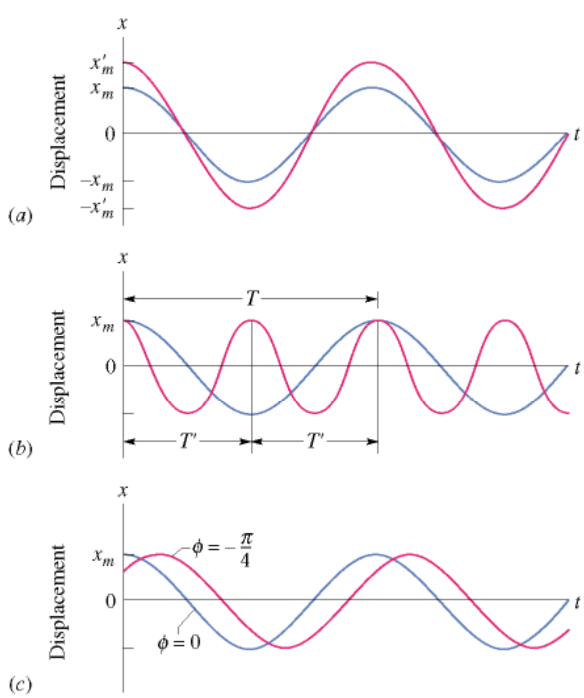
http://web.njit.edu/~sirenko/

Physics 103   Spring 2012

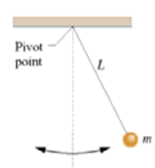


Simple Harmonic Motion (SHM)





1. Amplitude is different
2. Period (or frequency) is different.
3. Phase is different.



Displacemnt at time t

Phase

$$\underbrace{x(t)}_{} = \underbrace{x_m}_{} \cos(\omega t + \phi)$$

Amplitude

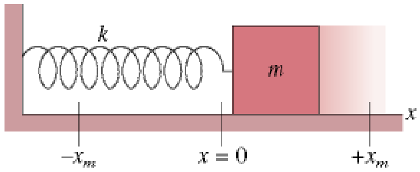
Angular frequency

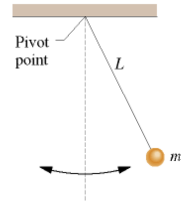
Time

Phase constant or phase angle

Simple Pendulum

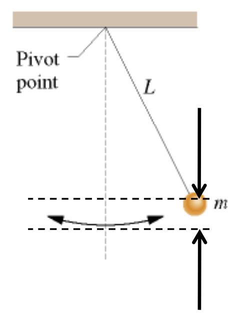
Simple pendulum follows SHM





Spring	Pendulum
$x = x_m\cos(\omega t+\phi)$	$\theta = \theta_m\cos(\omega t+\phi)$
$\omega = \sqrt{\frac{k}{m}}$	$\omega = \sqrt{\frac{g}{L}}$
$T = 2\pi\sqrt{\frac{m}{k}}$	$T = 2\pi\sqrt{\frac{L}{g}}$

Simple Pendulum: Questions



Q1. If we double θ_m the energy:

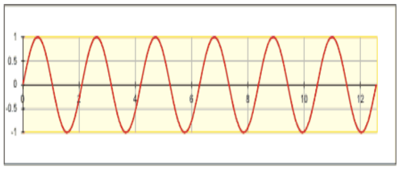
a) is half as large b) Stays the same
c) is twice as large d) is 4 times greater
e) is 16 times greater

Q2. If we double θ_m the period:

a) is half as large b) Stays the same
c) is twice as large d) is 4 times greater
e) is 16 times greater

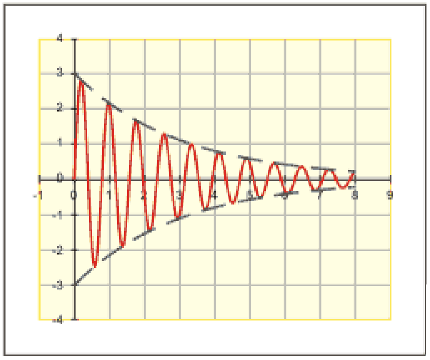
$$T = 2\pi\sqrt{\frac{L}{g}}$$

Damping of harmonic oscillations



Simple Harmonic Motion is an Idealization

Energy is constant ➡ Motion never decays



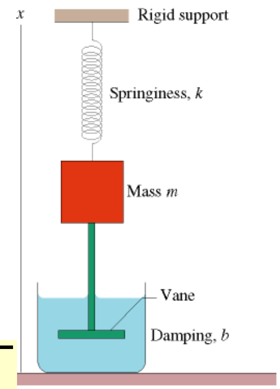
In real life the motion eventually stops

Friction

Air Resistance

….

Mechanical Energy → 0



$F_d = -bv$ Air resistance, etc.

Direction opposite to motion

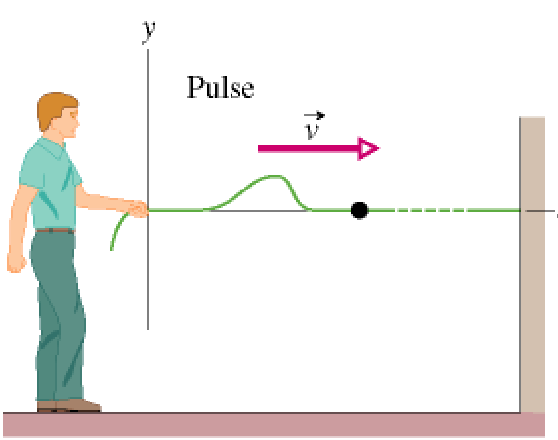
Magnitude proportional to velocity

$$x(t) = x_m e^{-bt/2m} \cos(\omega' t + \phi)$$

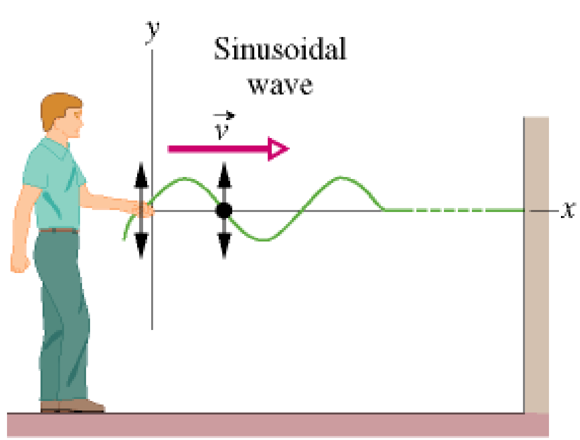
$$\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$$

WAVES

1. **Mechanical Waves**
2. Electromagnetic Waves $c = 299\ 792\ 458$ m/s (speed of light).
3. Material Waves (particles, like electrons, neutrons, *etc.*)

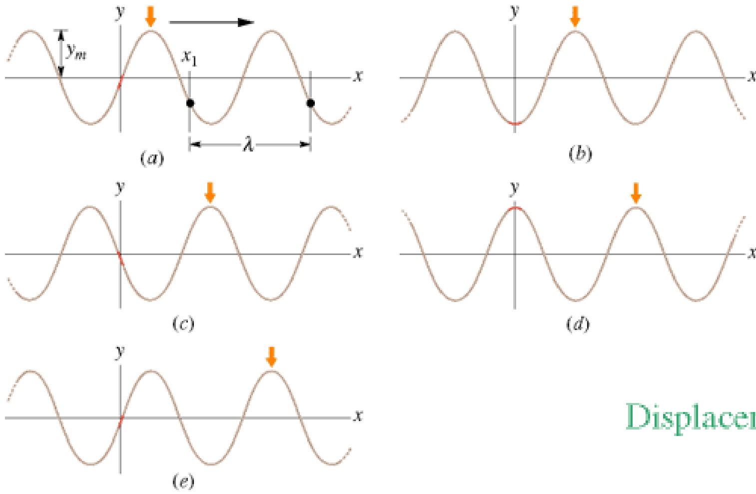


(a)



(b)

Transverse Sinusoidal Waves



$$k = \frac{2\pi}{\lambda} \qquad \text{(angular wave number).}$$

$$\omega = \frac{2\pi}{T} \qquad \text{(angular frequency).}$$

Amplitude

Displacement

Oscillating term

Phase

$$y(x,t) = y_m \sin(kx - \omega t)$$

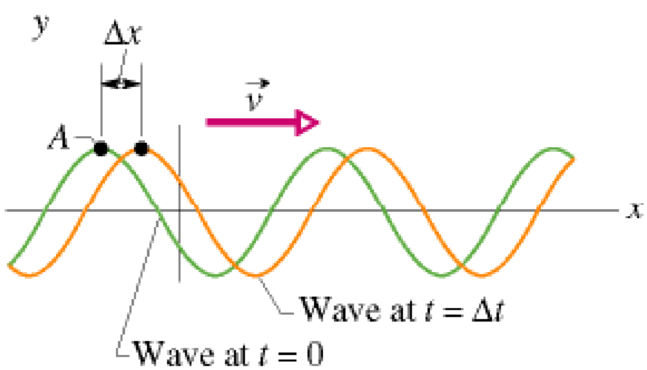
Angular wave number

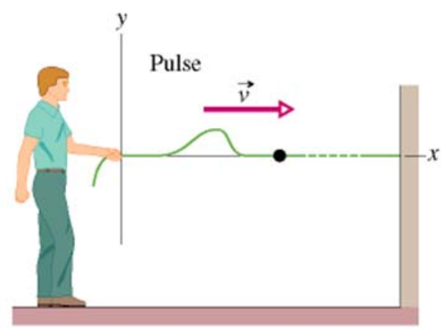
Position

Time

Angular frequency

The Speed of a Traveling Wave





$$v = \frac{\omega}{k} = \frac{\lambda}{T} = \lambda f \qquad \text{(wave speed).}$$

Wave Speed on a Stretched String

$$v = \sqrt{\frac{\tau}{\mu}} \quad \text{(speed)}, \qquad \mu = m/l$$

The speed of a wave along a stretched ideal string depends only on the tension and linear density of the string and not on the frequency of the wave.

Energy and Power of a Traveling String Wave

Chapter 17 - Microsoft Internet Explorer

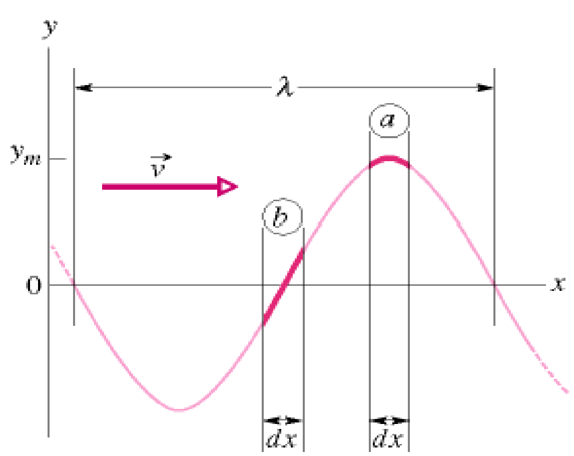
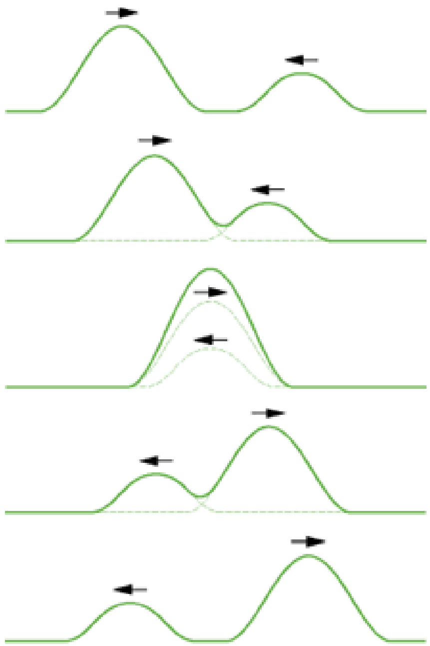


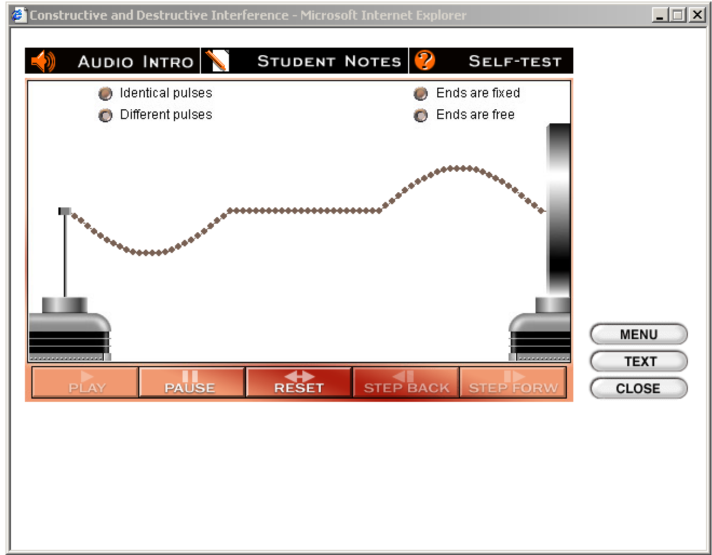
Fig. 17-11 A snapshot of a traveling wave on a string at time $t = 0$. String element a is at displacement $y = y_m$, and string element b is at displacement $y = 0$. The kinetic energy of the string element at each position depends on the transverse velocity of the element. The potential energy depends on the amount by which the string element is stretched as the wave passes through it.

$$P_{avg} = \tfrac{1}{2}\mu v \omega^2 y_m^2 \quad \text{(average power)}.$$

$$\Delta E = P_{avr} \times \Delta t$$

The Principle of Superposition for Waves





The Principle of Superposition for Waves

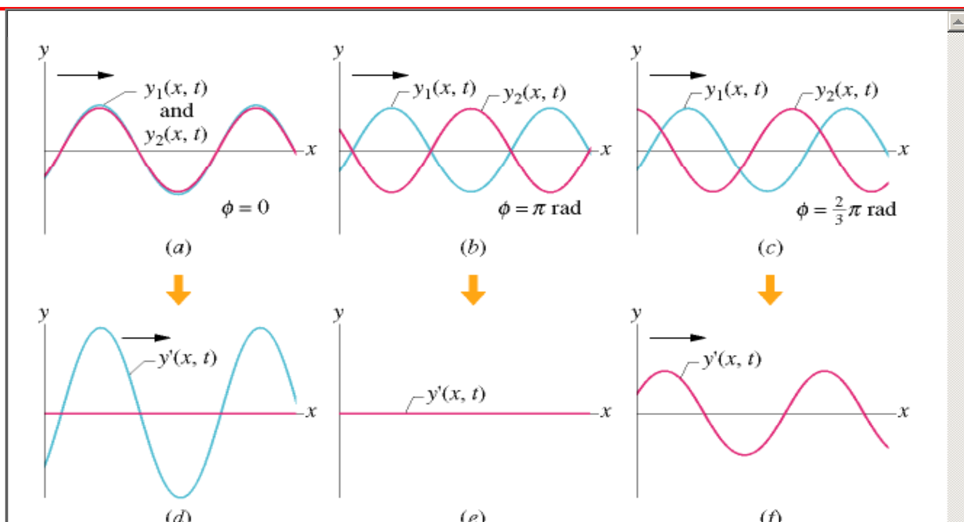


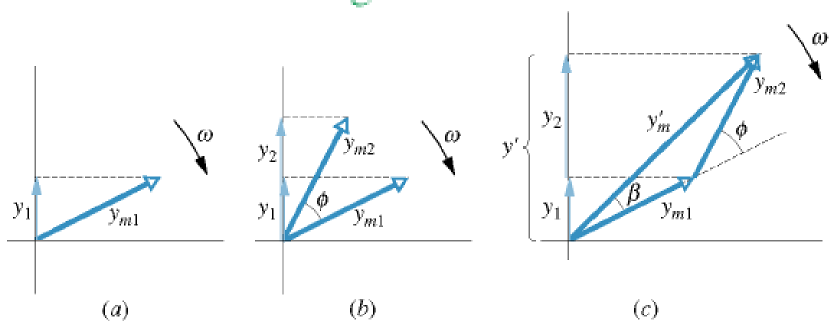
Fig. 17-14 Two identical sinusoidal waves, $y_1(x, t)$ and $y_2(x, t)$, travel along a string in the positive direction of an x axis. They interfere to give a resultant wave $y'(x, t)$. The resultant wave is what is actually seen on the string. The phase difference ϕ between the two interfering waves is (a) 0 rad or 0°, (b) π rad or 180°, and (c) $\frac{1}{2}\pi$ rad or 120°. The corresponding resultant waves are shown in (d), (e), and (f).

The Principle of Superposition for Waves

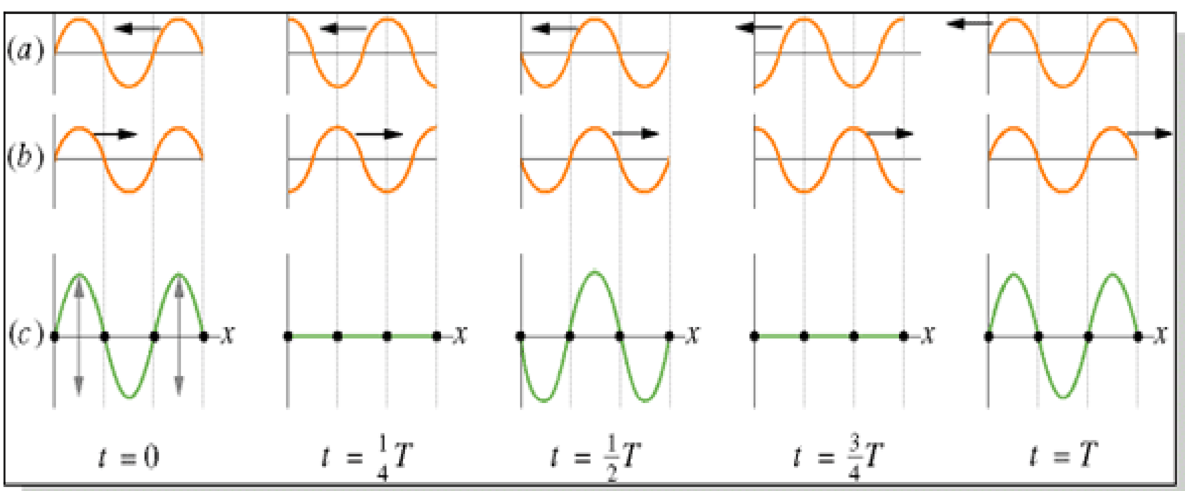
$$y_1(x,t) = y_m \sin(kx - \omega t)$$

$$y_2(x,t) = y_m \sin(kx - \omega t + \phi).$$

$$\overbrace{y'(x,t)}^{\text{Displacement}} = \underbrace{\left[2y_m \cos \tfrac{1}{2}\phi\right]}_{\text{Amplitude}} \underbrace{\sin(kx - \omega t + \tfrac{1}{2}\phi)}_{\text{Oscillating}}$$

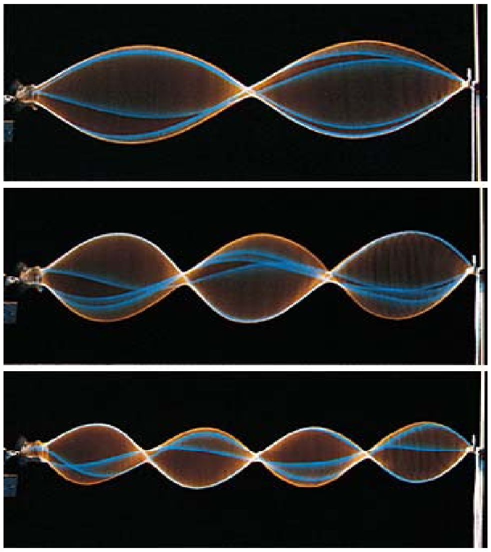


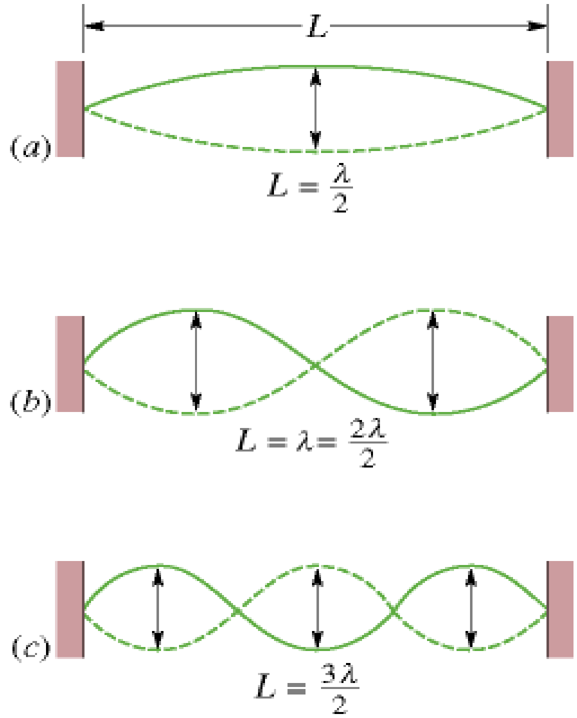
Standing Waves



$$\overbrace{y'(x,t)}^{\text{Displacement}} = \underbrace{[2y_m \sin kx]}_{\substack{\text{Amplitude} \\ \text{at position } x}} \underbrace{\cos \omega t}_{\substack{\text{Oscillating} \\ \text{term}}}$$

Standing Waves / Fixed ends

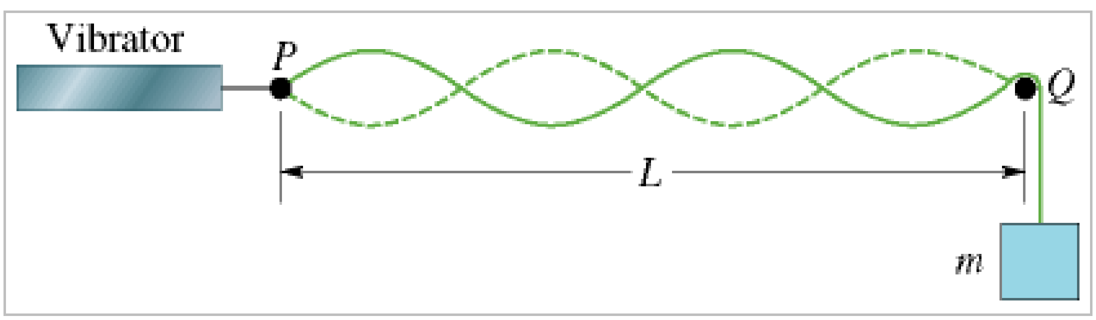




$$\lambda = \frac{2L}{n}, \qquad \text{for } n = 1, 2, 3, \ldots$$

$$f = \frac{v}{\lambda} = n\frac{v}{2L}, \qquad \text{for } n = 1, 2, 3, \ldots$$

Standing Waves / Fixed end / Resonance



$$\lambda = \frac{2L}{n}, \qquad \text{for } n = 1, 2, 3, \ldots \qquad v = \sqrt{\frac{\tau}{\mu}} = \sqrt{\frac{mg}{\mu}}.$$

$$f = \frac{v}{\lambda} = n\frac{v}{2L}, \qquad \text{for } n = 1, 2, 3, \ldots$$

SOUND WAVES:

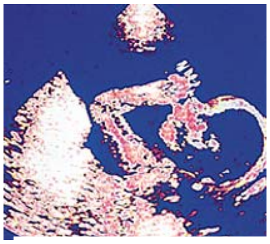
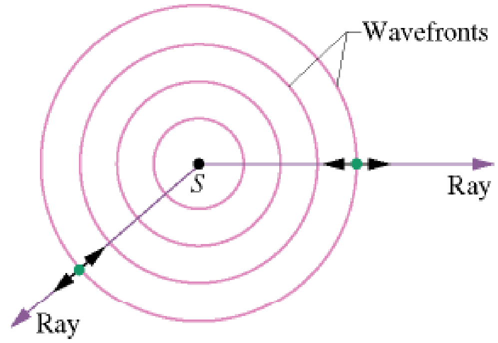


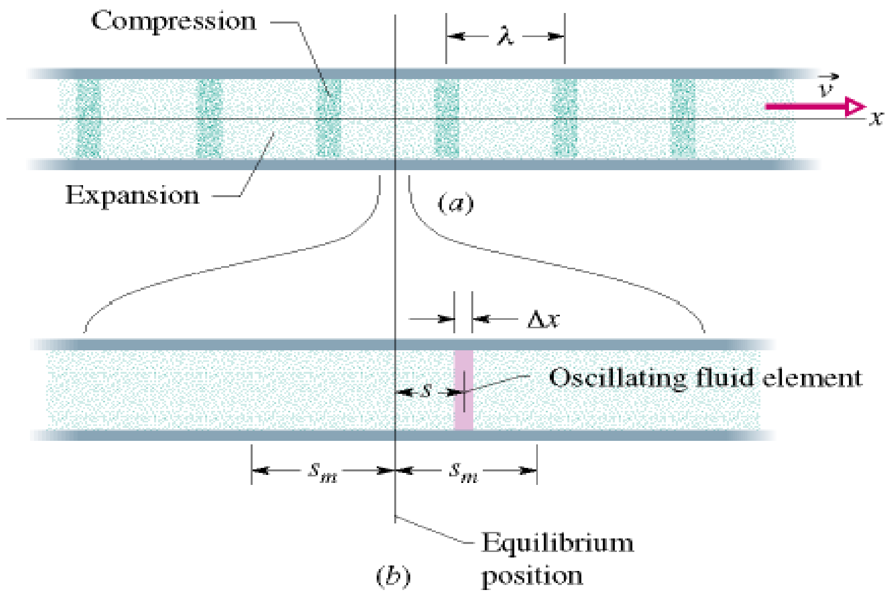
TABLE 18-1 **The Speed of Sound**[a]

Medium	Speed (m/s)
Gases	
Air (0°C)	331
Air (20°C)	343
Helium	965
Hydrogen	1284
Liquids	
Water (0°C)	1402
Water (20°C)	1482
Seawater[b]	1522
Solids	
Aluminum	6420
Steel	5941
Granite	6000



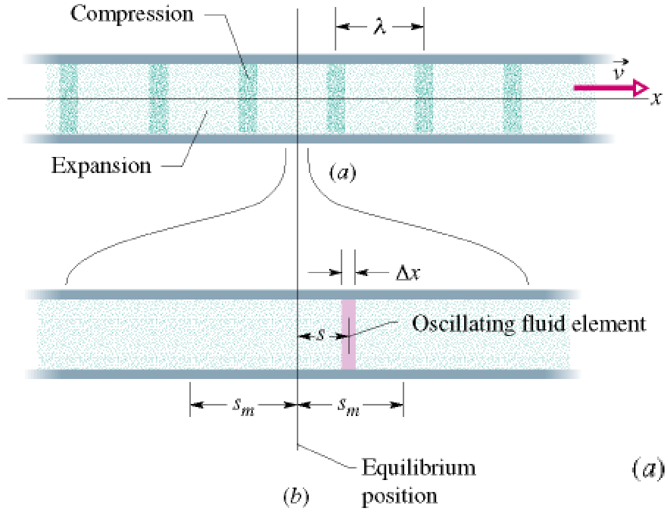
$$v = \sqrt{\frac{\tau}{\mu}} = \sqrt{\frac{\text{elastic property}}{\text{inertial property}}},$$

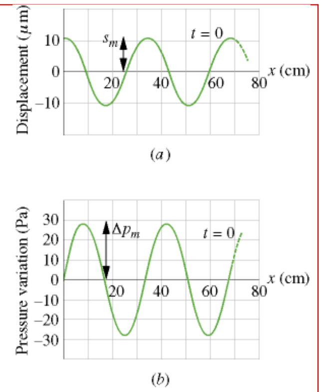
TRAVELLING SOUND WAVES:



A sound wave, traveling through a long air-filled tube with speed *v*, consists of a moving, periodic pattern of expansions and compressions of the air.

TRAVELLING SOUND WAVES:

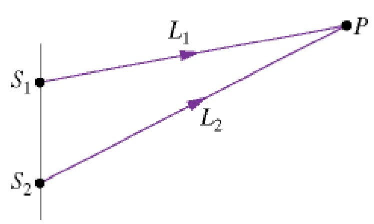




(a) $\underbrace{s(x,t)}_{\text{Displacement}} = \underbrace{s_m}_{\text{Amplitude}} \underbrace{\cos(kx - \omega t)}_{\text{Oscillating term}}$

(b) $\underbrace{\Delta p(x,t)}_{\text{Pressure variation}} = \underbrace{\Delta p_m}_{\text{Amplitude}} \underbrace{\sin(kx - \omega t)}_{\text{Oscillating term}}$

INTERFERENCE of the SOUND WAVES:



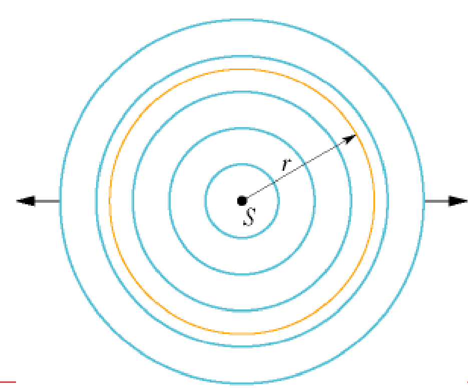
$$\frac{\Delta L}{\lambda} = 0, 1, 2, \ldots \qquad \text{(fully constructive interference).}$$

$$\frac{\Delta L}{\lambda} = 0.5, 1.5, 2.5, \ldots \qquad \text{(fully destructive interference).}$$

INTENSITY of the SOUND WAVES:

$$I = \frac{P_s}{4\pi r^2},$$

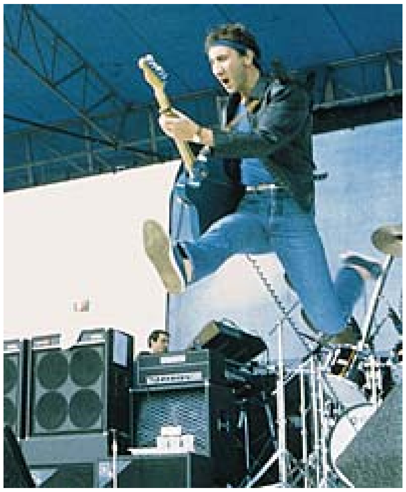
$$I = \tfrac{1}{2} \rho v \omega^2 s_m^2.$$

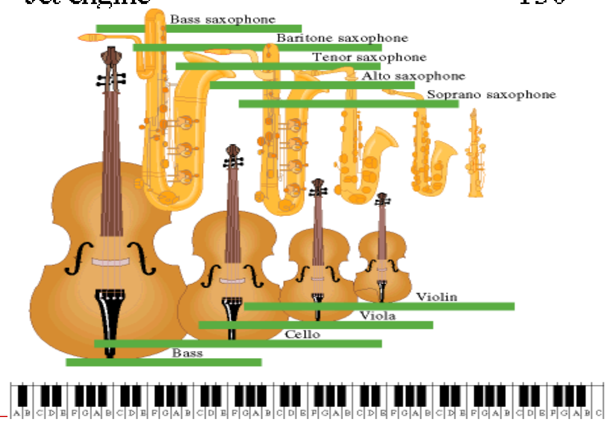


The Decibel Scale:

$$\beta = (10\ \text{dB}) \log \frac{I}{I_0}.$$

TABLE 18-2	Some Sound Levels (dB)
Hearing threshold	0
Rustle of leaves	10
Conversation	60
Rock concert	110
Pain threshold	120
Jet engine	130

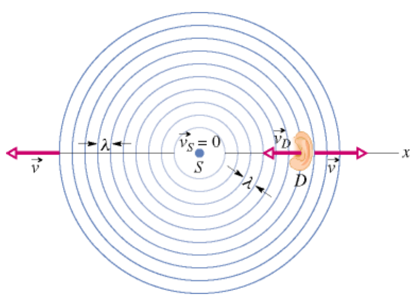


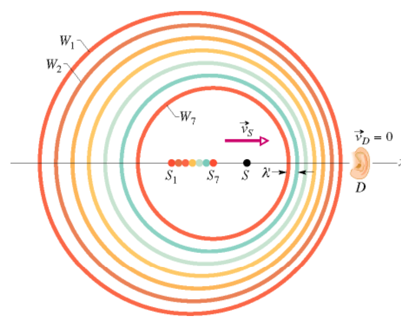


The Doppler Effect

$$f' = f\,\frac{v \pm v_D}{v \pm v_S}$$

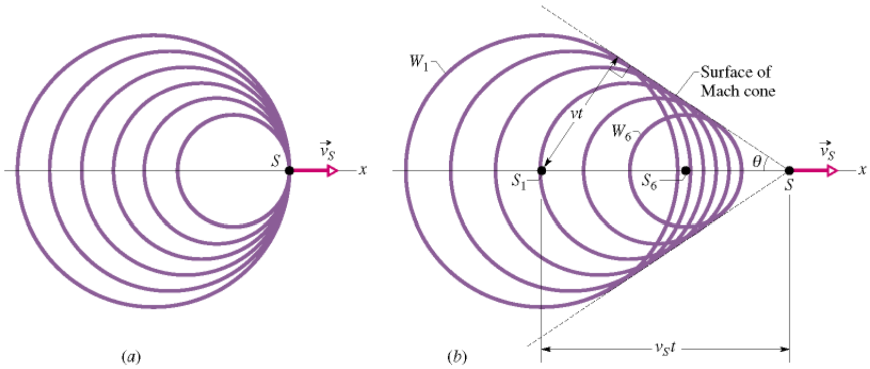
For speeds v less than the speed of sound (general Doppler effect),





When the motion of detector or source is toward the other, the sign on its speed must give an upward shift in frequency. When the motion of detector or source is away from the other, the sign on its speed must give a downward shift in frequency

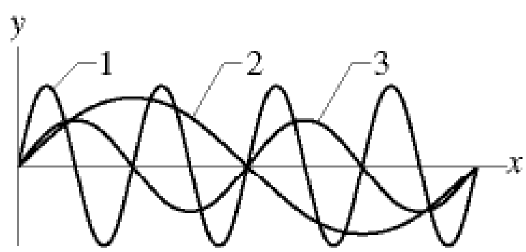
Supersonic Speeds; Shock Waves



$$\sin\theta = \frac{vt}{v_S t} = \frac{v}{v_S} \quad \text{(Mach cone angle).}$$

WAVES: QZ2

The figure is a composite of three snapshots, each of a wave traveling along a particular string. The phases for the waves are given by



(a) 2*x* - 4*t*,

(b) 4*x* - 8*t*, and

(c) 8*x* - 16*t*.

Which phase corresponds to which wave in the figure?

$$y(x,t) = y_m \sin(kx - \omega t).$$

$$k = \frac{2\pi}{\lambda} \quad \text{(angular wave number).}$$

Summary

Transverse and Longitudinal Waves

Mechanical waves can exist only in material media and are governed by Newton's laws. **Transverse** mechanical waves, like those on a stretched string, are waves in which the particles of the medium oscillate perpendicular to the wave's direction of travel. Waves in which the particles of the medium oscillate parallel to the wave's direction of travel are **longitudinal** waves.

Sinusoidal Waves

A sinusoidal wave moving in the positive x direction has the mathematical form

$$y(x,t) = y_m \sin(kx - \omega t), \tag{17-2}$$

where y_m is the **amplitude** of the wave, k is the **angular wave number**, ω is the **angular frequency**, and $kx - \omega t$ is the **phase.** The **wavelength** λ is related to k by

$$k = \frac{2\pi}{\lambda}. \tag{17-5}$$

The **period** T and **frequency** f of the wave are related to ω by

$$\frac{\omega}{2\pi} = f = \frac{1}{T}. \tag{17-9}$$

Finally, the **wave speed** v is related to these other parameters by

$$v = \frac{\omega}{k} = \frac{\lambda}{T} = \lambda f. \tag{17-12}$$

Summary

Equation of a Traveling Wave

Any function of the form

$$y(x,t) = h(kx \pm \omega t) \tag{17-16}$$

can represent a **traveling wave** with a wave speed given by Eq. 17-12 and a wave shape given by the mathematical form of h. The plus sign denotes a wave traveling in the negative x direction, and the minus sign a wave traveling in the positive x direction.

Wave Speed on Stretched String

The speed of a wave on a stretched string is set by properties of the string. The speed on a string with tension τ and linear density μ is

$$v = \sqrt{\frac{\tau}{\mu}}. \tag{17-25}$$

Power

The **average power**, or average rate at which energy is transmitted by a sinusoidal wave on a stretched string, is given by

$$P_{avg} = \tfrac{1}{2}\mu v \omega^2 y_m^2. \tag{17-32}$$

Summary

Superposition of Waves

When two or more waves traverse the same medium, the displacement of any particle of the medium is the sum of the displacements that the individual waves would give it.

Interference of Waves

Two sinusoidal waves on the same string exhibit **interference**, adding or canceling according to the principle of superposition. If the two are traveling in the same direction and have the same amplitude y_m and frequency (hence the same wavelength) but differ in phase by a **phase constant** ϕ, the result is a single wave with this same frequency:

$$y'(x,t) = \left[2y_m \cos\tfrac{1}{2}\phi\right]\sin(kx - \omega t + \tfrac{1}{2}\phi). \qquad (17\text{-}38)$$

If $\phi = 0$, the waves are exactly in phase and their interference is fully constructive; if $\phi = \pi$ rad, they are exactly out of phase and their interference is fully destructive.

Summary

Standing Waves

The interference of two identical sinusoidal waves moving in opposite directions produces **standing waves.** For a string with fixed ends, the standing wave is given by

$$y'(x,t) = \left[2y_m \sin kx\right]\cos \omega t. \qquad (17\text{-}47)$$

Standing waves are characterized by fixed locations of zero displacement called **nodes** and fixed locations of maximum displacement called **antinodes.**

Resonance

Standing waves on a string can be set up by reflection of traveling waves from the ends of the string. If an end is fixed, it must be the position of a node. This limits the frequencies at which standing waves will occur on a given string. Each possible frequency is a **resonant frequency,** and the corresponding standing wave pattern is an **oscillation mode.** For a stretched string of length L with fixed ends, the resonant frequencies are

$$f = \frac{v}{\lambda} = n\frac{v}{2L}, \qquad \text{for } n = 1, 2, 3, \ldots \qquad (17\text{-}53)$$

The oscillation mode corresponding to $n = 1$ is called the *fundamental mode* or the *first harmonic*; the mode corresponding to $n = 2$ is the *second harmonic*; and so on.

Summary

Sound Waves

Sound waves are longitudinal mechanical waves that can travel through solids, liquids, or gases. The speed v of a sound wave in a medium having **bulk modulus** B and density ρ is

$$v = \sqrt{\frac{B}{\rho}} \quad \text{(speed of sound)}. \tag{18-3}$$

In air at 20°C, the speed of sound is 343 m/s.

A sound wave causes a longitudinal displacement s of a mass element in a medium as given by

$$s = s_m \cos(kx - \omega t), \tag{18-13}$$

where s_m is the **displacement amplitude** (maximum displacement) from equilibrium, $k = 2\pi/\lambda$, and $\omega = 2\pi f$, λ and f being the wavelength and frequency, respectively, of the sound wave. The sound wave also causes a pressure change Δp of the medium from the equilibrium pressure:

$$\Delta p = \Delta p_m \sin(kx - \omega t), \tag{18-14}$$

where the **pressure amplitude** is

$$\Delta p_m = (v\rho\omega)s_m. \tag{18-15}$$

Summary

Interference

The interference of two sound waves with identical wavelengths passing through a common point depends on their phase difference ϕ there. If the sound waves were emitted in phase and are traveling in approximately the same direction, ϕ is given by

$$\phi = \frac{\Delta L}{\lambda} 2\pi, \tag{18-21}$$

where ΔL is their **path length difference** (the difference in the distances traveled by the waves to reach the common point). Fully constructive interference occurs when ϕ is an integer multiple of 2π,

$$\phi = m(2\pi), \quad \text{for } x = 0, 1, 2, \ldots, \tag{18-22}$$

and, equivalently, when ΔL is related to wavelength λ by

$$\frac{\Delta L}{\lambda} = 0, 1, 2, \ldots. \tag{18-23}$$

Fully destructive interference occurs when ϕ is an odd multiple of π,

$$\phi = (2m+1)\pi, \quad \text{for } m = 0, 1, 2, \ldots, \tag{18-24}$$

and, equivalently, when ΔL is related to λ by

$$\frac{\Delta L}{\lambda} = 0.5, 1.5, 2.5, \ldots. \tag{18-25}$$

Summary

Sound Intensity

The **intensity** I of a sound wave at a surface is the average rate per unit area at which energy is transferred by the wave through or onto the surface:

$$I = \frac{P}{A}, \qquad (18\text{-}26)$$

where P is the time rate of energy transfer (power) of the sound wave and A is the area of the surface intercepting the sound. The intensity I is related to the displacement amplitude s_m of the sound wave by

$$I = \tfrac{1}{2}\rho v \omega^2 s_m^2. \qquad (18\text{-}27)$$

The intensity at a distance r from a point source that emits sound waves of power P_s is

$$I = \frac{P_s}{4\pi r^2}. \qquad (18\text{-}28)$$

Sound Level in Decibels

The *sound level* β in *decibels* (dB) is defined as

$$\beta = (10\ \text{dB}) \log \frac{I}{I_0}, \qquad (18\text{-}29)$$

where I_0 (= 10^{-12} W/m^2) is a reference intensity level to which all intensities are compared. For every factor-of-10 increase in intensity, 10 dB is added to the sound level.

Summary

The Doppler Effect

The *Doppler effect* is a change in the observed frequency of a wave when the source or the detector moves relative to the transmitting medium (such as air). For sound the observed frequency f' is given in terms of the source frequency f by

$$f' = f\,\frac{v \pm v_D}{v \pm v_S} \quad \text{(general Doppler effect)}, \qquad (18\text{-}47)$$

where v_D is the speed of the detector relative to the medium, v_S is that of the source, and v is the speed of sound in the medium. The signs are chosen such that f' tends to be *greater* for motion (of detector or source) "toward" and *less* for motion "away."

Shock Wave

If the speed of a source relative to the medium exceeds the speed of sound in the medium, the Doppler equation no longer applies. In such a case, shock waves result. The half angle θ of the Mach cone is given by

$$\sin\theta = \frac{v}{v_S} \quad \text{(Mach cone angle)}. \qquad (18\text{-}57)$$