Chapter One

Logic

1.1 Propositions and Truth Values : العبارات والقيم الصحيحة

A proposition is declarative statement which is either true or false, but not both. (propositions are sometimes called '**statements** ') .

Examples : -

1. Triangles have four vertices .

2. 6 + 2 = 4 .

3. 5 < 24 .

The truth (**T**) or falsity (**F**) of a proposition is called **Truth Value** . proposition 3 has a truth value of true (T) , and propositions 1&2 have truth values of false (F) .

*Questions & demands are not propositions, since they can not be declared true or false .Thus the following are not propositions:

4. Keep off the cat.

5. Did you go to party?

6. Don't say that.

Sentences 4 – 6 are not propositions and therefore cannot be assigned truth values .

* Propositions are denoted using the letters p , q ,r , … . Any of these letters may be used to symbolize specific propositions.

***Compound proposition :**

A compound proposition is statement formed by connecting two or more statement, or by negating a simple proposition.

1 .2 Logical connectives :

1) Negation : (~) النفي

If p is any proposition , the negation of p denoted by ~p (or p or ¬ p) . And it's a proposition which is false when p is true, and true when p is false.

- We can summarize this in a table ,

p	~ p
T	F
F	T

2) Conjunction : (And) (Λ)   أداة الربط (و)

Let p & q be any two propositions , the compound proposition is called conjunction of p & q . And denoted by (p Λ q) .

The following table gives the truth values of p Λ q :

P	Q	p Λ q
T	T	T
T	F	F
F	T	F
F	F	F

From the table it can be seen that the conjunction p Λ q is true only when p and q are both true . Otherwise the conjunction is false .

3) Disjunction : (or) (V)   أداة الربط (أو)

Let p & q be any two propositions , compound proposition is called disjunction of p &q . And it's denoted by (p V q) .

The following table gives the truth value of (p V q) :

P	Q	p V q
T	T	T
T	F	T
F	T	T
F	F	F

From the previous table , one can notice that p V q is true when either or both of it's components are true and it's false otherwise.

4) Conditional Propositions : ($\rightarrow$) اذا كان...فان

The conditional connective (sometimes Called implication) is denoted by $\rightarrow$. And reed as if p then q, for any two propositions p & q .

The following is the truth table for $p \rightarrow q$:

P	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

* (marked row: T F F)

Notice that " the proposition " if p then q " is false only when p is true and q is false . i .e , a true statement can not imply a false one .

5) Biconditional Propositions :($\leftrightarrow$) (if and only if)

The biconditional connective is أداة الربط إذا وفقط إذا denoted by $\leftrightarrow$. and expressed by " if and only if …. Then … ". The truth table of $p \leftrightarrow q$ is ,

P	**q**	**p ↔ q**
T	T	T
T	F	F
F	T	F
F	F	T

Note that for p ↔ **q** to be true , when p and q must both have the same truth value . i . e , both must be true or both must be false .

Examples : -

A B

1. Construct a truth table for (q V p) Λ (~ p V ~ q) .

p	q	~ p	~ q	q V p	(~ p V ~ q)	A Λ B
T	T	F	F	T	F	F
T	F	F	T	T	T	T
F	T	T	F	T	T	T
F	F	T	T	F	T	F

2. Construct a truth table for (~ q Λ p) V (~q V ~ p) Λp.

A B

C

p	q	~ p	~ q	~ q Λ p	(~qV~p)	A VB	C Λ P
T	T	F	F	T	F	F	F
T	F	F	T	T	T	T	T
F	T	T	F	T	T	T	F
F	F	T	T	F	T	F	F

3. Construct a truth table for :

a) $\sim q \rightarrow p$. b) $\sim p \leftrightarrow \sim q$. c) $p \rightarrow (q \wedge r)$. d) $(\sim p \vee q) \leftrightarrow \sim r$.

a)

p	q	~q	~q→p
T	T	F	T
T	F	T	T
F	T	F	T
F	F	T	F

b)

p	q	~p	~q	~p↔~q
T	T	F	F	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

c)

P	q	r	q∧r	p→(q∧r)
T	T	T	T	T
T	T	F	F	F
T	F	T	F	F
T	F	F	F	F
F	T	T	T	T
F	T	F	F	T
F	F	T	F	T
F	F	F	F	T

d)

p	Q	r	~p	~r	~p Vq	(~p Vq)↔ ~r
T	T	T	F	F	T	F
T	T	F	F	T	T	T
T	F	T	F	F	F	T
T	F	F	F	T	F	F
F	T	T	T	F	T	F
F	T	F	T	T	T	T
F	F	T	T	F	T	F
F	F	F	T	T	T	T

نلاحظ هنا في المثالين c و d يوجد ثلاث متغيرات وهي p , q , r لذا يكون عدد الاحتمالات ثمانية ، حسب القاعدة :

(عدد المتغيرات)
عدد الاحتمالات = 2

$2^2 = 4$ → (b) , (a) أربع احتمالات كما في

$2^3 = 4$ → (d) , (c) ثماني احتمالات كما في

Exercises :

1) Draw the truth tables for the proposition :

1. ~p → q. 2. ~q Λ p. 3. (p V q) → (p Λ q) .

4. ~p↔ (p Λ q) .

2) Given the propositions . p , q & r , construct the truth tables for :

1. (p Λ q) → ~r . 2. p Λ (~q V r) . 3. ~ (p V q) ↔ (r V p).

1.3 Tautologies and Contradictions :

Definitions:

1) A **tautology** is a compound proposition which is true no matter what the truth values of it's simple components.

2) A **contradiction** is a compound proposition which is false no matter what the truth values of it's simple components.

* We shall denote a tautology by t and a contradiction by f.

Examples :

1) Show that $p \vee \overline{p}$ is a tautology ?

Sol :

Constructing the truth table for $p \vee \overline{p}$, we have :

p	$\overline{p}$	$p \vee \overline{p}$
T	F	T
F	T	T

Note that $p \vee \overline{p}$ is always true (no matter what proposition is substituted for p) and is therefore a Tautology .

2) Show that $(\overline{p \wedge q})$ is tautology.

Sol :

The truth table of $(p \wedge q) \vee (\overline{p \wedge q})$ is given below :

p	q	$p \wedge q$	$\overline{p \wedge q}$	$(p \wedge q) \vee (\overline{p \wedge q})$
T	T	T	F	T
T	F	F	T	T
F	T	F	T	T
F	F	F	T	T

The last column of the truth table contains only the truth value T and hence we can deduce that $(p \wedge q) \vee (\overline{p \wedge q})$ is a Tautology.

3) Show that $(p \wedge \overline{q}) \wedge (\overline{p} \vee q)$ is a contradiction .'

Sol :

P	q	$\overline{p}$	$\overline{q}$	$p \wedge \overline{q}$	$p \vee q$	$(p \wedge \overline{q}) \wedge (\overline{p} \vee q)$
T	T	F	F	F	T	F
T	F	F	T	T	F	F
F	T	T	F	F	T	F
F	F	T	T	F	T	F

The last column shows that $(p \wedge \overline{q}) \wedge (\overline{p} \vee q)$ is always false , no matter what the truth values of p & q .

Hence $(p \wedge \overline{q}) \wedge (\overline{p} \vee q)$ is a contradiction .

Exercises:

Determine whether each of the following is a tautology, a contradiction or neither:

1) $p \rightarrow (p \vee q)$. 2) $(p \rightarrow q) \land (\overline{p} \vee q)$. 3) $(p \land q) \land \overline{(p \vee q)}$.

1.4 Logical Equivalence: التكافؤ المنطقي

Two propositions are said to be <u>logically equivalent</u> if they have the same truth values . Using P and Q to denote (possibly) compound propositions , we write $P \equiv Q$ if P&Q are logically equivalent .

Example :- Show that $\bar{p} \vee \bar{q}$ and $\overline{p \land q}$ are logically equivalent . i .e , that $(\bar{p} \vee \bar{q}) \equiv (\overline{p \land q})$.

<u>Sol :</u>

p	Q	$\bar{p}$	$\bar{q}$	$\bar{p} \vee \bar{q}$	$p \land q$	$\overline{p \land q}$
T	T	F	F	F	T	F
T	F	F	T	T	F	T
F	T	T	F	T	F	T
F	F	T	T	T	F	T

متكافئة

Comparing the columns for $\bar{p} \vee \bar{q}$ & $\overline{p \land q}$ we not that the truth values are the same . Hence , $\bar{p} \vee \bar{q}$ & $\overline{p \land q}$ are logically equivalent.

Exercises :

1. Prove that $(p \rightarrow q) \equiv (\bar{p} \vee q)$.
2. Prove that $(p \wedge q)$ and $\overline{(p \rightarrow q)}$ are logically equivalent propositions.
3. Show that the biconditional proposition $p \leftrightarrow q$ is logically equivalent to the conjunction of the two conditional propositions $p \rightarrow q$ and $q \rightarrow p$.

1.5 The Algebra of propositions: الجبر القضائي

The following is a lit of some important logical equivalences , all in which can be verified using one of the techniques described in (1.4) .

These laws hold for any simple propositions p , q and r .

*** Idempotent laws: قوانين الجمود**

$P \wedge P \equiv P$

$P \vee P \equiv P$

*** Commutative laws: قوانين الإبدال**

$p \wedge q \equiv q \wedge p$

$p \vee q \equiv q \vee p$

*** Associative laws: قوانين التجميع**

$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$.

$(p \vee q) \vee r \equiv p \vee (q \vee r)$.

*** Distributive laws: قوانين التوزيع**

$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$.

$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$.

*** Involution law: قانون الالتفاف**

$\overline{\overline{P}} \equiv P.$

*** De Morgan's law: قانون دي موركان**

$\overline{p \vee q} \equiv \overline{p} \wedge \overline{q}$

$\overline{p \wedge q} \equiv \overline{p} \vee \overline{q}$

*** Complement laws: قوانين الإكمال (المتممة)**

$P \vee \overline{P} \equiv t$

$P \wedge \overline{P} \equiv f$

$\overline{f} \equiv t$

$\overline{t} \equiv f$

where ***t*** = tautology and ***f*** = contradiction.

*** Absorption laws : قوانين الاختزال**

$P \wedge (p \vee q) \equiv p .$

$P \vee (p \wedge q) \equiv p .$

*** Identity laws:**

$P \vee f \equiv p.$

$P \wedge t \equiv p.$

$P \vee t \equiv t.$

$P \wedge f \equiv f.$

Example :

Prove that $(\bar{p} \wedge q) \vee (\overline{p \vee q}) \equiv \bar{p}$.

Sol :

$$
\begin{aligned}
(\bar{p} \wedge q) \vee (\overline{p \vee q}) &\equiv (\bar{p} \wedge q) \vee (\bar{p} \vee \bar{q}) && (\text{De Morgan law}) \\
&\equiv \bar{p} \wedge (q \vee \bar{q}) && (\text{Distributive law}) \\
&\equiv \bar{p} \wedge t && (\text{complement law}) \\
&\equiv \bar{p} && (\text{identity law})
\end{aligned}
$$

Exercises : prove each of the following logical equivalences :

1. $(p \wedge p) \vee (\bar{p} \vee \bar{p}) \equiv t$.
2. $(p \wedge q) \wedge q \equiv p \wedge q$.
3. $\bar{p} \wedge (\overline{p \wedge q}) \equiv \bar{p}$.
4. $p \wedge [(p \vee q) \vee (p \vee r)] \equiv p$
5. $q \wedge \left[(p \vee q) \wedge (\overline{\bar{q} \wedge \bar{p}}) \right] \equiv q$

1.6 Mathematical Induction: الاستقراء الرياضي

Let S(n) be a proposition concerning a positive integer n. If:

a) P (1) is true , and

b) For every $k \geq 1$, the truth of P(k) implies the truth of P(k + 1) , then P(n) is true for all positive integers n.

Example(1): prove that by Mathematical Induction , $1 + 3 + 5 \ldots + (2n - 1) = n^2$, is true for all $n \geq 1$.

Sol :

1) first step : show that P(1) is true ,

$P(1) = (2(n) - 1) = n^2$

$= (2(1) - 1) = 1^2$

$1 = 1 \quad \therefore P(1)$ is true .

2) Assume P(k) is true for some $k \geq 1$,

$1 + 3 + 5 + \ldots + (2k - 1) = k^2$

3) To prove P(k + 1) is true ,

$1 + 3 + 5 \ldots + (2k - 1) + (2k + 1) = (k + 1)^2 \ldots\ldots (*)$

Now , notice the left hand side ,

$1 + 3 + 5 \ldots + (2k - 1) = k^2$, if we sub . in (*) we get ,

$K^2 + 2k + 1 = (k + 1)^2$

$\because k^2 + 2k + 1 = (k + 1)^2$ إكمال مربع

$\therefore$ P (k + 1) is true , since the L.H.S equals the R.H.S .

Example (2) : By Mathematical Induction , show that for all $n \geq 1$, $1 + 2 + \ldots + n = n \; \frac{n(n+1)}{2}$.

Sol :

1) P(1) is true ?

$P(1) = n = \frac{n(n+1)}{2}$

$\rightarrow 1 = \frac{1(1+1)}{2}$

$\rightarrow 1 = \frac{2}{2} = 1$

$\therefore$ P (1) is true for n =1

2) Assume P(k) is true for some k ≥ 1

Since , $1 + 2 + \dots + k = \dfrac{k(k+1)}{2}$, sub in (*)

L.H.S R.H.S

$$\frac{k(k+1)}{2} + k + 1 = \frac{(k+1)(k+2)}{2}$$

Let's work on the L.H.S , we get ,

$$\frac{k(k+1)+2(k+1)}{2}$$

$$= \frac{k^2+k+2k+2}{2}$$

$$= \frac{k^2+3k+2}{2}$$

Now the R.H.S ,

$$\frac{(k+1)(k+2)}{2} = \frac{k^2+2k+k+2}{2}$$

$$= \frac{k^2+3k+2}{2}$$

$\therefore$ L.H.S = R.H.S → P(k+1) is true .

Chapter Two

Sets and Subsets

2.1 Sets:

A set is to be thought of as any collection of objects whatsoever . The object can also be anything and they are called elements of the set .

The elements contained in a given set need not have anything in common ((other than the obvious common attribute that they all belong to the given set)) , there is no restriction on the number of elements allowed in a set ; there may be an infinite number , a finite number or even no elements at all .

Examples (1) :

1. A set could be defined to contain Picasso , the Babylon Tower and the number π. This is a finite set .
2. The set containing all the positive, even integers is clearly an infinite set.

Notations :

1. We shall generally use upper – case letters to denote sets and lower – case letters to denote elements.

2. The symbol $\in$ denotes ' **belongs to** ' or ' is an element of '.

- Thus $a \in A$ means (the element) ***a*** belongs to (the set) **A**. And $a \notin A$ means ***a*** does not belong to **A**.

3. Sets can be defined is various ways :

a) The simplest is by listing (**Enumerate**) it's elements, for example A = $\{1,2,3,4,5\}$ defines the set consisting of the first five positive integers, the order in which the elements are listed is not important.

c) The other way has the form A = $\{x : P(x)\}$, which read as " the set of all x such that P(x) is true " . Thus, A={***x:x is an integer and 1≤ x ≤5***}

*** Finite Set:** A set is said to be finite if it's consist of exactly (n) elements where (n) is some positive integers , otherwise it's infinite .

Example (2) :

1) A = $\{x : x \geq 5\} \rightarrow$ A = $\{5,6,7,\ldots..\}$ infinite set.

2) B = $\{x : x - 1 = 0\} \rightarrow$ B = $\{1\}$ finite set .

* Null Set (Empty Set): المجموعة الخالية

The null set or (empty set) , which contains no elements . And it's denoted by ϕ or $\{\ \}$.

Example (3) :

$$\phi = \{x : x, is\, a\, green\, rabbit\}.$$

* Equality of sets : تساوي المجموعات

Two sets are said to be equal if and only if the contain the same elements; that is,

A = B if $\forall$ x $[x \in A \leftrightarrow x \in B]$ is a true proposition and conversely.

Example (4) :

A = $\{1,-12,1000,\pi\}$, **B** = $\{-12,\pi,1000,1\}$

And **C** = $\{1,-12,-12,\pi,1000,-12,1\}$.

We should note here that the order and repeating elements in a set is not important. Thus,

A = B = C.

* The Cardinality: عدد عناصر المجموعة

If A is a finite set it's cardinality, $|A|$, is the number of (distinct) elements which the set contains .

If A has an infinite number of elements , we say it has infinite cardinality , and write $|A| = \infty$.

Examples (5) :

1. $\phi \rightarrow |\phi| = 0$, since ϕ contains no elements .
2. A = $\{\phi\} \rightarrow |A| = 1$, * بالنسبة للمجموعة الخالية تعتبر عنصر واحد اذا كانت عنصر لمجموعة اخرى
3. A = $\{\pi, 2, Ali\} \rightarrow |A| = 3$.
4. If X = $\{0, 1..., n\} \rightarrow |X| = n + 1$.
5. A = $\{2, 4, 6, ...\} \rightarrow |A| = \infty$.
6. Let A = $\{1, \{1,2\}\}$.

   * نلاحظ هنا انه A يحتوي على عنصرين العدد 1 والمجموعة $\{1,2\}$ لذا عدد عناصر المجموعة هو ٢ .

   $\rightarrow |A| = 2$.

7. Similarly ,

   a) $|\{1, 2, \{1,2\}\}| = 3$.

b) $|\{\phi, \{1,2\}\}| = 2$

c) $|\{\phi, \{\phi\}\}| = 2$.

d) $|\{\phi, \{\phi\}, \{1,2\}\}| = 3$.

e) $|\{\phi, \{\phi, \{\phi\}\}\}| = 2$.

Exercises (2.1) :

1. List the elements of each of the following sets, using the '… ' notion where necessary :

a) $\{x : x.is.aniteger , and -3 < x < 4\}$.

b) $\{x : (3x-1)(x+2) = 0\}$.

c) $\{x : x \geq 0 and (3x-1)(x+2) = 0\}$.

d) $\{x : x.is.an\,\text{int}\,eger, and (3x-1)(x+2) = 0\}$

2. Let X $= \{0,1,2\}$. List the elements of each of the following sets :

a) $\{z : z = 2x \,\&\, x \in X\}$.

b) $\{z : z = x + y.where.x \in X \,\&\, y \in X\}$.

c) $\{z : z \in X.or - z \in X\}$.

d) $\{z : z^2 \in X\}$.

e) $\{z : z\, is\, an\, \text{integer}, and\ z^2 \in X\}$.

3. Determine the cardinality of each of the following :

a) $\{x : x, is\ an\ \text{integer}, and\ 1/8 < x < 17/2\}$

b) $\{a, b, c\{a, b, c\}\}$.

c) $\{a, \{b, c\}, \{a, b, c\}\}$.

d) $\{\{a, b, c\}, \{a, b, c\}\}$.

e) $\{a, \{a\}, \{\{a\}\}, \{\{\{a\}\}\}\}$.

2.2 Subsets:

The set **A** is said to be a subset of the set **B** , if every element of A is also an element of B , and denoted by $A \subseteq B$. symbolicall,

$A \subseteq B$ if $\forall$ x $\{x \in A \rightarrow x \in B\}$. Is true.

$\in$ or $\notin$ *علاقة العنصر بالمجموعة هي

$\subseteq$ or $\not\subset$ * علاقة المجموعة بالمجموعة هي

Examples :

1. A = $\{1,2,3,5\}$ & B = $\{2,1,3,5\}$

$\therefore$ A $\subseteq$ B

But,

If A = $\{1,2,4\}$ & B = $\{2,1,3,5\}$

A $\not\subset$ B

2. Let X = $\{1, \{2,3\}\} \rightarrow \{1\} \subseteq$ X but ,

$\{2,3\} \not\subset$ X , However , $\{2,3\}$ is an element of X , so $\{\{2,3\}\} \subseteq$ X .

* Proper Subset :

If $A \subseteq B$ but $A \neq B$ then we say A is a proper subset of B and denoted by $A \subset B$.

ملاحظات :

- كل مجموعة هي مجموعة جزئية من نفسها . $(B \subseteq B)$.

- المجموعة الخالية (ϕ) هي جزئية من كل مجموعة مثلاً $(\phi \subseteq A)$

Exercises :

1. State whether each of the following statements is true or false:

a) $2 \in \{1,2,3,4,5\}$.

b) $\{2\} \in \{1,2,3,4,5\}$.

c) $2 \subseteq \{1,2,3,4,5\}$.

d) $\{2\} \subseteq \{1,2,3,4,5\}$.

e) $\phi \subseteq \{\phi, \{\phi\}\}$.

f) $0 \in \phi$.

g) $\{1,2,3,4,5\} = \{5,4,3,2,1\}$.

2. list all the subsets of :

a) $\{a,b\}$. b) $\{a,b,c\}$. c) $\{a\}$.

2.3 Operations On Sets :

Given two sets A & B :

1. The intersection of sets : ($\cap$) التقاطع

$A \cap B = \{x : x \in A \,\&\, x \in B\}$

Is the set of all elements which belong to both A and B. and it's denoted by $A \cap B$**.**

Example :

$A = \{2,3,5\}$ & $B = \{1,6,2\}$

$\rightarrow A \cap B = \{2\}$.

2. The Union of sets : ($\cup$)

$A \cup B = \{x : x \in A, or\ x \in B\ or\ both\}$

Is the set of all elements which belong to A or B or both. and it's denoted by $A \cup B$.

Example :

$A = \left\{\frac{1}{3}, 2, 6\right\}$ & $B = \{2,1,10\}$

$A \cup B = \left\{\frac{1}{3}, 1, 2, 6, 10\right\}$.

* نلاحظ عدم التكرار .

Example :

$A = \{4\}$ & $B = \{3,5,6\}$

$A \cap B = \phi$ * لا توجد عناصر مشتركة

3. The Complement of sets : المتممة

It consists of all those elements in U , that is not belong to A . and it's denoted by

($\bar{A}$ or A^c or A').

$$A^c = \{x : x \in U, and\ x \notin A\}$$

*** U represents the Universal set . (المجموعة الشاملة)**

Examples:

(1) Le $U = \{1,2,3,4,5,6,7\}, B = \{1,2,6\}\ and\ A = \{2,3,5\}$

$\bar{A} = A^c = \{1,4,6,7\}$

كل العناصر الموجودة في U وليست موجودة في A .

$B^c = \bar{B} = \{3,4,5,7\}$

كل العناصر الموجودة في U وليست موجودة في B

(2) Find $(\overline{A \cup B}) \cap$ A from the previous example?

* First we find A$\cup$**B =** $\{1,2,3,5,6\}$.

* Second we find $\overline{A \cup B} = \{4,7\}$

كل العناصر في U وليست في (B$\cup$A):

$\rightarrow (\overline{A \cup B}) \cap = \{\ \} = \phi$.

4. The difference of sets :

The difference of two sets A and B denoted by **A-B** or **A/B**. This set contains all the elements of A which do not belong to B:

$$\mathbf{A - B} = \{x : x \in A, and .x \notin B\} = A \cap \bar{B} .$$

* Note that the complement of A is given by,

$\overline{A}$ **= U – A**

Example :

Consider the sets, $A = \{2,3,5\}$ & $B = \{1,6,2\}$

$\rightarrow A - B = \{3,5\}$.

كل العناصر الموجودة في A ولا توجد في B.

$\rightarrow B - A = \{1,6\}$

كل العناصر الموجودة في B ولا توجد في A.

Example :

Let $U = \{1,2,3,\ldots,10\}$, $A = \{1,2,3,4,5,6\}$

And $B = \{3,6,9\}$

There fore ,

$\rightarrow A \cap B = \{3,6\}$.

$\rightarrow A \cup B = \{1,2,3,4,5,6,9\}$

$\rightarrow A - B = \{1,2,4,5\}$.

$\rightarrow B - A = \{9\}$

$\rightarrow \overline{A} = \{7,8,9,10\}$

$\rightarrow \overline{B} = \{1,2,4,5,7,8,10\}$

$\rightarrow \overline{A \cup B} = \{7,8,10\} = \overline{A} \cap \overline{B}$

$\rightarrow \overline{A \cap B} = \{1,2,4,5,7,8,9,10\} = \overline{A} \cup \overline{B}$.

$\rightarrow \overline{A - B} = \{3,6,7,8,9,10\} = \overline{A} \cup B$

Exercises : Let U = $\{n : n \in N \wedge n < 10\}$

A = $\{2,4,6,8\}$, B = $\{2,3,5,7\}$ & C = $\{1,4,9\}$.

Define (for example, by listing elements) each of the following:

1) A $\cap$ B 2) A $\cup$ B 3) A – B

4) B $\cap$ C 5) $\bar{A} \cap$ B 6) A $\cap$ (B $\cup$ C).

7) $\bar{B} \cup$ B 8) $\bar{B} \cap$ B 9) $\overline{A \cup C}$.

10) (A – C) – B .

* Where N is the natural no.'s, N = $\{0,1,2,....\}$.

2.4 Power Set: مجموعة القوى

Let A any set. The power set of A, denoted **P(A)** , is the set of all subsets of A :

P(A) = $\{B : B \subseteq A\}$

- لايجوز التكرار بالمجموعات والتبديل لا يؤثر .
- المجموعة الخالية تكون دائماً مجموعة جزئية من المجموعات.

Example :

Consider the set, A = $\{1,2,3\}$.

P(A) = $\{\{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}, \phi\}$

*** $2^3 = 8$**

Theorem: If $|A| = n$ then $|P(A)| = 2^n$

Example :

Consider the set, B= $\{a,b,c,d\}$, $2^4 = 16$

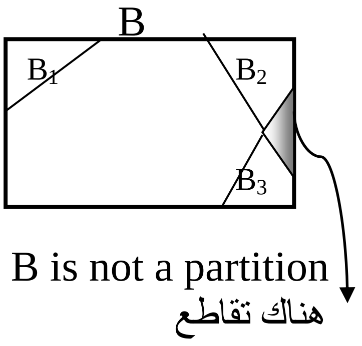
$$P(B) = \begin{Bmatrix} \{a\},\{b\},\{c\},\{d\},\{a,b\},\{a,c\},\{a,d\},\{c,d\},\{b,c\}, \\ \{b,d\},\{a,b,c\},\{a,b,d\},\{a,b,c,d\},\{b,c,d\},\{a,c,d\},\phi \end{Bmatrix}$$

2.5 Partitions of a Set : التجزئة

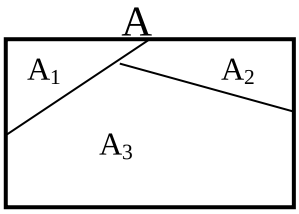
Let A be a set, there are **two** conditions, if it's true then it's a partition:

1. $A_1 \cup A_2 \cup A_3 \cup \ldots = A.$
2. $A_1 \cap A_2 \cap A_3 \cap \ldots = \phi.$

$\rightarrow$ اذا تحققا الشرطان فهي تمثل partition



B is not a partition



A is a partition

Example : consider the set, $A = \{1,2,3,4\}$

$P_1(A)= \{\{1\},\{2,3\}\{4\}\} \rightarrow$ is a partition.

$P_2(A)= \{\{1,2,3\},\{4\}\} \rightarrow$ is a partition .

$P_3(A)= \{\{1\}\{2,3\},\{3,4\}\} \rightarrow$ is Not a partition since,

$\{2,3\} \cap \{3,4\} \neq \phi$

2.6 The Cartesian product: الضرب الديكارتي

The Cartesian product , X×Y of two sets X and Y is the set of all ordered pairs (x,y) where x belongs to X and y belongs to Y :

$$X \times Y = \{(x,y) : x \in X \text{ , } and \ y \in Y\} .$$

*** Remark :**

When X = Y , it is usually denote X × X by X^2 .

This is read as " X two " and not " X squared " .

Theorem :

If $X_1, X_2, X_3, \ldots X_n$ are finite sets then ,

$$|X_1 \times X_2 \times X_3 \times \ldots \times X_n| = |X_1| \times |X_2| \times |X_3| \times \ldots \times |X_n| .$$

Example: Consider the two sets, A = $\{1,2,3\}$ and B = $\{a,b\}$

$|A| = 3$, $|B| = 2$ $\rightarrow$ $|A \times B| = 3 \times 2 = 6$.

A × B = $\{(1,a),(1,b),(2,a),(2,b),(3,a),(3,b)\}$.

Example: A = $\{1,2\}$, B = $\{a,b\}$, C = $\{k,m\}$.

$$A \times B \times C = \left\{ \begin{matrix} (1,a,k),(1,a,m),(1,b,k),(1,b,m),(2,a,k), \\ (2,a,m),(2,b,k),(2,b,m) \end{matrix} \right\}.$$

Exercises :

1. List the elements of P(A) for the following :

   A = $\{\{1\},\{1,2\}\}$.

2. Which of the following are partitions of the set

   $\{2,3,7,9,10\}$?

a) $\{\{2,3\},\{3,7,9\},\{10\}\}$.

b) $\{\{2,10\},\{3,7\},\{9\}\}$.

c) $\{\{2,3,4\},\{7,9,10\}\}$.

d) $\{\{2\},\{3\},\{7\},\{9\},\{10\}\}$.

e) $\{2,3,7,9,10\}$.

f) $\{\{2,3,7,9,10\}\}$.

g) $\{\{10,3\},\{7,2\}\}$.

h) $\{\{2,9,10\},\{3,7\},\phi\}$.

3. Let A = $\{1,2,3,4\}$, B = $\{3,4,5\}$, X = $\{a,b\}$, Y = $\{b,c,d\}$.

List the elements of each of the following sets :

a) $(A \cap B) \times (X \cap Y)$.

b) $(A \times X) \cap (B \times Y)$.

c) $(A \cap X) \times Y$.

Chapter Three

" Relations"

3.1 Relations :

Let A and B be sets . A relation from A to B (or between A and B) is a subset of the Cartesian product **A×B**.

Remark : The elements of the relation is an ordered pairs .

* (أي أن عناصر العلاقة عبارة عن أزواج مرتبة) .

* We shall use a R b to denote " a is related to b ". And a $\not R$ b to denote (a,b) ∉ R or " a is not related to b"

Example (1) :

A = $\{1,2,3\}$, B = $\{1,2,3\}$

A × B = $\{(1,1),(1,2),(1,3),(2,1),(2,2),(2,3),(3,1),(3,2),(3,3)\}$.

1) Find the elements of R_1 iff a = b

→ $R_1 = \{(1,1),(2,2),(3,3)\} \subseteq A \times B$

2) Find the elements of R_2 iff a < b

→ $R_2 = \{(1,2),(2,3),(2,3)\}$

3) Find the elements of R_3 iff $a \geq b$

$\rightarrow R_3 = \{(1,1),(2,2),(3,3),(3,1),(3,2),(2,1)\}$.

Example (2) :

Let A = B = $\{1,2,3,4,5,6\}$ and

R = $\{(a,b): a \text{ divides } b\}$,

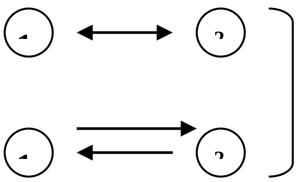
$$R = \left\{\begin{matrix}(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(2,2),(2,4), \\ (2,6),(3,3),(3,6),(4,4),(5,5),(6,6)\end{matrix}\right\}$$

3.2 Relations and Digraphs :

To represent relations using graphs, there are two basic ways to do it,

1) Digraph (directed graph) : (المخطط المتجه)

If two elements ***a*** and ***b*** are such that a ***R*** b and b ***R*** a, we will usually connect their points in the digraph by a single bi-directional arrow rather than two directed arrows .



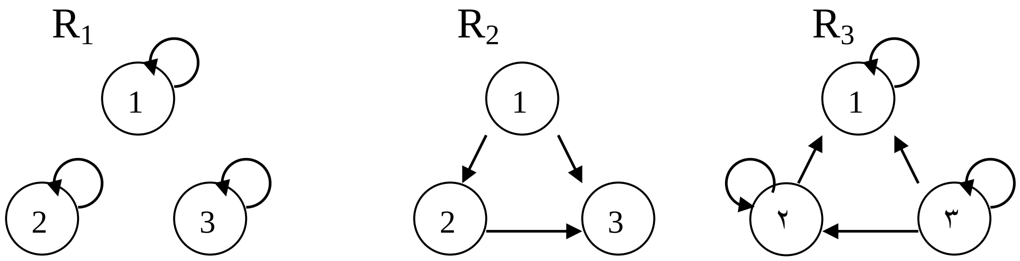
نفس المعنى

* Now , get back to **Example(1)** and let's represent it in digraph :

* هناك خطوتين :

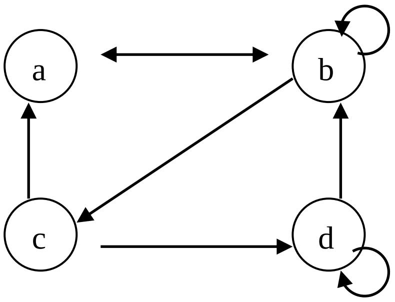
١. ضع عناصر المجموعة داخل دوائر .

٢. أرسم عنصر ينتمي للعلاقة بسهم يبدأ من العنصر الأول وينتهي بالعنصر الثاني .



Example : (3)

From the following digraph, list the elements of R,



Sol :

A = $\{a,b,c,d\}$

R = $\{(b,b),(d,d),(a,b),(b,a),(d,b),(c,d),(b,c),(c,a)\}$.

Example (4) :

Let A = $\{1,2,3\}$, B = $\{1,2,4\}$.

Find: R_1 , R_2 , R_3 , R_4 and R_5 and represent them in digraphs :

$A \times B = \{(1,1),(1,2),(1,4),(2,1),(2,2),(2,4),(3,1),(3,2),(3,4)\}.$

1. Find R_1 iff $a = b$

$\rightarrow R_1 = \{(1,1),(2,2)\} \subseteq A \times B$

2. Find R_2 iff $a \leq b$

$\rightarrow R_2 = \{(1,1),(1,2),(1,4),(2,2),(2,4),(3,4)\}.$

3. Find R_3 iff $a > b$

$\rightarrow R_3 = \{(2,1),(3,1),(3,2)\}$

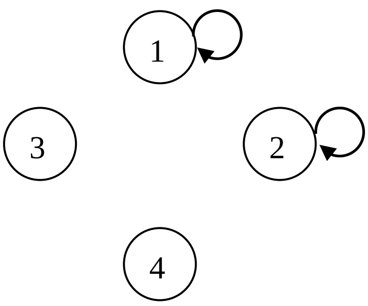
4. Find R_4 iff $a=2b$ (الأول = ضعف الثاني)

$\rightarrow R_4 = \{(1,2),(2,4)\}.$

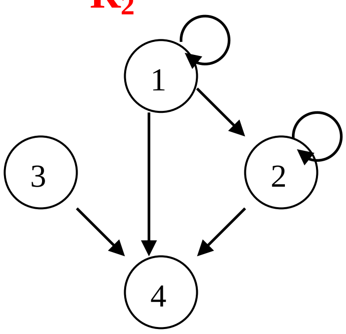
5. Find R_5 iff $a= 4b$ (الأول = أربع أضعاف الثاني)

$\rightarrow R_5 = \{(1,4)\}.$

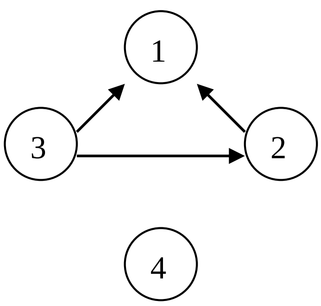
R_1



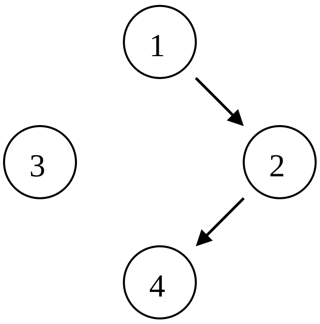
R_2



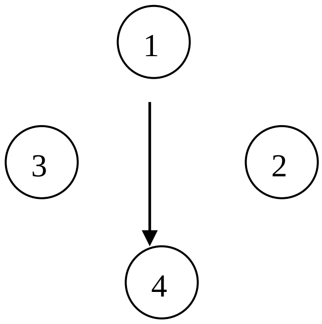
R_3



R_4



R_5



2. Binary Matrix : (المصفوفة الثنائية)

Let A = $\{a_1, a_2, \ldots, a_n\}$ and B = $\{b_1, b_2, \ldots, b_n\}$ be finite sets and let **R** be a relation from A to B.

The binary matrix of R is a rectangular array of (zeros and ones) with ***n*** rows and ***m*** columns. The rows correspond to the elements of A and columns correspond to the elements of B. At the intersection of **i^{th}** row and **j^{th}** column we place (**1**) if $a_i R b_j$ or (**0**) if $a_i \not R b_j$.

i.e,

$$(a,b) \in R \Rightarrow 1$$

$$(a,b) \notin R \Rightarrow 0$$

Example :

Back to **example(4)** construct the relations by using binary matrix (Boolean Matrix) ,

Sol :

$$A = \{1,2,3\} \quad , B = \{1,2,4\}$$

R_1

$$\begin{array}{c|c} & \begin{array}{ccc} b_1=1 & b_2=2 & b_3=4 \end{array} \\ \begin{array}{c} a_1=1 \\ a_2=2 \\ a_3=3 \end{array} & \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \end{array}$$

R_2

$$\begin{array}{cc} & \begin{array}{ccc} 1 & 2 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \end{array}$$

R_3

$$\begin{array}{cc} & \begin{array}{ccc} 1 & 2 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \end{pmatrix} \end{array}$$

R_2

$$\begin{array}{cc} & \begin{array}{ccc} 1 & 2 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \end{array}$$

R_5

$$\begin{array}{cc} & \begin{array}{ccc} 1 & 2 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \end{array}$$

3.3 Properties of Relations :

Let R be a relation on set A. We say that R is:

1. Reflexive: الانعكاس

A relation is said to be reflexive if and only if ***a*** R ***a*** for every a ∈ A.

Example : $A = \{1,2,3\}$

$R_1 = \{(1,1),(1,2),(1,3),(2,2),(2,3),(3,3,)\}$

R_1 is reflexive أي كل عنصر يرتبط مع نفسه

$R_2 = \{(1,1),(2,3),(2,2),(3,1)\}$

$3 \in A$ but $(3,3) \notin R_2$

$\therefore$ R_2 is not reflexive.

2. Symmetric : متناظرة

A relation is said to be symmetric if and only if ***a*** R ***b*** implies ***b*** R ***a*** for every a,b $\in$ A ;

Example : $A = \{a,b,c\}$

$R_1 = \{(a,a),(a,b),(b,a),(c,c)\}$

R_1 is symmetric

*أي أن كل عنصر موجود في R_1 يجب أن يكون عكسه موجود أيضاً .

Example :

$A = \{1,2,4\}$

$R_1 = \{(1,1),(2,4),(4,2),(1,2),(2,2),(4,4)\}$

$\because$ $(1,2) \in R$ but $(2,1) \notin R$

$\therefore$ R is not symm.

3. Transitive : متعدية

A relation is said to be transitive if and only if ***a*** R ***b*** and ***b*** R ***c*** implies ***a*** R ***c*** for every a,b,c $\in$ A .

Example : $A = \{1,2,3\}$

$R_1 = \{(1,1),(1,2),(2,3),(1,3)\}$

R_1 is transitive

$R_2 = \{(1,1),(1,2),(2,3),(3,2),(2,1),(2,2),(3,3)\}$

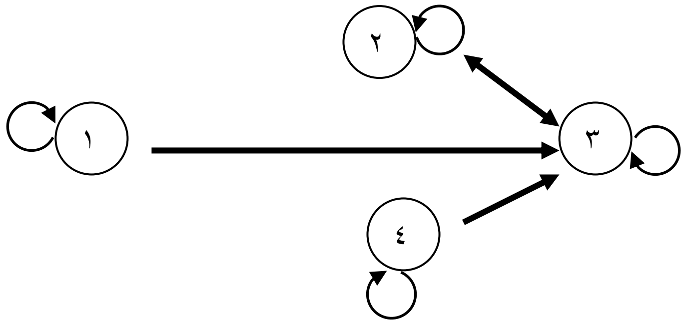
$\because (\overset{a}{1},\overset{b}{2}) \wedge (\overset{b}{2},\overset{c}{3}) \in R_2, but (\overset{a}{1},\overset{c}{3}) \notin R_2$

$\therefore$ R_2 is not transitive.

Exercises :

1. Represent the relation "less than (<)" on $A = \{1,2,3,4,5\}$ by using Boolean Matrix.

2. From the following digraph , list the elements of R,



3. Find the digraph representation and the relation of R corresponding to the following Boolean Matrix,

$$\mu = \begin{pmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \end{pmatrix}$$

4. Let A = $\{a, b, c, d, e\}$ for each of the following relations R on A, determine which of the three properties (reflexive, symm., trans.) are satisfied by the relation . Justify your answers :

a) R = $\{(a,a),(a,b),(a,c),(b,a),(b,b),(b,c),(c,a),(c,b),(c,c)\}$

b) R = $\{(a,a),(b,b),(c,c),(d,d),(e,e),(a,b),(b,c)\}$.

c) R = $\{(a,a),(a,d),(b,b),(c,c),(d,d),(d,e),(e,a),(e,e)\}$.

d) R= $\{(a,b),(b,c),(c,d),(d,e),(e,a)\}$.

e) R = $\{(a,b),(b,a),(b,d),(d,a),(c,e),(e,c),(e,e)\}$.

5. Let R = $\{(a,a),(a,b),(a,c),(b,b),(b,c)\}$. Be a relation on the set $\{a,b,c,d\}$. What is the minimum number of elements which need to be added to R in order that it becomes :

(1) Reflexive; (2) symmetric.

(3) Transitive ?

3.4 Composition of Relations : -

Let **R** be the relation between the set A and the set B and let **S** be a relation between B and C . We can then define a new relation, the **composition** of R and S, written RoS. The relation RoS is a relation from A to C, and is defined as follows:

$$\text{SoR} = \{(x,z) \in A \times C, \exists y \in B; (x,y) \in R \Lambda (y,z) \in S\}.$$

Example (1):- Consider the relations R and S defined as follows: -

R = $\{(1,1),(1,3),(2,1),(3,4)\}$.

S = $\{(1,1),(3,1),(3,2)\}$.

Then,

SoR = $\{(1,1),(1,2),(2,1)\}$.

Example (2):- Let R $=\{(x,y)\in N\times N; x+y=12\}$.

$$S=\{(x,y)\in N\times N; 2x+y=25\}.$$

Find SoR and RoS as an order pairs?

Solution :

$$R=\left\{\begin{matrix}(1,11),(2,10),(3,9),(4,8),(5,7),(6,6),(7,5),(8,4),(9,3),\\(10,2),(11,1)\end{matrix}\right\}.$$

$$S=\left\{\begin{matrix}(1,23),(2,21),(3,19),(4,17),(5,15),(6,13),(7,11),\\(8,4),(9,7)(10,5),(11,3),(12,1)\end{matrix}\right\}.$$

SoR $=\left\{\begin{matrix}(1,3),(2,5),(3,7),(4,9),(5,11),(6,13),(7,15),(8,17),\\(9,19),(10,21),(11,23)\end{matrix}\right\}.$

RoS $=\{(7,1),(8,3),(9,5),(10,7),(11,9),(12,11)\}$

3.5 Composition by logical matrix product :-

the matrix representation of the composition of two relations is equal to the matrix product of the matrix representations of these relation. Where addition corresponds to logical OR and multiplication to logical AND.

Example :

Let A = $\{a,b,c\}$, and let R and S be relations on A whose matrices are:

$$M_R = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad , \qquad M_S = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix}$$

And the logical matrix product gives:

$$M_{SoR} = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}$$

Chapter Four

Matrices

*Linear Equations : -

Let a_1 , a_2 , …, a_n , b $\in$ R and X_1 , X_2 , …,X_n are variables (unknowns) then the from :

$$a_1 x_1 + a_2x_2 + \ldots + a_nx_n = b_n .$$

Called linear equation, a_1,a_2,..a_n are coefficients and b is the absolute value.

And the form :

$$\begin{aligned} a_{11} x_1 + a_{12}x_2 + \ldots + a_{1n}x_n &= b_1 \\ a_{21} x_1 + a2_2x_2 + \ldots + a_{2n}x_n &= b_2 . \\ \vdots \qquad \vdots \qquad \ddots \qquad &\vdots \quad \vdots \\ a_{n1} x_1 + a_{n2}x_2 + \ldots + a_{nn}x_n &= b_n . \end{aligned}$$

Called system of linear Equations

4.1 Matrices :

The matrix is a rectangular arrangement form consists of orthogonal rows and columns. The coefficients of the linear system are elements of the matrix,

$$A=\begin{pmatrix} a_{11} & a_{12} & a_{13} \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} \cdots & a_{1n} \\ \vdots & \vdots & \vdots \quad \ddots & \vdots \\ a_{n1} & a_{n2} & a_{n3} \cdots & a_{nn} \end{pmatrix}$$

This matrix is called the coefficients matrix and it's usually denoted by capital letters. And it can be represented by :

$A = [a_{ij}]$, $1 \leq i \leq n$, $1 \leq j \leq n$.

Examples: -

1. $2x_1 + 3x_2 + 8x_3 = 6$

   $x_1 + x_2 - 3x_3 = 2.$

It's a system consists of 2 equations and 3 unknowns.

Can be represented in a matrix as :

$$\left(\begin{array}{ccc|c} 2 & 3 & 8 & 6 \\ 1 & 1 & -3 & 2 \end{array}\right)$$

2. The matrix $B=\left(\begin{array}{cc|c} 2 & 5 & 1 \\ 3 & 6 & 2 \\ 1 & 2 & 0 \end{array}\right)$, represent the following

System :

$2x_1 + 5x_2 = 1$
$3x_1 + 6x_2 = 2$
$x_1 + 2x_2 = 0$

Note : This matrix is of degree 3×2. In general : the matrix is of degree m×n, where m = is the no. of equations and n= is the no. of variables , in the linear system.

4.2 Types of Matrices :

1) When $m = n$, then the matrix called Square Matrix.
2) If $a_{ij} = 0$, $\forall$ i , j , then the matrix called Zero Matrix, and it's denoted by O.
3) The square matrix is called the Identity Matrix , If :

$$a_{ij} = \begin{cases} 1 & \text{for } i = j \\ 0 & \text{elsewhere} \end{cases}$$

and it's denoted by **I**.

Example :

1. $\mathbf{O} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}_{2\times3}$, is a zero matrix.

2. $\mathbf{I} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}_{3\times3}$, is an identity matrix.

4) A^T is the transpose of the matrix A, when the rows of A are the columns of A^T, and so as for the columns of A.

Example:-

$$A = \begin{pmatrix} 1 & 2 & 0 \\ 5 & -1 & 2 \end{pmatrix}_{2\times 3} \Longrightarrow A^T = \begin{pmatrix} 1 & 5 \\ 2 & -1 \\ 0 & 2 \end{pmatrix}_{٣\times ٢}$$

5) If $A = A^T$, then A is called Symmetric Matrix.

6) The square matrix is called lower triangle matrix if $a_{ij} = 0$, $\forall$ i< j and called upper triangle matrix if $a_{ij} = 0$, $\forall$ i> j .

Example:

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 5 & 0 \\ 1 & 3 & 2 \end{pmatrix}_{3\times 3} \qquad B = \begin{pmatrix} 1 & 6 & 3 \\ 0 & 5 & 1 \\ 0 & 0 & 2 \end{pmatrix}_{3\times 3}$$

7) The matrix is called Diagonal Matrix if

$$a_{ij} = \begin{cases} 0 \quad , \quad \text{if} \quad \forall\ i \neq j \\ \text{not all zero , if } i = j \end{cases}$$

4.3 Operations on Matrices : -

1. (Equality): The two matrices $A_{m\times n}=B_{m\times n}$ if $a_{ij}=b_{ij}$, $\forall$ i,j

2. Multiplied by scalar :

If k is scalar and $A_{m\times n} = [a_{ij}]_{m\times n}$, then $kA = [ka_{ij}]_{m\times n}$

Example:-

$$3* \begin{pmatrix} 1 & 2 \\ 0 & -3 \end{pmatrix}_{2\times 2} = \begin{pmatrix} 3 & 6 \\ 0 & -9 \end{pmatrix}_{2\times 2}$$

Note : The division by scalar is like multiplying by $\frac{1}{k}$.

3. Addition of Matrices :

Let $A = [\, a_{ij} \,]_{m\times n}$ and $B = [\, b_{ij} \,]_{m\times n}$ be two matrices , then

$A + B = [\, a_{ij} \,]_{m\times n} + [\, b_{ij} \,]_{m\times n} = C = [\, a_{ij} + b_{ij}]_{m\times n}$

Example:-

$$A = \begin{pmatrix} 2 & -1 & 3 \\ 0 & 2 & -3 \end{pmatrix}_{2\times 3} \qquad B = \begin{pmatrix} 1 & -1 & 0 \\ 1 & 0 & 2 \end{pmatrix}_{2\times 3}$$

then,

$$A+B = \begin{pmatrix} 3 & -2 & 3 \\ 1 & 2 & -1 \end{pmatrix}_{2\times 3}$$

4. Multiplication of Matrices:

If **A** of degree $m \times k$ then **B** must be of degree $k \times n$ to be able of multiplying A by B.

The result matrix **C** is of degree $m \times n$.

$$A \times B = [\boldsymbol{a_{ij}}]_{m\times n} \times [\boldsymbol{b_{ij}}]_{m\times n}$$

$$= \left[\sum_{t=1}^{k} (a_{it} \times b_{tj})\right]_{m\times n} = [\boldsymbol{c_{ij}}]_{m\times n} = C_{m\times n}$$

Example:-

$$A = \begin{pmatrix} 2 & 1 & 3 \\ 1 & 2 & 1 \end{pmatrix}_{2\times 3} \qquad B = \begin{pmatrix} 3 & 0 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix}_{3\times 3}$$

$\longrightarrow$ $\mathbf{A_{2\times 3} \times B_{3\times 3}}$

$$\mathbf{C_{2\times 3}} = \begin{pmatrix} 2*3+\ 1*1+\ 3*1 & 2*0+\ 1*2+\ 3*1 & 2*1=\ 1*1+\ 3*2 \\ 1*3+\ 2*1+\ 1*1 & 1*0+\ 2*2+\ 1*1 & 1*1+\ 2*1+\ 1*2 \end{pmatrix}$$

$$= \begin{pmatrix} 10 & 5 & 9 \\ 6 & 5 & 5 \end{pmatrix}_{2\times 3}$$

Note :

1. If A and B are square matrices from $\boldsymbol{n \times n}$, then C is from the same degree, i.e, $A_{n\times n} \times B_{n\times n} = C_{n\times n}$.
2. $A_{n\times n} \times I_{n\times n} = I_{n\times n} \times A_{n\times n} = A_{n\times n}$

5. Power of matrices :

If $A_{n\times n}$ is a square matrix, then $A^2_{n\times n} = A_{n\times n} \times A_{n\times n}$, In general:

$$A^m_{n\times n} = \underbrace{A_{n\times n} \times A_{n\times n} \times \ldots \times A_{n\times n}}_{\text{m-times}}$$

Example:-

$$A_{2\times2} = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} \longrightarrow A^2_{2\times2} = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}_{2\times2}$$

H.W:

1) Find [$kA_{2\times3}$], $A = \begin{pmatrix} 2 & 3 & 4 \\ 5 & 1 & 0 \end{pmatrix}$

Where, $k = \frac{1}{2}$.

2) If $A = \begin{pmatrix} 1 & 0 & 2 \\ 4 & 3 & 1 \\ 5 & 0 & -1 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 0 \\ 2 & 2 \\ 4 & 5 \end{pmatrix}$

Find $A \times B = ?$

3) If $A = \begin{bmatrix} 2 & 1 & 3 \\ -1 & 0 & 2 \\ 1 & 2 & 1 \end{bmatrix}$, B= $\begin{bmatrix} 3 & 0 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$

Fine A – B= ?

4) Given the square matrices ,

Where, A= $\begin{bmatrix} 1 & 3 \\ -4 & 2 \end{bmatrix}$ and B= $\begin{bmatrix} 3 & 0 \\ 1 & -2 \end{bmatrix}$

Verify by direct computation that $AB \neq BA$.